1✓

Chapter 9 : PLANE E. M. WAVES...
... AND PROPAGATION IN MATTER

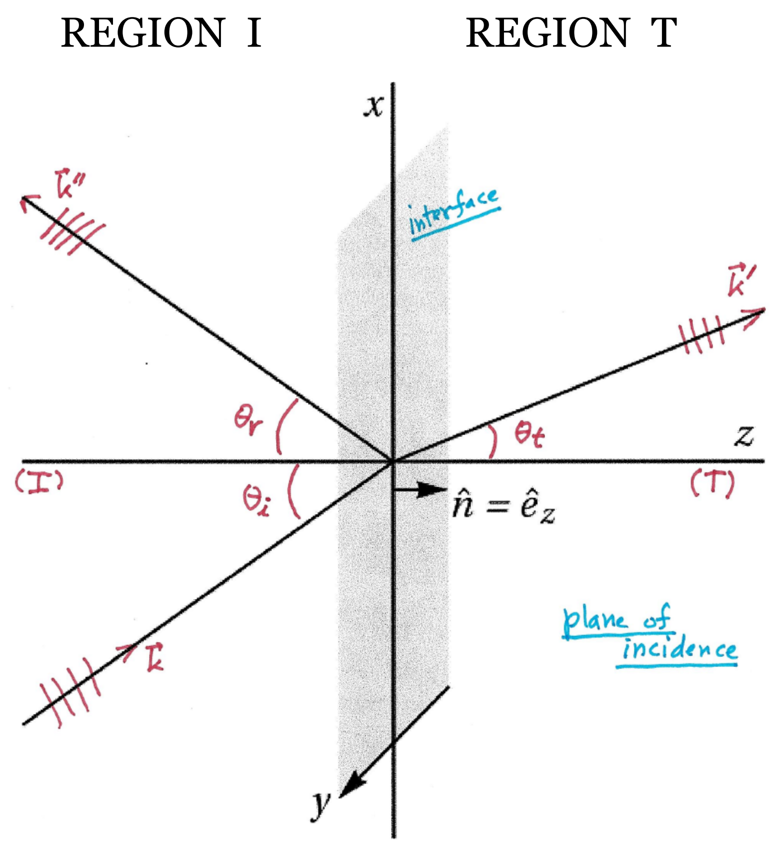
★ Waves in matter ; reflection and refraction, part 2

★ Section 9.4
Brewster's Angle
and Total Internal Reflection

■1

In[◦]:= W1F1

Out[◦]=



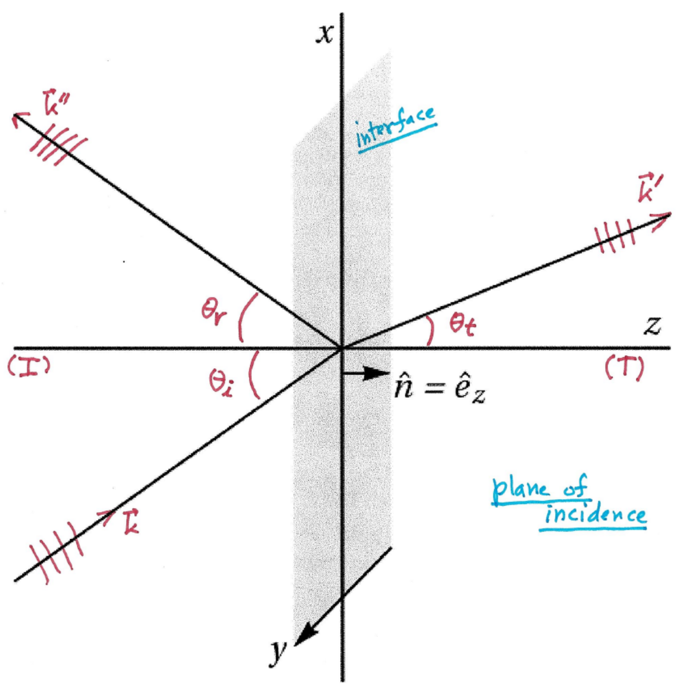
2✓

Reflection and Refraction from a Dielectric Interface

In[•]:= W1F1

—I— —T—

Out[•]=



In either dielectric we have plane waves with

▪2

Fresnel's equations

TE waves have

$$E^{\text{trans}} / E^{\text{inc}} =$$
$$2\, n_T \cos\theta_i \,/\, (n_I \cos\theta_t + n_I \cos\theta_i)$$

$$E^{\text{refl}} / E^{\text{inc}} =$$
$$(n_I \cos\theta_i - n_T \cos\theta_t) / (n_T \cos\theta_t + n_I \cos\theta_i)$$

TM waves have

$$B^{\text{trans}} / B^{\text{inc}} =$$
$$2\, n_T \cos\theta_i \,/\, (n_T \cos\theta_i + n_T \cos\theta_t)$$

$$B^{\text{refl}} / B^{\text{inc}} =$$
$$(n_T \cos\theta_i - n_I \cos\theta_t) / (n_T \cos\theta_i + n_I \cos\theta_t)$$

We are assuming that $\mu_I = \mu_T = 1$.
Then n = $\sqrt{\epsilon}$ in each material.

3 ✓

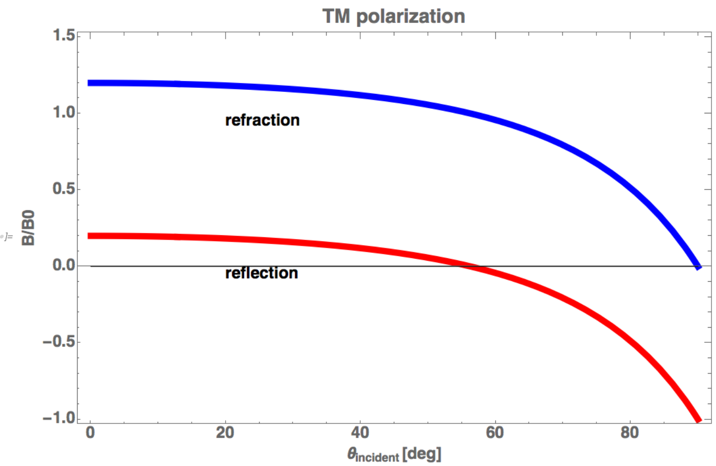
Section 9.4: Brewster's angle and Total Internal Reflection

● Brewster's Angle

Consider TM polarization, with $n_T > n_I$.
The reflection coefficient is zero at Brewster's angle.
There is no reflection at θ_B , and very little reflection at angles near θ_B .

```
TMplot (* for air
 (n=1) ⟶ glass (n=1.5) *)
```


Out[•]=



■3

That’s why you should wear polarized sun-glasses when you go fishing.

◆ Calculation of Brewster' s angle

Recall, for transverse magnetic waves ...

$$\frac{B^{\text{refl}}}{B^{\text{inc}}} = \frac{n_T \cos\theta_i - n_I \cos\theta_t}{n_T \cos\theta_i + n_I \cos\theta_t}$$

$$\textit{and} \quad n_I \sin\theta_i = n_T \sin\theta_t$$

The ratio is zero at $\theta_i = \theta_{\text{Brewster}}$.

So, solve these equations,

$$n_T \cos\theta_B = n_I \cos\theta_t$$
$$n_I \sin\theta_B = n_T \sin\theta_t$$

The result is

$$\tan\theta_B = n_T / n_I$$

Example. Calculate θ_B for the surface of a lake.

In[2]:=
```
ArcTan[1.33 / 1] * 180 / Pi
ArcTan[1 / 1.33] * 180 / Pi
```

Out[2]= 53.0612

Out[3]= 36.9388

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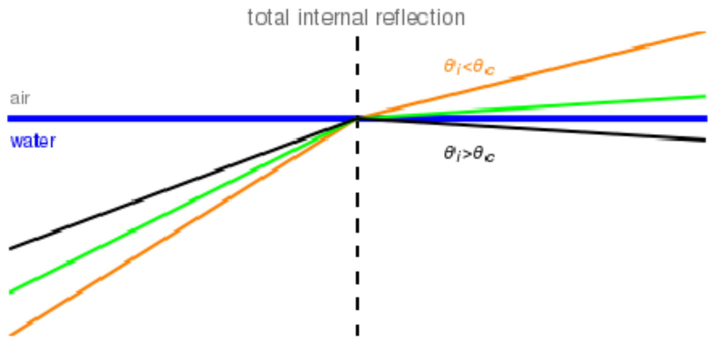
● Total Internal Reflection

For the case $n_I > n_T$ [e.g., light going from water (I) into air (T)] the transmitted wave vanishes if $\theta_i > \theta_{\text{critical}}$. In other words, the light cannot escape from the material for incident angles greater than θ_{critical} .

In[•]:= fisheye

Out[•]=



▪4

◆ Calculate the critical angle.

It does not depend on polarization, so we just use Snell's law.

$$n_I \sin\theta_i = n_T \sin\theta_t$$

$$\sin\theta_t = (n_I / n_T) \sin\theta_i$$

There is no solution if $\sin\theta_t > 1$;

therefore $\theta_{\text{critical}} = \arcsin(n_T / n_I)$.

■ For example, for light incident from water into air, the critical angle is

$\theta_{\text{critical}} = \arcsin(1/1.33) =$ 48.7 degrees.

■ This explains the term "fisheye lens" used in photography.

■ Applications of total internal reflection

5▪

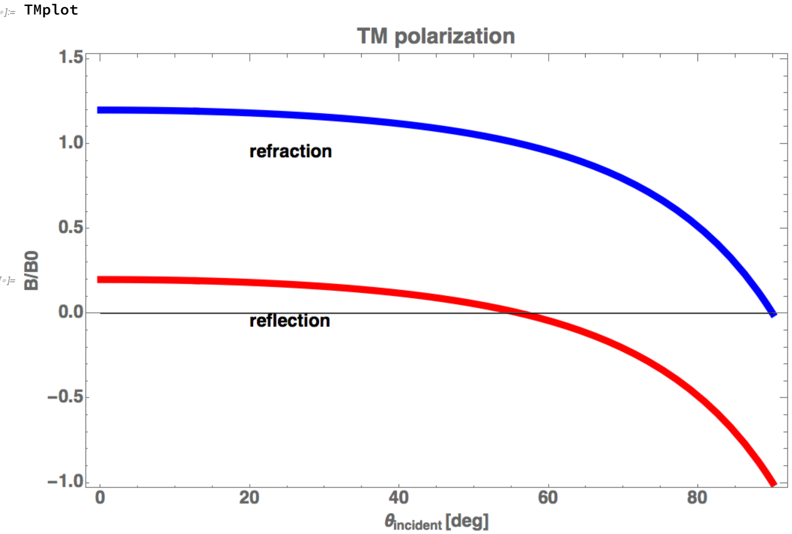
Conservation of Energy in Reflection and Refraction

We know that energy is conserved, in general.
Let's see how it comes about in the process of reflection and refraction.

Consider TM polarization ; air (n=1) ⟶ glass (n=1.5) ; at normal incidence ; i.e., $\theta_{\text{inc}} = 0$.

In[•]:= TMplot

Out[•]=



At normal incidence, $B_0{}' = 1.2\ B_0$ and $B_0{}'' = 0.2\ B_0$.
[(2*1.5)/(1.5+1) = 1.2 ; (1.5-1)/(1.5+1) = 0.2]

Is energy conserved?

Calculate the energy fluxes.

▪ (1) Incident wave only:
$S_1 = c/(4\pi)\ E_0\ B_0 \cos^2(kz\text{-}\omega t)$;
average = $c\,/\,(8\,\pi)\ B_0^2$

▪ (2) Transmitted wave:
$S_2 = c/(4\pi)\ E_0{}'\ B_0{}' \cos^2(kz\text{-}\omega t)$;
average = $c\,/\,(8\,\pi)\ B_0'^2/1.5 = 1.2^2/1.5 \times S_1 = 0.96\ S_1$

▪ (3) Reflected wave only:
$S_3 = c/(4\pi)\ E_0{}''\ B_0{}'' \cos^2(kz\text{-}\omega t)$;
average = $c\,/\,(8\,\pi)\ B_0''^{\,2} = 0.2^2\ S_1 = 0.04\ S_1$.

But what about *interference* between the incident and reflected waves?

Exercise: Calculate $(\vec{E} + \vec{E}'') \times (\vec{B} + \vec{B}'')$;   the result is, no interference.