1-

Diffraction

What is "diffraction"?

Look up the word in a dictionary.

.

from vocabulary.com:

"

Diffraction is the process of light bending around an obstacle or spreading out after it moves through a small space.

(Any kind of wave can experience diffraction, including sound, radio, and water.)

The root of diffraction is the Latin word *diffringere*, "break into pieces," from *dis*, "apart," and *frangere*, "to break."

"



from The Oxford Dictionary of Physics

"

The spreading or bending of waves as they pass through an aperture or round the edge of a barrier. The diffracted waves subsequently interfere with each other producing regions of reinforcement and weakening. First noticed as occurring with light by Francesco Grimaldi (1618-63) the phenomenon gave considerable support to the wave theory of light. Diffraction also occurs with beams of particles because of the quantum mechanical wave nature of such particles.

"

.

Fraunhofer diffraction

Fresnel diffraction

2-

## History of the science of diffraction

Grimaldi (1618-1663)
Newton (1642-1727)
Huygens (1629-1695)
Young (1773-1829)
Fresnel (1788-1827)
Fraunhofer (1787-1826)
Kirchhoff (1824-1887)
many others

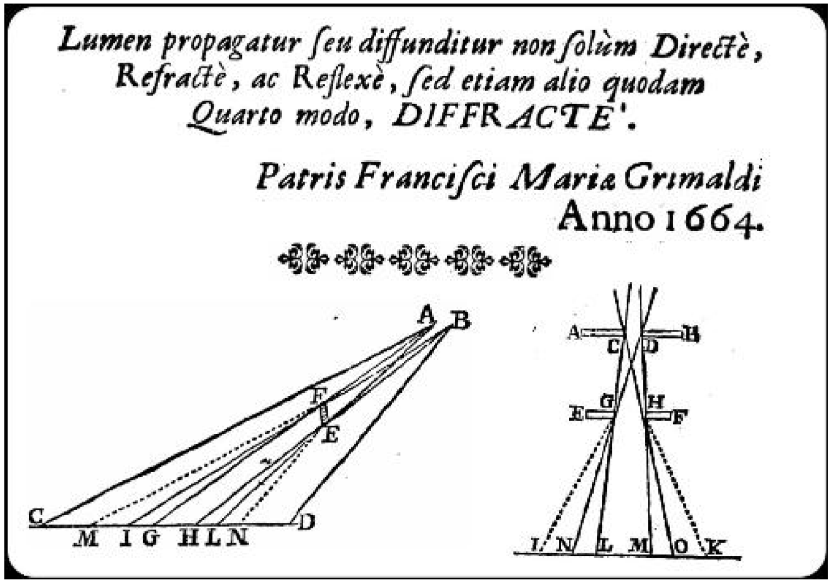


## Grimaldi' s discovery of diffraction

(unreferenced websites)

In[500]:= gr1

Out[500]=




Shadows projected by opaque bodies illuminated by solar light or by point sources built in the laboratory lead to the conclusion that, in a transparent homogeneous medium, light propagates along straight lines (the law of rectilinear propagation). This conclusion, however, does not hold rigorously and without limits.

Grimaldi was the first to realize it.

In order to check this idea, Grimaldi drew a little hole in the shutter of a window, thus letting the sun's light into a room (here we have almost his own words). Propagation of light in the room would then take place along a cone "that shall be visible in the air if there be some dust or if some smoke is produced". Interposing an opaque body along the light's path will produce a shadow on a sheet of white paper

– "the limit of the shadow remains in some way ill-defined, due to a certain penumbra, characterized by a perceptible graduation [...] in the regions between total shadow and full light [...]". Moreover, the overall shadow appeared consistently wider than it ought to be on the assumption that "everything took place in terms of straight lines".

```
In[501]:= gr2
```



Out[501]=

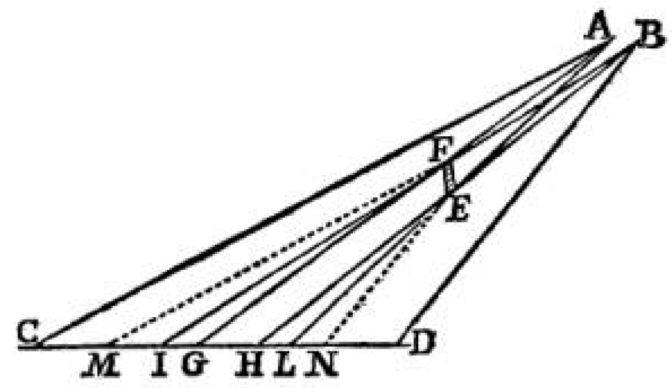



Fig. 1: *AB is the hole from which light penetrates into the room: EF is the obstacle across the light cone; MN is the projected shadow, sensibly wider than that foreseen by th law of rectilinear propagation; IG and HL are the zones of penumbra, GH that of full darkness.*

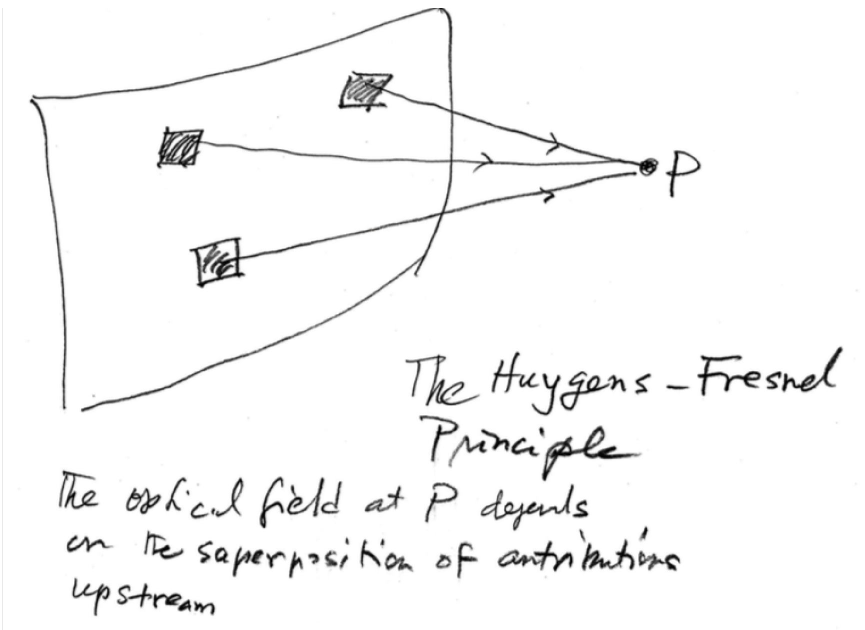
3-

## The Huygens-Fresnel Principle

*Every point on a wave front is a source of spherical waves, and the superposition of these wavelets is a later wave front.*

In[580]:= `scan1`

Out[580]=




## The Huygens-Fresnel Integral

$$\Phi(P) = \Phi_0 \frac{e^{ikr_0}}{r_0} \iint da \frac{e^{ikr}}{r} \beta(\alpha)$$



$\Phi_0$ = amplitude;

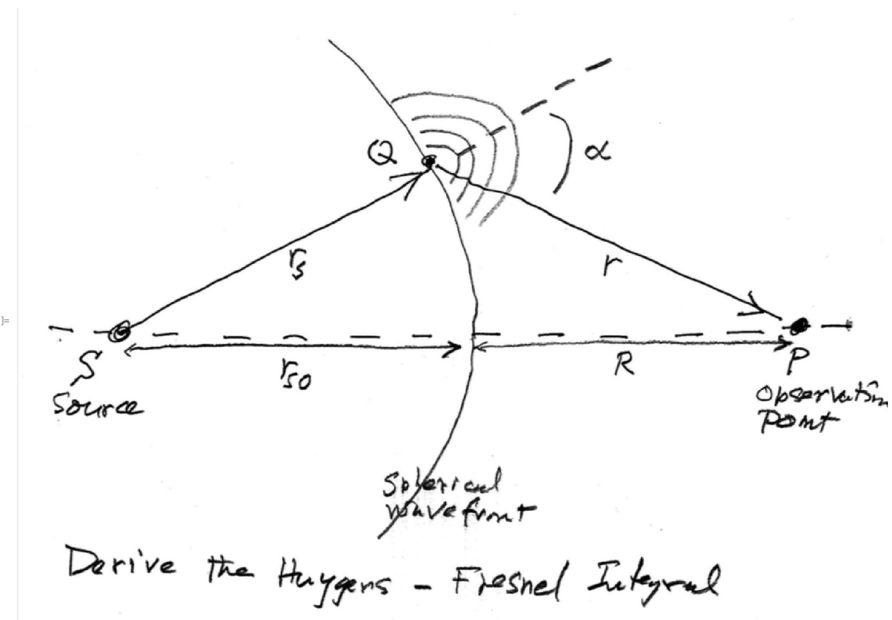
$e^{ikr_0}/ r_0$ = spherical wavelet;

$\iint da$ = integral over the aperture;

$\beta(\alpha)$ = “obliquity factor” ; might take it to be a constant;

In[581]:= `scan2`

Out[581]=



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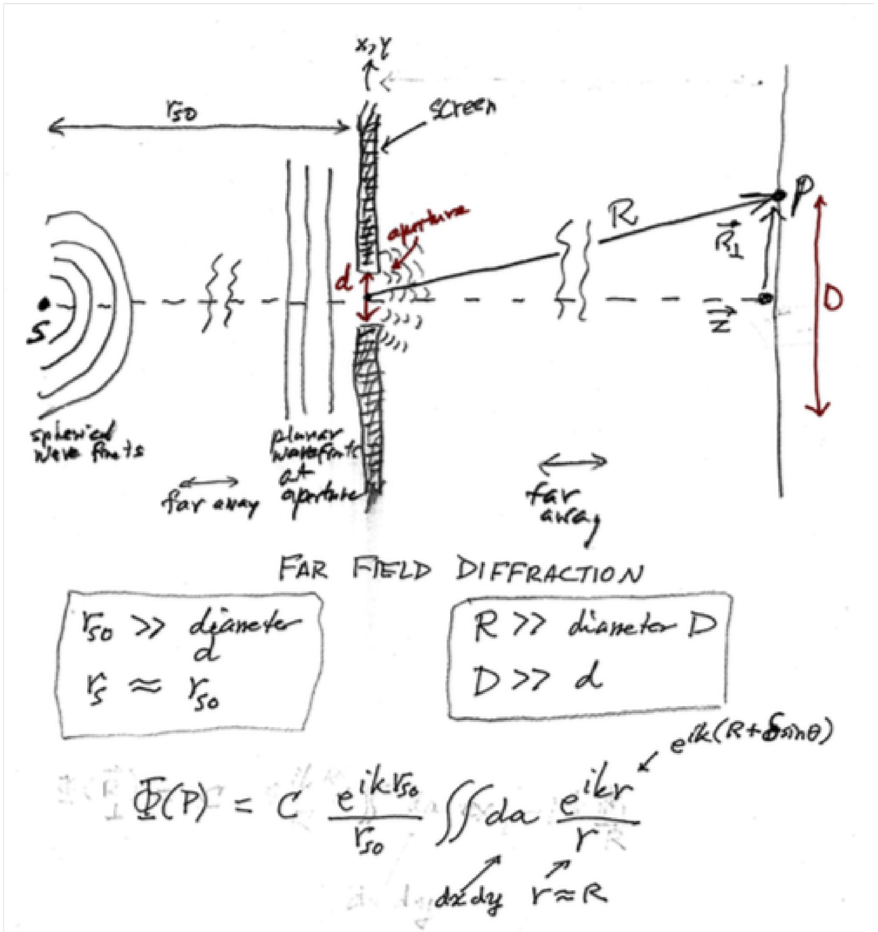
# Fraunhofer diffraction = *far-field diffraction*

In[582]:= scan3

Out[582]=






r ≈ R + $\delta$r where $\delta$r ≪ R and $\delta$r is linear in x and y, the coordinates on the aperture. Based on the figure,

$$\Phi(P) = \frac{[C]}{R} \iint da\, e^{ik(R+\delta r)}$$

where $[C] = \Phi_0 \, (e^{ikr_o} / r_0)\, \beta$

The far zone approximation

$$r = [\,(R_x - x)^2 + (R_y - y)^2 + R_z{}^2\,]^{1/2}$$

$$R = [\,R_x^2 + R_y^2 + R_z^2\,]^{1/2}$$

$$\delta r = -(\,x\,R_x + y\,R_y\,)\,/\,R$$

$$\Phi(\vec{R}) =$$

$$\frac{[C]}{R}\, e^{ikR} \iint dx\, dy\, \exp\left[-i\,k\,(x\,R_x + y\,R_y)\,/\,R\right]$$

## Case of circular symmetry

$$dx\,dy = 2\pi\, a\, da$$

$$\Phi(\vec{R}) = 2\pi \frac{[C]}{R}\, e^{i\,kR} \int a\, da\, J_0(\, k\, a\, R_\perp / R)$$



## Fraunhofer diffraction for a single slit

$$\Phi\,(X) = \frac{[CL]}{R}\, e^{i\,kR} \int_{-a/2}^{a/2} dx\, \exp[-i\, k\, x\, \sin\theta]$$

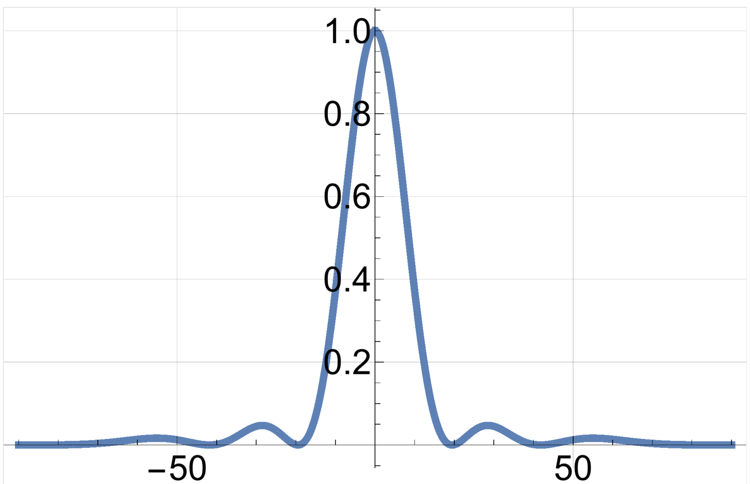
$$\Phi(X) = \frac{[CL]}{R}\; e^{i\,kR}\; a\; \frac{\sin\left(\frac{1}{2}\, ka \sin\theta\right)}{\frac{1}{2}\, ka \sin\theta}$$

$$I(x) = I0 \left( \frac{\sin\left(\frac{1}{2}\, ka \sin\theta\right)}{\frac{1}{2}\, ka \sin\,\theta} \right)^2$$

Figures showing Fraunhofer diffraction for a single slit with a = 3$\lambda$

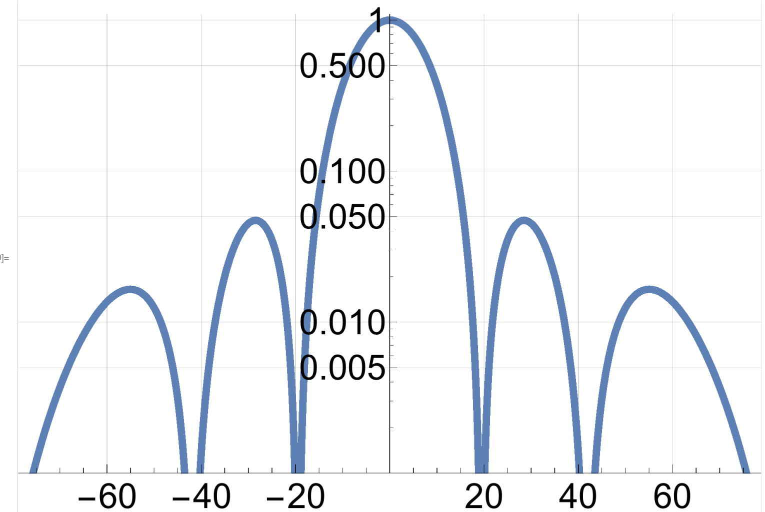
In[527]:=
```
ka = 6 Pi; (* i.e., a = 3λ *)
f[θ_] = Power[
   ( Sin[0.5 * ka * Sin[θ]] / (0.5 * ka * Sin[θ]) ), 2];
Plot[f[ang / 180 * Pi], {ang, -90, 90},
 PlotRange → {All, All},
 BaseStyle → 30, ImageSize → 640,
 PlotStyle → Thickness[0.01],
 GridLines → Automatic]
LogPlot[f[ang / 180 * Pi], {ang, -90, 90},
 PlotRange → {All, {0.001, 1.1}},
 BaseStyle → 30, ImageSize → 640,
 PlotStyle → Thickness[0.01],
 GridLines → Automatic]
```

Out[529]=






Out[530]=



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## Fraunhofer diffraction for a circular aperture

$$\Phi(\theta) = 2\pi \frac{[C]}{R} e^{ikR} \int_0^a a' da' J_0(k a' \sin\theta)$$

$$\Phi(\theta) = 2\pi \frac{[C]}{R} e^{ikR} \frac{J_1(ka \sin\theta)}{ka \sin\theta}$$

$$I(\theta) = I_0 \left(\frac{2 J_1(ka \sin\theta)}{ka \sin\theta}\right)^2$$

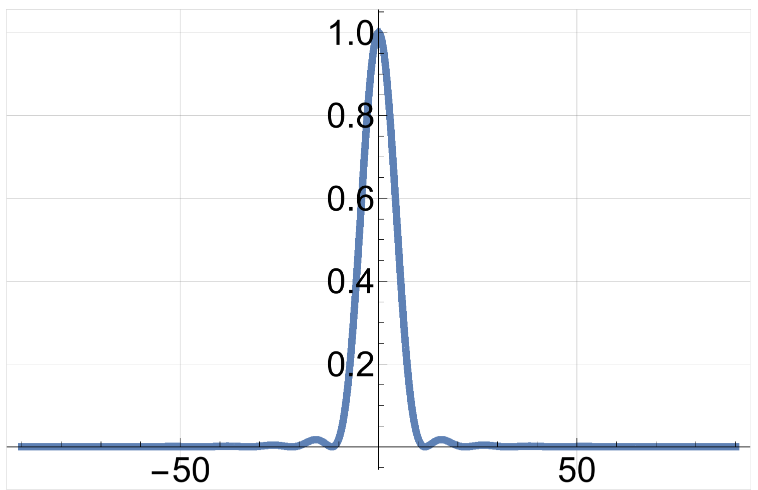
Figures showing Fraunhofer diffraction for a circular aperture



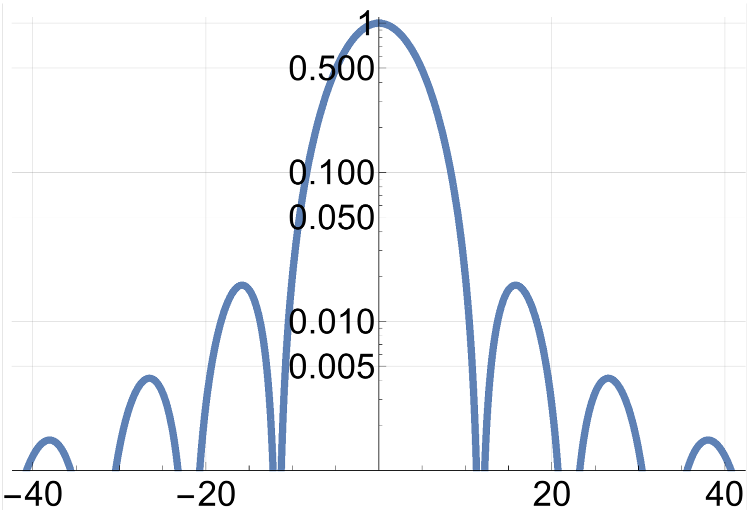
In[531]:=
```
ka = 6 Pi; (* i.e., a = 3λ *)
g[θ_] =
 Power[2 BesselJ[1, ka * Sin[θ]] / (ka * Sin[θ]) , 2]
Plot[g[ang / 180 * Pi], {ang, -90, 90},
 PlotRange → {All, All},
 BaseStyle → 30, ImageSize → 640,
 PlotStyle → Thickness[0.01],
 GridLines → Automatic]
LogPlot[g[ang / 180 * Pi], {ang, -90, 90},
 PlotRange → {All, {0.001, 1.1}},
 BaseStyle → 30, ImageSize → 640,
 PlotStyle → Thickness[0.01],
 GridLines → Automatic]
```

Out[532]= $\frac{\text{BesselJ}[1, 6\pi \,\text{Sin}[\theta]]^2 \,\text{Csc}[\theta]^2}{9\pi^2}$

Out[533]=



Out[534]=




The Airy pattern; Airy disk and Airy rings.

The "diffraction limit" = $\theta_{\text{DIFF LIMIT}}$

Calculate the radius of the Airy disk.

$J_1(\,ka \sin\theta_1\,) = 0 \implies ka \sin\theta_1 = x_{11}$

In[506]:=
```
x11 = x /. FindRoot[BesselJ[1, x], {x, 4}]
sθ = x11 / (2 * Pi * a / λ)
2 * sθ
```

Out[506]= 3.83171

Out[507]= $\dfrac{0.609835\,\lambda}{a}$

Out[508]= $\dfrac{1.21967\,\lambda}{a}$



Assume $\theta_1 \ll 1$ ; approximate $\sin\theta \approx \theta$ ;

$$\theta_{DL} = 2\,\theta_1 = 1.22\,\frac{a}{\lambda}$$

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## Fresnel diffraction

is more difficult to calculate and to observe, but it is described in textbooks on Optics.

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What is missing from this theory?

Light is an electromagnetic phenomenon, and electromagnetic waves are ***vector waves***. The scalar theory may qualitatively describe diffraction, but for quantitative accuracy we need a theory of ***vector diffraction.***

$\Longrightarrow$ ***The Kirchhoff identity for diffraction***

Jackson shows how the Kirchhoff identity can be applied to scalar waves, and can also be extended to vector waves.