R9

In[107]:=
```
Remove[dir0, dir1]
dir0 = "/Users/OurMacBookAir/Documents"
dir1 =
 "/Teaching.2018.current/chapter11.current/RadLecs
   .999"
SetDirectory[StringJoin[dir0, dir1]]
FileNames["*.png"]
```

Out[108]= /Users/OurMacBookAir/Documents

Out[109]= /Teaching.2018.current/chapter11.current/RadLecs
.999

Out[110]= /Users/OurMacBookAir/Documents/Teaching.2018.
current/chapter11.current/RadLecs.999

Out[111]= {1191.png, R9.shot1.png,
R9.shot2.png, R9.shot3.png, R9.shot4.png}

In[170]:=
```
shot1 =
  Show[Import["R9.shot1.png"], ImageSize → 768];
shot2 = Show[Import["R9.shot2.png"],
   ImageSize → 768];
shot3 = Show[Import["R9.shot3.png"],
   ImageSize → 768];
shot4 = Show[Import["R9.shot4.png"],
   ImageSize → 768];
figR91 = Show[Import["1191.png"], ImageSize → 768];
scanR91 = Show[Import["sc1.png"], ImageSize → 768];
scanR92 = Show[Import["sc2.png"], ImageSize → 768];
```

Import: File not found during Import.

Show: Symbol is not a type of graphics.

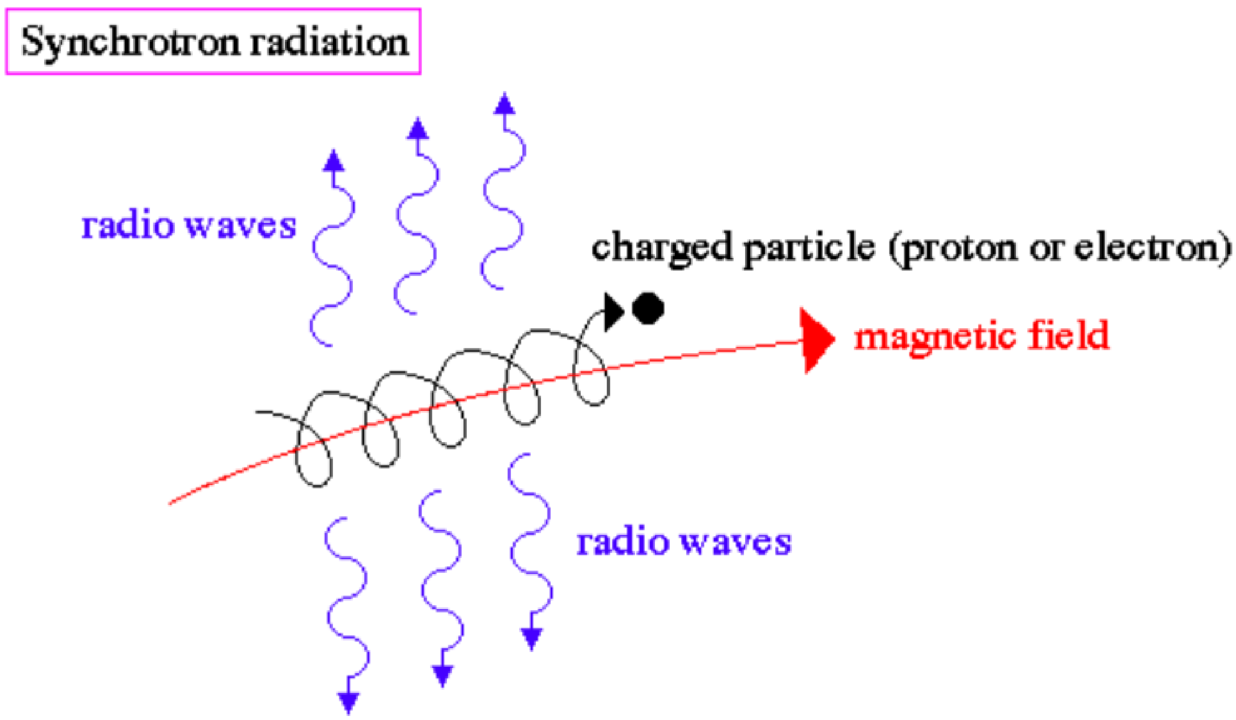
Import: File not found during Import.

Show: Symbol is not a type of graphics.

Synchrotron Radiation

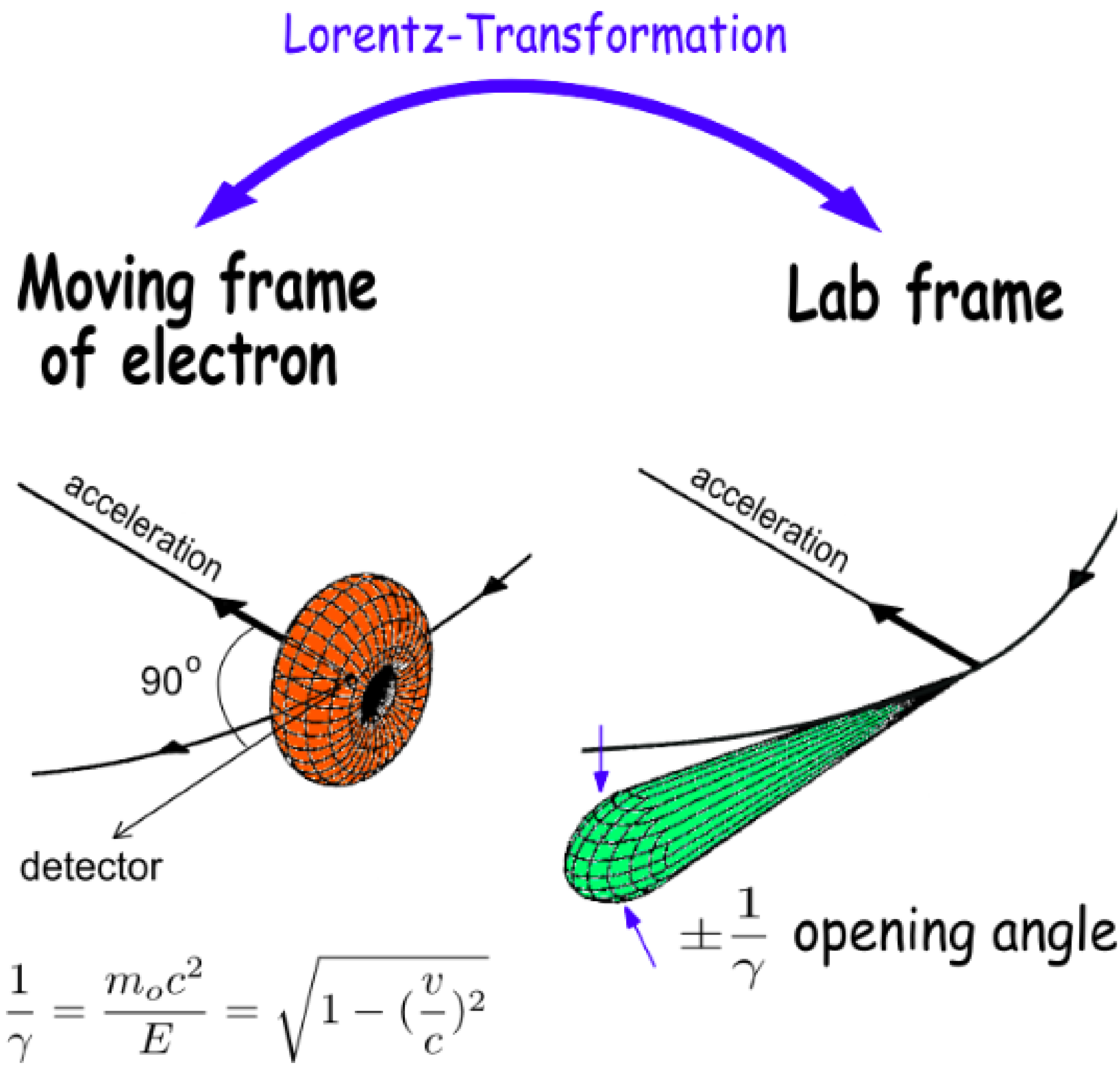
In[166]:=
```
shot1
shot2
shot3
shot4
```

Out[166]=

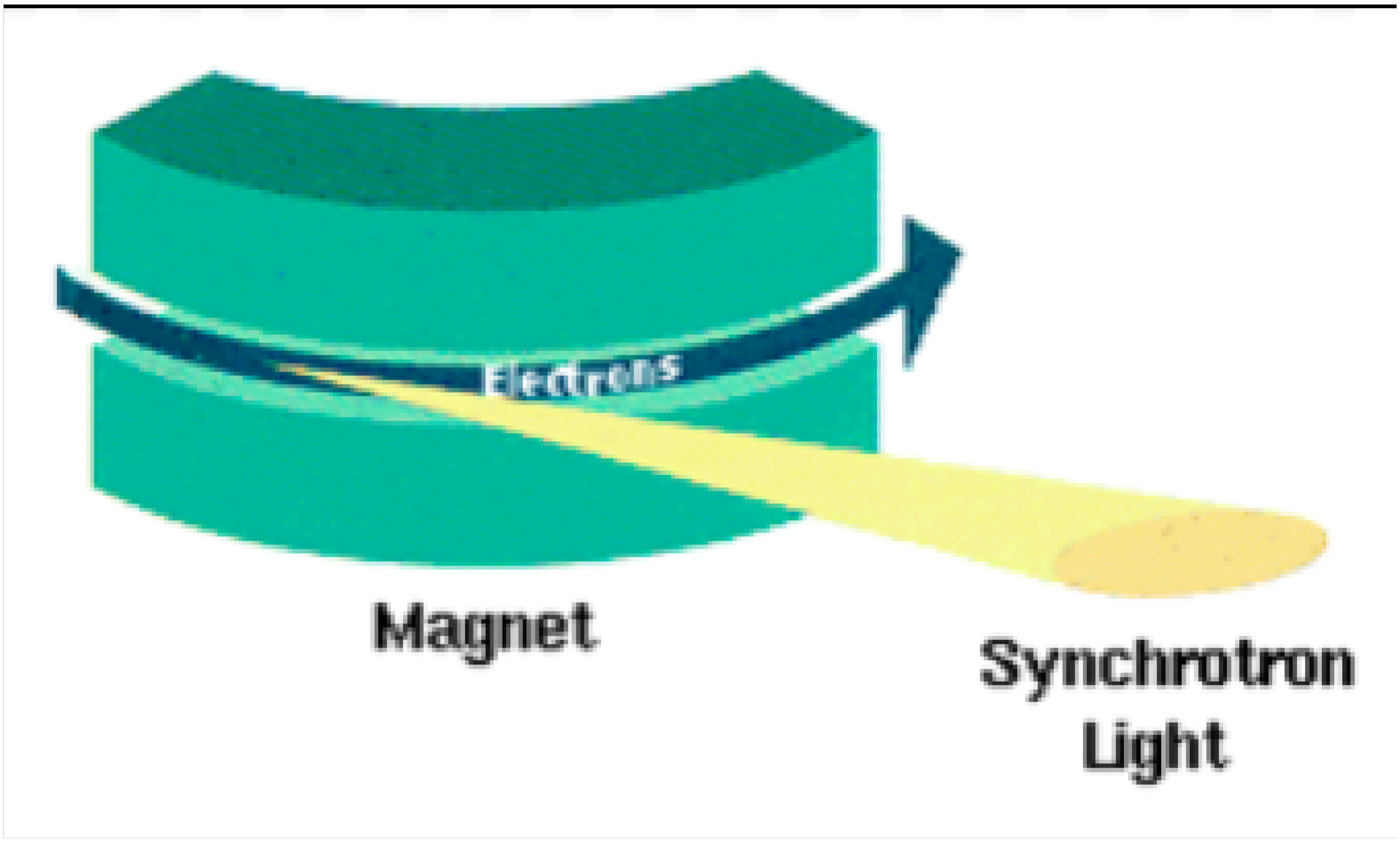


synchrontron radiation occurs when a charged particle encounters a strong magnetic field – the particle is accelerated along a spiral path following the magnetic field and emitting radio waves in the process – the result is a distinct radio signature that reveals the strength of the magnetic field

Out[167]=

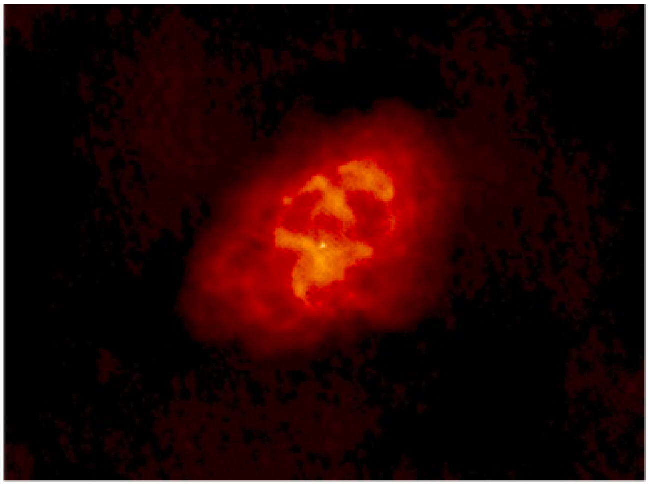

Lorentz-Transformation
Moving frame
of electron
Lab frame
acceleration
90°
detector
acceleration
$\pm\frac{1}{\gamma}$ opening angle
$\frac{1}{\gamma} = \frac{m_o c^2}{E} = \sqrt{1 - (\frac{v}{c})^2}$

Out[168]=



Out[169]=

7–3



Crab nebula at 90 cm (NRAO 300′ telescope), resolution 1.3″

The characteristic frequency was

$$\nu_c = \frac{\omega_c}{2\pi} \qquad (6.25)$$

$$= 6300 \left(\frac{B}{10^{-7}\,\mathrm{T}}\right)\left(\frac{E/m_e c^2}{10^3}\right)^2 \mathrm{MHz} \qquad (7.1)$$

Optical light has $\nu \sim 10^8$ MHz. To emit this frequency, the electrons must have $\gamma \sim 10^6$ for a typical B-field!

Life time of electrons with $\gamma = 10^6$ (per Eq. 6.20): 16 years.

Diameter of Crab: ~2 pc $\Longrightarrow$ it is not a problem to deliver all energy by accelerating electrons at the center of the neutron star in the center of the nebula.

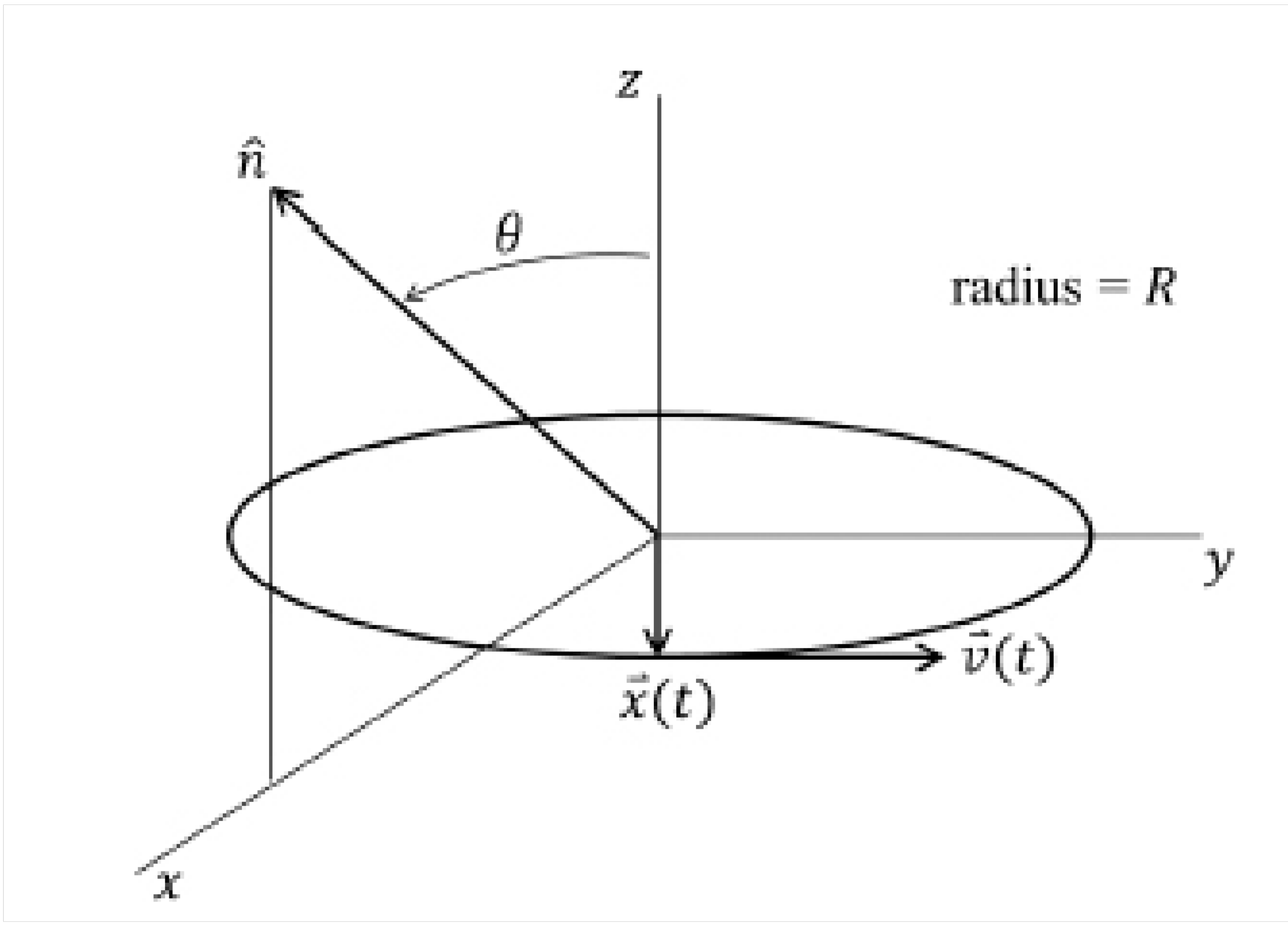
1-

Synchrotron Radiation
Section 11.9

A charged particle (e) in circular periodic motion ...
See Figure 11.12.

In[177]:= figR91

Out[177]=



$\vec{r}(t) = R\,\{\cos(\omega_0 t), \sin(\omega_0 t), 0\}$

$\vec{v}(t) = R\omega_0\,\{-\sin(\omega_0 t), \cos(\omega_0 t), 0\}$

.

By cylindrical symmetry around the z axis, W.L.O.G. we can let the field observation point be in the xz plane, i.e., $\phi = 0$.

.

$\hat{n} = (\sin\theta, 0, \cos\theta)$

2-

- **The average power radiated (= E/T = energy radiated in one cycle / period) ; the angular distribution . . .**

.

From the last lecture, the power distribution for the m-th harmonic is

In[178]:= scanR91

Out[178]= Show[$Failed, ImageSize → 768]

$\frac{dP_m}{d\Omega}$ =

(250)

The integral can be separated into components

(251)

where

T1 =

T2 =

We get cylindrical Bessel functions. Recall,

$$J_m(z) = (-i)^m \int_0^{2\pi} d\phi\, e^{i m (\phi - z \cos\phi)};$$

$$\int_0^{2\pi} d\phi\, e^{i m (\phi - z \cos\phi)} = (-i)^m J_m(z)$$

Let z ≡ m (ω_0R/c) sinθ = m β sinθ,

where $\beta = \omega_0 R / c$.

⇒ First term:

$$T1 = (2\pi / \omega_0) \int_0^{2\pi} d\phi \cos\phi\, e^{i (m\phi - z \cos\phi)}$$

$$= (-i)^m (\pi / \omega_0) [J_{m+1}(z) - J_{m-1}(z)]$$

⇒ Second term:

$T2 = (-i)^m \, (\pi / \omega_0) \, [\, J_{m+1}(z) + J_{m-1}(z) \,]$

.

So, $\tilde{v}_m =$

$= \int_0^{2\pi} dt \, \vec{v}(t) \, e^{i m \omega_0 \left(t - \hat{n}.\vec{r}(t)/c\right)}$

$= v_0 \, (-i)^m \, (\pi / \omega_0)$

$\{ \hat{e}_x \, [J_{m+1} + J_{m-1}] - i \, \hat{e}_y \, [J_{m+1} - J_{m-1}] \}$

⇒ A bunch of substitutions ...

- Calculate $\hat{n} \times \tilde{v}\, m$
- Calculate $\left| \hat{n} \times \tilde{v}\, m \right|^2$
- Apply a Bessel function recurrence relation,

$J_{m+1}(z) + J_{m-1}(z) = (2m/z)\, J_m(z)$

- Use another recurrence relation,

$J_{m+1}(z) - J_{m-1}(z) = -2\, J_m'(z)$

' = derivative w.r.t. z

Putting it all together ⟹

$\frac{dP_m}{d\Omega} =$

(267)

This is the differential power radiated into the m-th harmonic.

3-

- **The total power radiated into the m-th harmonic, during one period.**

Define $P_m = \int d\Omega \, \frac{dP_m}{d\Omega}$

According to WR the integral is too difficult, so they do the calculation in a different way.

Go back to Eq. (250)

$$\frac{dP_m}{d\Omega} = \left| \hat{n} \times \vec{I} \right|^2$$

We have $\left| \hat{n} \times \vec{I} \right|^2 = I^2 - (\hat{n} \cdot \vec{I})^2$

where

$$\vec{I} = \int_0^T dt \, \vec{v}(t) \, e^{i m \omega_0 \left(t - \hat{n} \cdot \vec{r}(t)/c\right)}.$$

After some clever analysis,

$$P_m = \frac{e^2}{R} \int d\phi \cos(m\phi) \left(\beta^2 \cos\phi - 1\right)$$

$$\square \; \frac{\sin(2\, m\beta \sin(\phi/2))}{\sin(\phi/2)}$$

Now it turns out...

$$\int_0^x dz\, J_{2\,m}(z) = \frac{1}{\pi}\int_0^{\pi} d\phi\; \cos(m\phi)\; \frac{\sin(x \sin(\phi/2))}{\sin(\phi/2)}$$

So

$$P_m = \frac{e^2\, m\, \omega_0}{R} \left\{ 2\, \beta^2 J'_{2\,m}(2 m\beta) - \left(1-\beta^2\right) \int_0^{2\, m\beta} dz\, J_{2\,m}(z) \right\}$$

(287)

4-

▪ The total radiated power

Can we calculate $\sum_{m=1}^{\infty} P_m$?

According to WT we can't calculate that.

But we can calculate something different.

In[85]:=
```
scanR92
```

Out[85]=
```
Show[$Failed, ImageSize → 768]
```