1✓

Chapter 9 : PLANE E. M. WAVES...
... AND PROPAGATION IN MATTER

★ Waves in matter ; reflection and refraction ; part 1

★ Sections 9.2 and 9.3

Reflection and Refraction from a Dielectric Interface

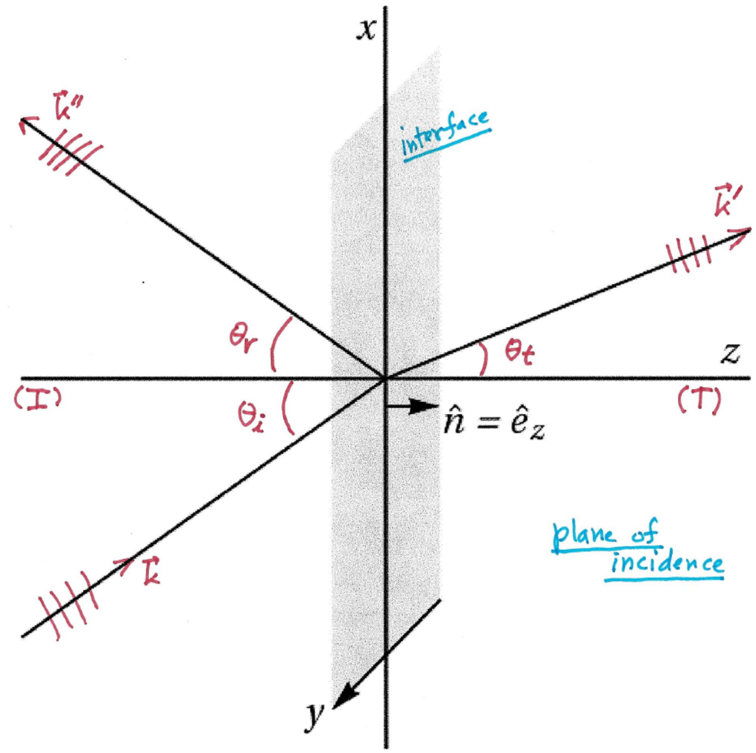
In either dielectric we have plane waves with frequency ω. Recall the properties of plane waves (Sec. 9.1.) *Now all we need to do is satisfy the four boundary conditions.*

WT use a different and more difficult method with Green's functions. (See page 444.)

■1

In[•]:= W1F1

Out[•]=



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Be careful when referring to Wilcox and Thron, because they use a bad notation.
In Sections 9.2 ~ 9.4 there are two regions.

Region I = the region of the incident and reflected waves; WT calls this "region 2".

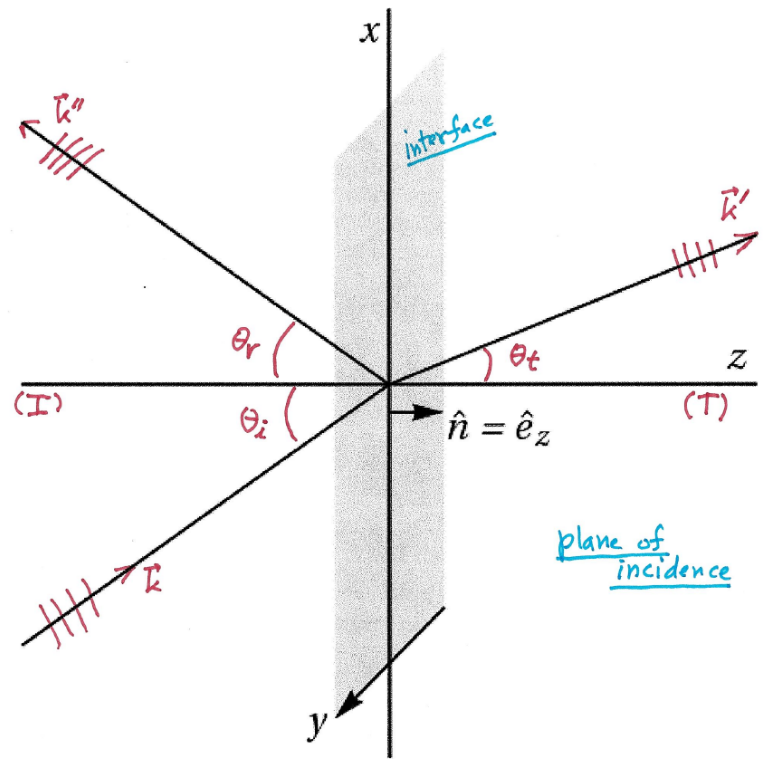
Region T = the region of the transmitted wave; WT calls this "region 1".

In the lecture, I'll use ϵ_I and ϵ_T.
To compare to WT,
set $\epsilon_I = \epsilon_2$ and $\epsilon_T = \epsilon_1$.
To compare to Jackson,
set $\epsilon_I = \epsilon_1$ and $\epsilon_T = \epsilon_2$.

■2

In[∘]:= `W1F1`

Out[∘]=



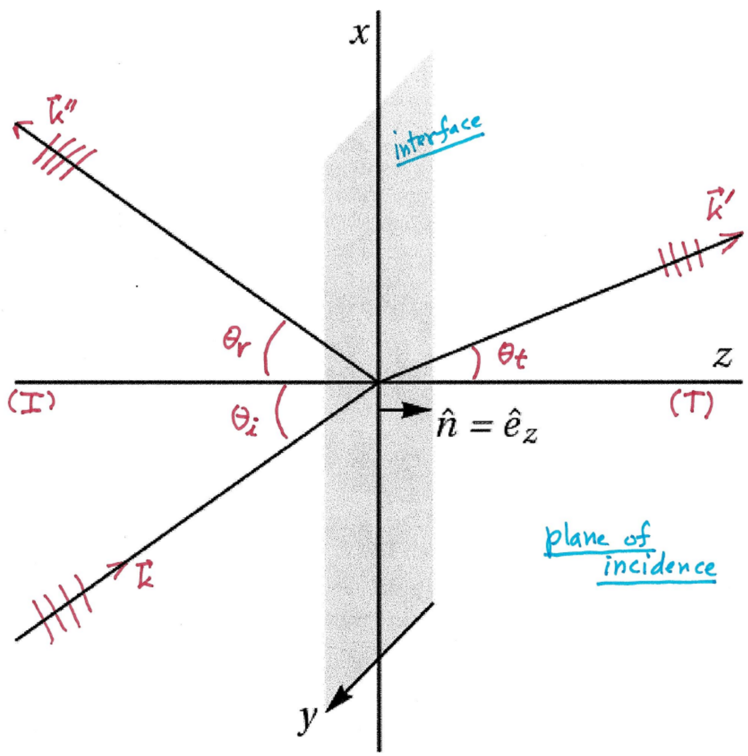
Interface = xy plane■
Plane of incidence = xz plane■

3 ✓

Section 9.2: TE polarization

In[ ]:= W1F1

Out[ ]=



The TE waves are polarized with the electric fields transverse to the plane of incidence.

3

In[ ]:= scTE1

Out[ ]=

$$\vec{E}(\vec{x},t) = E_0\, \hat{e}_y\, e^{i(\vec{k}\cdot\vec{x} - \omega t)}$$
$$\vec{k} = (k_x, 0, k_z)$$
$$\vec{E}'(\vec{x},t) = E_0'\, \hat{e}_y\, e^{i(\vec{k}'\cdot\vec{x} - \omega t)}$$
$$\vec{k}' = (k_x, 0, k_z')$$
$$\vec{E}''(\vec{x},t) = E_0''\, \hat{e}_y\, e^{i(\vec{k}''\cdot\vec{x} - \omega t)}$$
$$\vec{k}'' = (k_x, 0, k_z'')$$

Magnetic fields.

By Faraday's Law, $\vec{k} \times \vec{E} = (\omega/c)\, \vec{B}$;

also, $\omega/k = v_{phase} = c/n$;

$\therefore\ \vec{B} = n\, \hat{k} \times \vec{E}$

$= n\, \{ -\cos\theta_i\, E_y\, ,\, 0\, ,\, \sin\theta_i\, E_y \}$

4.

Boundary conditions at the interface

(z=0)

- $E_{\text{tang.}}$: $E_0 + E_0'' = E_0'$
- D_{normal} : 0 + 0 = 0
- B_{normal} (*z comp.*) :

$n_I \sin\theta_i\,(E_0 + E_0'') = n_T \sin\theta_t\, E_0'$

i.e., $E_0 + E_0'' = E_0'$

- $H_{\text{tang.}}$ (*x comp.*) :

$(1/\mu_I)\ n_I \cos\theta_i\,(E_0 - E_0'')$

$= (1/\mu_T)\, n_T \cos\theta_t\, E_0'$

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scTE2; scTE3; scTE4; scTE5

Out[ ]=

Two equations for two unknowns
(E_0' and E_0'')

$$E_0 + E_0'' = E_0'$$

$$\left(\frac{n}{\mu}\right)_I (E_0 - E_0'')\cos\theta_i = \left(\frac{n}{\mu}\right)_T E_0' \cos\theta_t$$

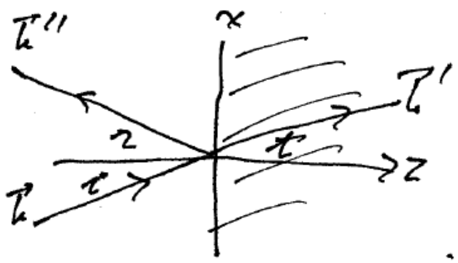
a bit of algebra →

$$\frac{E_0'}{E_0} = \frac{2\,(n/\mu)_I \cos\theta_i}{(n/\mu)_I \cos\theta_i + \left(\frac{n}{\mu}\right)_T \cos\theta_t}$$

$$\frac{E_0''}{E_0} = \frac{(n/\mu)_I \cos\theta_i - (n/\mu)_T \cos\theta_t}{(n/\mu)_I \cos\theta_i + (n/\mu)_T \cos\theta_t}$$

Section 9.3: TM polarization

In[▫]:= scTM1



Out[▫]=

TM polarization. Start with

$$\vec H = H_0 \hat e_y e^{i(\vec k\cdot\vec x - \omega t)}$$

$$\vec k = (k_x \hat e_x + k_z \hat e_z)$$

$$\vec H' = H_0' \hat e_y e^{i(\vec k'\cdot\vec x - \omega t)}$$

$$\vec k' = k_x \hat e_x + k_z' \hat e_z$$

$$\vec H'' = H_0'' \hat e_y e^{i(\vec k''\cdot\vec x - \omega t)}$$

$$\vec k'' = k_x \hat e_x + k_z'' \hat e_z$$

The boundary conditions for $H_{tang.}$ and B_{normal} ⇒

$$H_0 + H_0'' = H_0' \qquad (1)$$

$$0 + 0 = 0 \quad \text{(automatic)}$$

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In[▫]:= scTM2

B and E

$\nabla\times\vec H = \frac{1}{c}\frac{\partial \vec D}{\partial t}$ so $\vec k\times\vec H = -\frac{\omega}{c}\vec D$

$$\vec D = -\frac{1}{\omega}\, c\vec k\times\vec H$$

$$= -\frac{c}{\omega}(k_x \hat e_x + k_z \hat e_z)\times H_y \hat e_y$$

$$= -\frac{c}{\omega}H_y(-k_z\hat e_x + k_x \hat e_z)$$

$$= \frac{c}{\omega}H_y(k_z\hat e_x - k_x\hat e_z)$$

Out[▫]=

D_{normal} is continuous at $z=0$, so

$$\frac{c}{\omega}k_x(H_0 + H_0'') = \frac{c}{\omega}k_x H_0'$$

$H_0 + H_0'' = H_0'$ (same as before).

$E_{tang.}$ is continuous, so

$$\frac{c}{\omega}k_z(H_0 - H_0'') = \frac{c}{\omega}k_z' H_0'$$

Note $\frac{ck_z}{\omega} = \frac{ck}{\omega}\cos\theta_i = n_I\cos\theta_i$

and $ck_z'/\omega = n_T\cos\theta_t$

$$\therefore \left(\frac{n}{\epsilon}\right)_I \cos\theta_i (H_0 - H_0'') = \left(\frac{n}{\epsilon}\right)_T \cos\theta_t H_0' \qquad (2)$$

In[•]:= scTM3

Solve (1) and (2)

Note: $\frac{n}{\epsilon} = \frac{\mu}{n}$

$$\frac{H_0''}{H_0} = \frac{\left(\frac{\mu}{n}\right)_I \cos\theta_i - \left(\frac{\mu}{n}\right)_T \cos\theta_t}{\left(\frac{\mu}{n}\right)_I \cos\theta_i + \left(\frac{\mu}{n}\right)_T \cos\theta_t}$$

Out[•]=

$$\frac{H_0'}{H_0} = \frac{2\left(\mu/n\right)_I \cos\theta_i}{\left(\frac{\mu}{n}\right)_I \cos\theta_i + \left(\frac{\mu}{n}\right)_T \cos\theta_t}$$

or

1. $$\frac{H_0''}{H_0} = \frac{\left(\frac{n}{\mu}\right)_T \cos\theta_i - \left(\frac{n}{\mu}\right)_I \cos\theta_t}{\left(\frac{n}{\mu}\right)_T \cos\theta_i + \left(\frac{n}{\mu}\right)_I \cos\theta_t}$$

$$\frac{H_0'}{H_0} = \frac{2\left(n/\mu\right)_T \cos\theta_i}{\left(n/\mu\right)_T \cos\theta_i + \left(\frac{n}{\mu}\right)_I \cos\theta_t}$$

■6

Fresnel's Equations

For simplicity, assume $\mu_I = \mu_T = 1$.
(This is a very reasonable assumption, physically. Why?)

TE waves have (9.79)

$E^{\text{trans}} / E^{\text{inc}} =$

$2\, n_I \cos\theta_i \,/\, (n_I \cos\theta_i + n_T \cos\theta_t)$

$E^{\text{refl}} / E^{\text{inc}} =$

$(n_I \cos\theta_i - n_T \cos\theta_t) / (n_I \cos\theta_i + n_T \cos\theta_t)$

TM waves have (9.102)

$B^{\text{trans}} / B^{\text{inc}} =$

$2\, n_T \cos\theta_i \,/\, (n_T \cos\theta_i + n_I \cos\theta_t)$

$B^{\text{refl}} / B^{\text{inc}} =$

$(n_T \cos\theta_i - n_I \cos\theta_t)/(n_T \cos\theta_i + n_I \cos\theta_t)$

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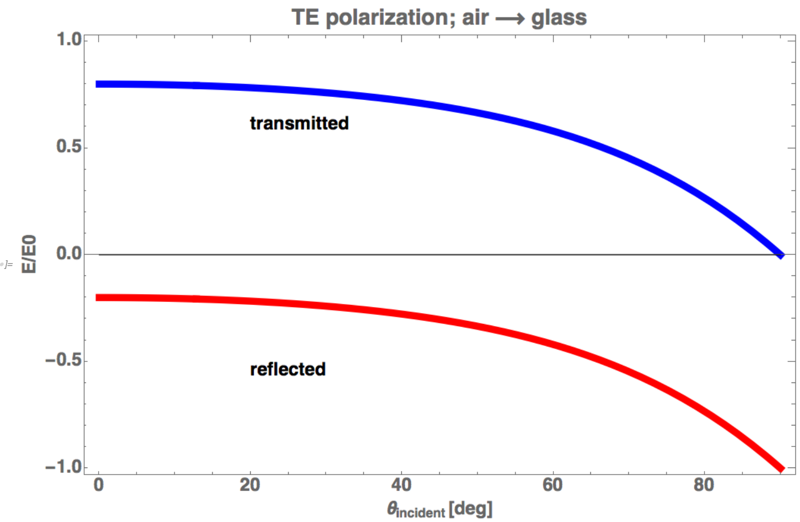
Now plot graphs of some cases

```
bs = {FontFamily → "Helvetica",
    FontWeight → Bold, FontSize → 24};
(* TE case; I = vacuum; T = glass *)
nI = 1; nT = 1.5;
β[α_] = ArcSin[(nI / nT) * Sin[α]]; (* Snell's law *)
trans[α_] =
   (2 * nI * Cos[α]) / (nI * Cos[α] + nT * Cos[β[α]]);
refl[α_] = (nI * Cos[α] - nT * Cos[β[α]]) /
    (nI * Cos[α] + nT * Cos[β[α]]);
epi = {Line[{{0, 0}, {90, 0}}],
    Text["transmitted", {20, 0.65}, {-1, 1}],
    Text["reflected", {20, -0.5}, {-1, 1}]};
plotTE =
 Plot[{trans[θi * Pi / 180], refl[θi * Pi / 180]},
    {θi, 0, 90},
    PlotRange → {{-2, 92}, {-1.03, 1.03}},
    PlotStyle → {{Blue, Thickness[0.01]},
      {Red, Thickness[0.01]}},
    Frame → True, FrameLabel →
     {"θincident [deg]", "E/E0"},
    BaseStyle → bs, ImageSize → 2^8 * 4,
    PlotLabel → "TE polarization; air ⟶ glass",
    Epilog → epi] // Rasterize
```

When light is incident from air into glass, the reflected ***electric field wave*** undergoes a 180^o phase change. (What about the ***magnetic field wave***?) When light is incident from glass into air, the reflected ***electric field wave*** has no phase change. (Homework problem 3-7.) *(Recall Newton's Rings; the central spot is dark because of destructive interference.)*

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In[•]:=
```
(* TM case; I = vacuum; T = glass *)
nI = 1; nT = 1.5;
β[α_] = ArcSin[(nI / nT) * Sin[α]]; (* Snell's law *)
trans[α_] =
  (2 * nT * Cos[α]) / (nT * Cos[α] + nI * Cos[β[α]]);
refl[α_] = (nT * Cos[α] - nI * Cos[β[α]]) /
   (nT * Cos[α] + nI * Cos[β[α]]);
epi = {Line[{{0, 0}, {90, 0}}],
   Text["transmitted", {20, 1.0}, {-1, 1}],
   Text["reflected", {20, 0.}, {-1, 1}]};
plotTM =
 Plot[{trans[θi * Pi / 180], refl[θi * Pi / 180]},
   {θi, 0, 90},
   PlotRange → {{-2, 92}, {-1.03, 1.53}},
   PlotStyle → {{Blue, Thickness[0.01]},
     {Red, Thickness[0.01]}},
   Frame → True, FrameLabel →
    {"θincident [deg]", "B/B0"},
   BaseStyle → bs, ImageSize → 2^8 * 4,
   PlotLabel → "TM polarization ; air ⟶ glass",
   Epilog → epi] // Rasterize
```


Out[•]=