Cherenkov Radiation

Jackson, Section 13.4

Introduction

In free space, a charged particle moving with constant velocity does not radiate electromagnetic waves. How do we know this? Give two reasons based on Maxwell's theory.

In a dielectric material, a particle moving with constant velocity ***will*** radiate electromagnetic waves, if the velocity is greater than the velocity of light in the medium.

Interesting history

1934 discovery by Pavel Cherenkov
1938 theory developed by Ilya Frank and Igor Tamm
1958 Cherenkov, Ftrank, and Tamm shared the Nobel prize in ohysics

Cherenkov grew up on a farm, in remote Russia.
He went to an non-elite college in remote Russia.
He went to graduate school in Moscow.
His faculty supervisor was Vavilov a distinguished Russian physicist.
Cherenkov decided to do his research on an effect that had been known for a long time, but never really studied in much detail.
pre-history: When a radioactive sample, like radium or uranium, is immersed in a liquid, a faint blue light is observed coming from the liquid.
Marie and Pierre Curie knew the effect, using their samples of radium.
A French physicist Mallet studied the effect in some detail, but lost interest.

Vavilov suggested to do experiments with gamma rays incident on various liquids, and to observe the light that is produced.

By detailed (and difficult!) experiments, Cherenkov discovered:
-- the effect can be observed for many liquids, including water;
-- it could also be observed with various transparent dielectrics.;
-- the radiation is not isotropic;
-- there are polarization effects;
-- the spectrum of light is a continuum stronger in the blue

The theory offered by Vavilov was that this is an example of fluorescence. However, Cherenkov ruled that out by (i) the spectrum is continuous, and (ii) there is no time delay between gamma-ray and visible light. So, he claimed that this is a new, previously unknown mechanism for the radiation of light.

Theory of Cherenkov radiation

Cherenkov had a friend -- Ilya Frank -- also a student at the time, and they worked together on some experiments. So Frank knew the phenomenon in detail, and he tried to come up with a theory. He got advice from Igor Tamm.

The key is the anisotropy of the radiation. Tamm and Frank thought this must imply a coherent radiation ***produced by an electron*** and coming from a volume around the path of the electron. So it was not an atomic radiation like fluorescence. And the explanation must come out of the index of refraction.

Preliminary Derivation

Consider a charged particle moving on a straight line, with velocity v = cβ.

Huygens picture: the electron continuously creates wavelets in the E.M. field, which travel away from the particle as spherical waves. The spherical waves spread out with velocity c/n where n = $\sqrt{\epsilon}$. c/n = the phase velocity of light in the medium.

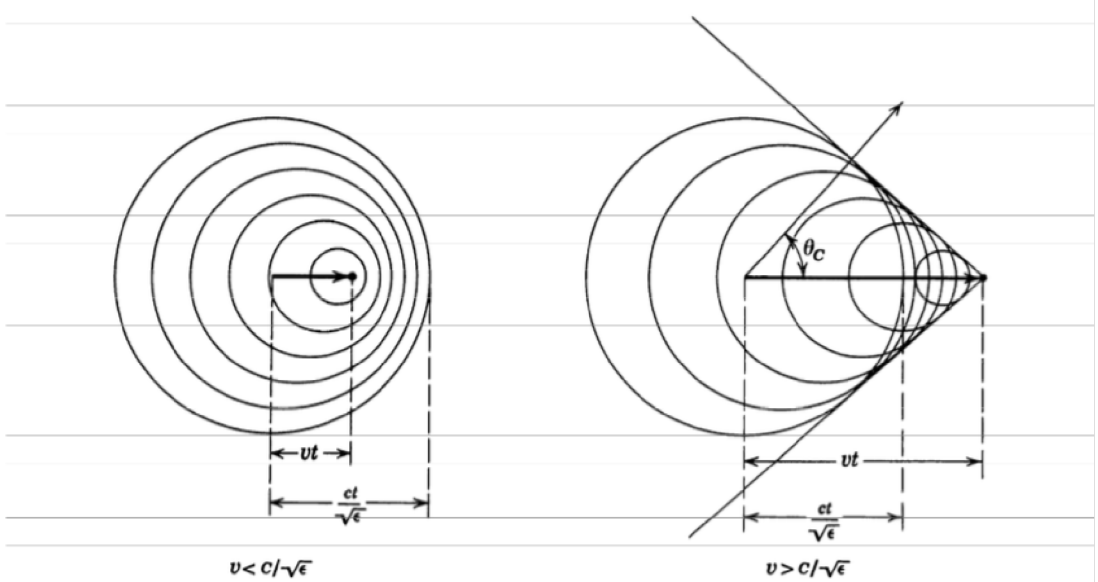

Figure 13.5 Cherenkov radiation. Spherical wavelets of fields of a particle traveling less than and greater than the velocity of light in the medium. For $v > c/\sqrt{\epsilon}$, an electromagnetic "shock" wave appears, moving in the direction given by the Cherenkov angle θ_C.

If v < c/n then the wavelets undergo destructive interference and there is no escaping radiation (figure a).

But if v > c/n then the wavelets form a shock wave (figure b) which escapes to infinity.

The Cherenkov angle

It's just trigonometry.

Look at figure b.

The particle has traveled distance vt from the back of the arrow (≡ time 0). That is the hypotenuse. The spherical wave emitted at time t = 0 has traveled distance (c/n)t. That is the distance traveled by the shock wave.
The shock wave is a cone and

$$\cos\,\theta_c = \frac{c/n\,t}{v\,t} = \frac{1}{\beta\,n}$$

$$\cos\,\theta_c = \frac{1}{\beta\, n}$$

- Cherenkov radiation requires β n > 1.

That is, v > c/n = v_{light} .

- The half-angle of the radiation cone is

$\frac{\pi}{2} - \theta_c$ where $\theta_c = \arccos \frac{1}{\beta\, n}$.

- θ_c decreases as energy increases.
- For an ultrarelativistic particle the Cherenkov angle is

$\arccos\,\frac{1}{n}$.

Frequency dependence

$$\cos\theta_c = \frac{1}{\beta\, n(\omega)}$$

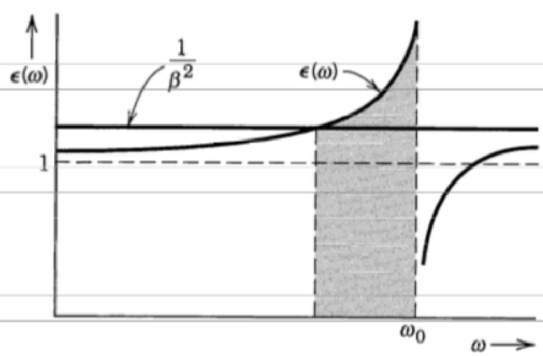

Figure 13.3 Cherenkov band. Radiation is emitted only in shaded frequency range, where $\epsilon(\omega) > \beta^{-2}$.

Quiz

(a) In free space, a charged particle moving with constant velocity does not radiate electromagnetic waves. How do we know this? Give two reasons based on Maxwell’s theory. (b) What are the fields? Be precise and accurate.

Homework Assignment #13
assigned Friday Nov 16 due Wednesday Nov 21

Problem 13-1. (A) Exercise 12.7.3.

(B) Plot the function for an interesting choice of ka.

Problem 13-2. Jackson Exercise 13.9.

13.9 Assuming that Plexiglas or Lucite has an index of retraction of 1.50 in the visible region, compute the angle of emission of visible Cherenkov radiation for electrons and protons as a function of their kinetic energies in MeV. Determine how many quanta with wavelengths between 4000 and 6000 Å are emitted per centimeter of path in Lucite by a 1 MeV electron, a 500 MeV proton, and a 5 GeV proton.

Problem 13-3.

Your answers should be accurate, precise, readable, and sufficiently complete — at least three well written paragraphs for each part.

(a) Describe an experiment, in which MSU faculty and grad students are involved, that makes use of Cherenkov radiation.

(b) Describe another experiment, in which MSU faculty and grad students are involved, that makes use of Cherenkov radiation.

(c) Describe a third experiment, in which MSU faculty and grad students are involved, that makes use of Cherenkov radiation.

Problem 13-4.

A charged particle moves in free space with constant velocity;
$\vec{v}(t) = v_0\, \hat{e}_x$.

(A) Write down the electric and magnetic fields in Cartesian coordinates.

(B) Show that $\vec{E}$ and $\vec{B}$ are carried along in the x direction by the charge.

(C) Now suppose $\beta = 0.9$.

Draw the xz-plane, and let t_0 be the instant of time in the lab frame of reference when the particle is at the origin. Draw the point P_1 in the xz-plane with position vector $(d/\sqrt{2},\ 0,\ d/\sqrt{2})$. Draw the point P_2 with position vector $(0, 0, d)$. Draw the electric field vector at P_1 and at P_2 . (Be precise and accurate.) Also, draw on the graph in a different color the positions of the particle at the retarded time for P_1 and P_2 .