Bremsstrahlung introduction

Suppose a particle with charge e is moving in free space, initially with constant velocity, $\vec{v} = v\,\hat{e}_x$.

At time t = 0, it reaches the origin.

But for t > 0 there is a braking force, and the particle rapidly decelerates to rest.

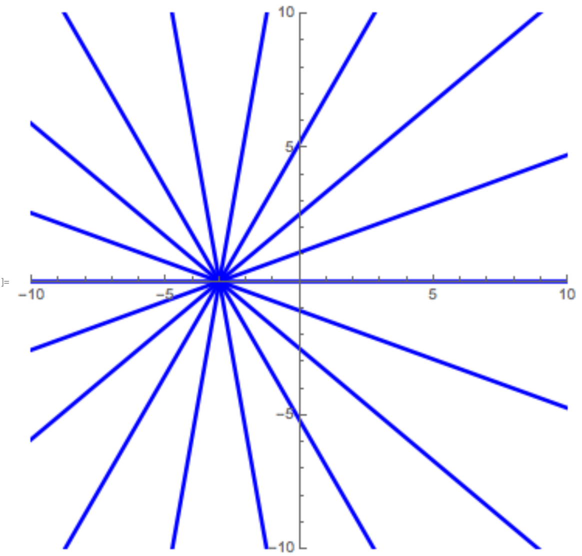
Calculate the radiation that is emitted.

These pictures illustrate what happens.

■ For t < 0, the electric field lines are shown in blue, in a volume of space centered at the origin.

In[1771]:=
```
pic["sh1a.png"]
```

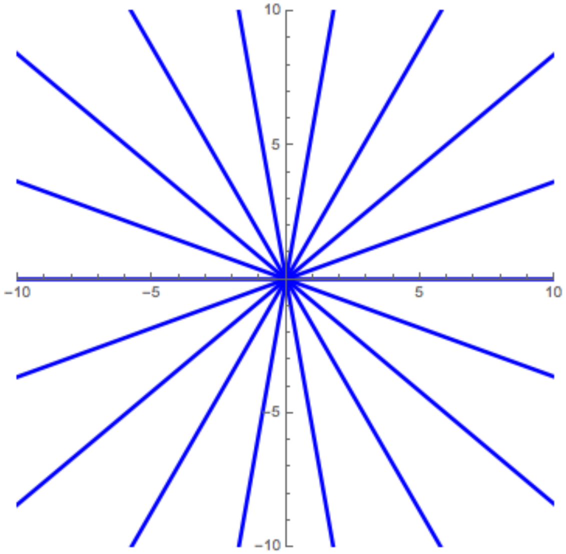
Out[1771]=



- For t = 0, the electric field lines are shown in blue, in the same volume.

In[1772]:= `pic["sh1b.png"]`

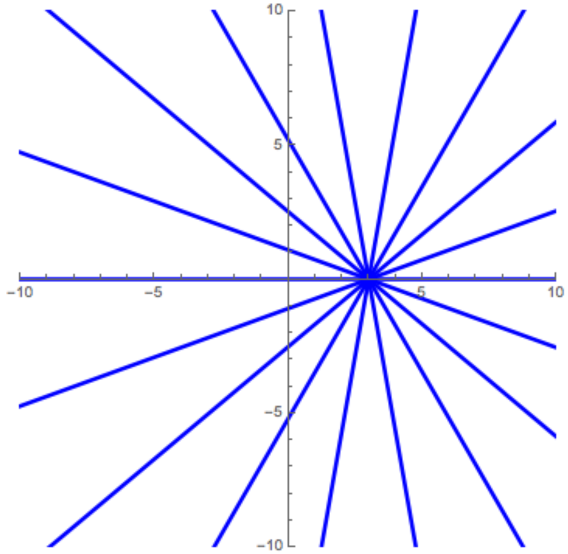
Out[1772]=



- For t > 0, the electric field lines are shown in blue, in the same volume, ***if there is no braking***

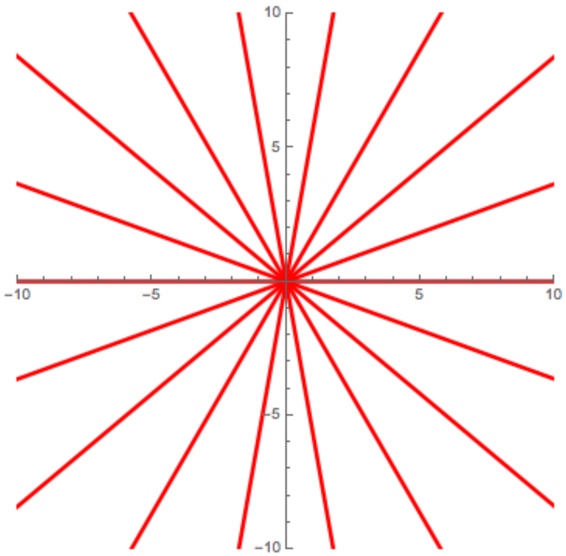
In[1773]:= `pic["sh1c.png"]`

Out[1773]=



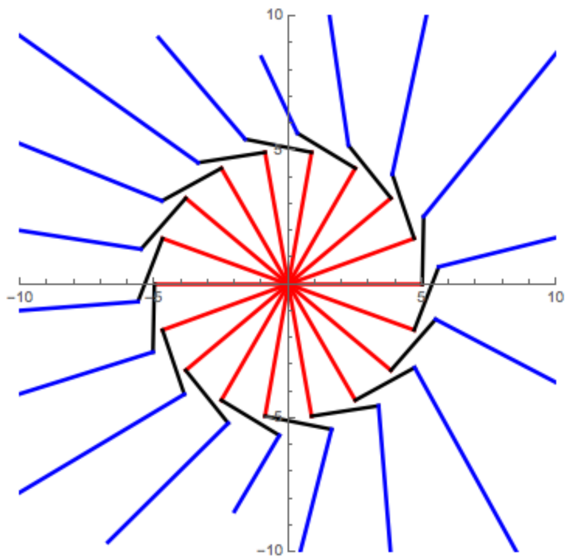
■ For t ⟶ ∞, the electric field lines are shown in red, in the same volume, ***if the braking has occurred.***

In[1774]:= `pic["sh1d.png"]`

Out[1774]=


■ For t > 0 , the electric field lines are shown in blue and red, in the same volume, a short time after the particle comes to rest. The information that the particle has come to rest travels at the speed of light. For the time shown, c t = the radius. For r > ct. the information hasn't yet reached to field.

In[1775]:= `pic["sh2.png"]`

Out[1775]=


The outgoing sphere is the radiation called ***Bremsstrahlung.***

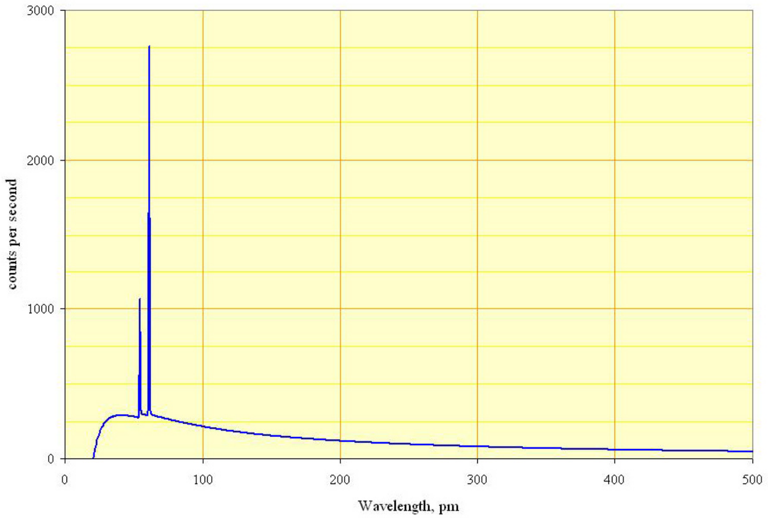
History of the science of Bremsstrahlung

DISCOVERY

•X-rays [Roentgen, 1895] ; using a “Crookes tube”;

•There exists a line spectrum and a continuum; see Wikipedia; Rhodium cathode; K-lines of Rhodium;

In[1776]:=



•Sommerfeld named the continuous spectrum “Bremsstrahlung”; bremsen = to

brake; strahlung = radiation ;

•Coincidence measurements of electrons and photons (Nakel, 1966)

THEORY

■Classical and semi-classical calculations

•Kramers [1923]

•Wentzel {1923]

■Quantum mechanical calculations

•Sommerfeld [1931] nonrelativistic electrons and quantum mechanical; QED= the theory of the photon!

•Bethe and Heitler [1934] using the Dirac equation

•Bethe and Maxion [1954] using coulomb waves function for electron scattering

•Tseng and Pratt [1971] complete numerical (computer) calculations

Jackson, Section 15.1:
Radiation Emitted during Collisions

In a collision there is acceleration and therefore radiation. We'll assume that the projectile is much lighter than the target, so we'll treat the collision as a particle moving in a fixed potential.
There will be some approximations which must be justified.

■ Recall the classical theory of radiation by a nonrelativistic particle

The intensity per unit solid angle and per unit frequency interval is

In[1777]:= eq["15.1.png"]

Out[1777]=
$$\frac{dI(\omega)}{d\Omega} = \frac{e^2}{4\pi^2 c}\left|\int \mathbf{n}\times(\mathbf{n}\times\dot{\boldsymbol{\beta}})e^{i\omega\left(t-\frac{\mathbf{n}\cdot\mathbf{r}(t)}{c}\right)}\,dt\right|^2$$

15.1

We may approximate
$t - \hat{n}\cdot\vec{r}/c \sim t - v\,t/c \approx t$
because $v/c \ll 1$; this is called the " dipole approximation".

In[1778]:= eq["15.2.png"]

Out[1778]=
$$\frac{dI(\omega)}{d\Omega} = \frac{e^2}{4\pi^2 c}\left|\int \mathbf{n}\times(\mathbf{n}\times\dot{\boldsymbol{\beta}})e^{i\omega t}\,dt\right|^2$$

15.2

Now consider the collision process. There is a finite collision time $\tau = a/v$ where a is the characteristic distance over which the force is significant. The integral over t is over an interval of order τ.
Now we can separate low frequencies ($\omega \ll 1/\tau$) and high frequencies ($\omega \gg 1/\tau$).

For low frequencies ...

$\int dt\, \dot{\boldsymbol{\beta}} \approx \Delta\vec{\beta} = \vec{\beta}_2 - \vec{\beta}_1$

= the total change of $\vec{\beta}$ in the collision; and so the radiation intensity is

In[1779]:= eq["15.5.png"]

Out[1779]=

$$\frac{dI(\omega)}{d\Omega} \simeq \frac{e^2}{4\pi^2 c}|\Delta\boldsymbol{\beta}|^2 \sin^2\Theta, \qquad \omega\tau \ll 1$$

15.5

Here Θ is the angle between the outgoing radiation and the change of velocity; $\hat{n}\cdot\Delta\vec{\beta} = |\Delta\vec{\beta}| \cos\Theta$. The total intensity (i.e., integrated over directions) is

In[1780]:= eq["15.6.png"]

Out[1780]=

$$I(\omega) \simeq \frac{2}{3\pi}\frac{e^2}{c}|\Delta\boldsymbol{\beta}|^2, \qquad \omega\tau \ll 1$$

15.6

independent of ω.

For low frequencies the spectrum is a constant, i.e., *independent of frequency*.

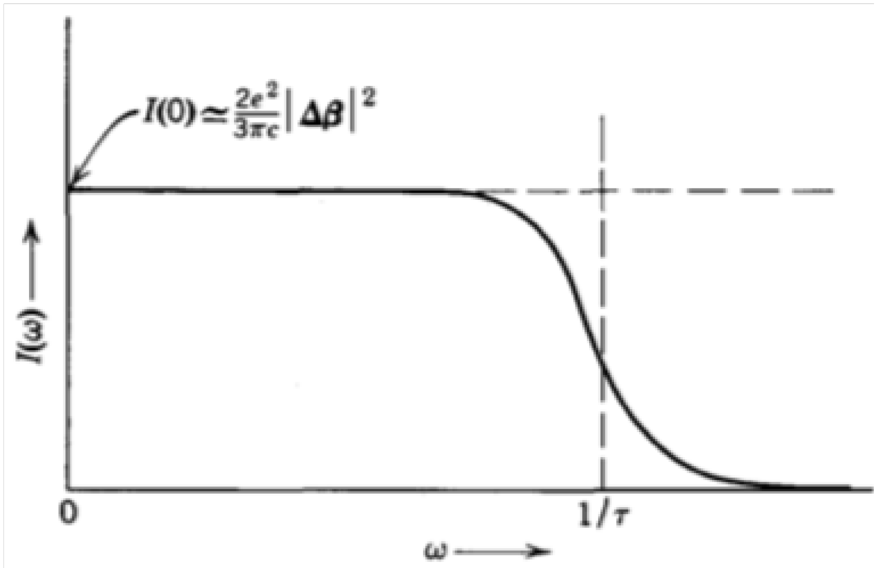
For high frequencies ...

... the exponential factor $e^{i\omega t}$ oscillates rapidly compared to the time dependence of $\dot{\boldsymbol{\beta}}$ so the integral is small; and thus we can neglect the radiation for $\omega\tau \gg 1$.

★ Figure 15.1. shows the frequency spectrum of radiation emitted in a collision of *duration* τ and *velocity change* $\Delta\vec{\beta}$.

In[1781]:= eq["fig15.1.png"]

Out[1781]=



A crude but useful approximation is just a step function,

In[1782]:= eq["15.7.png"]

Out[1782]=

$$I(\omega) = \begin{cases} \frac{2e^2}{3\pi c}|\Delta\beta|^2, & \omega\tau < 1 \\ 0, & \omega\tau > 1 \end{cases}$$

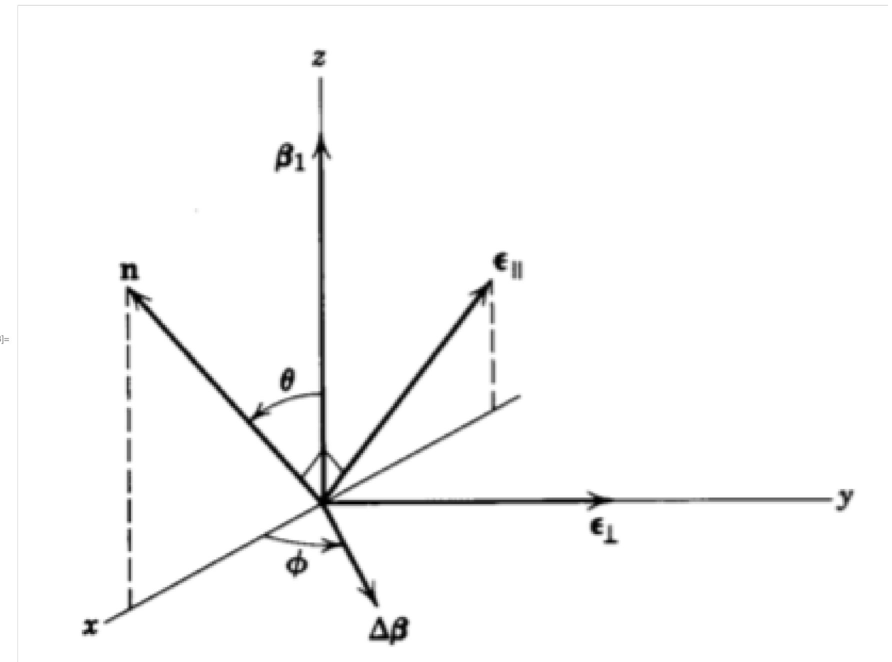
15.7

Polarization

Equations 15.1 and 15.5 are summed over polarizations. An interesting question is, what is the intensity for a specified polarization?

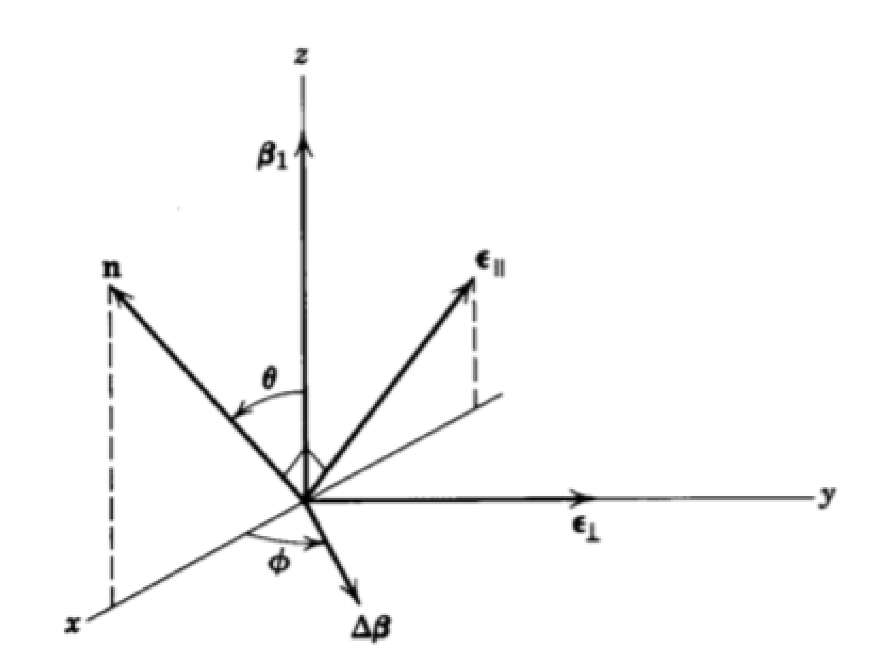
In[1783]:= eq["fig15.2.png"]

Out[1783]=



Consider the plane P spanned by the initial velocity $\vec{\beta}_1$ and the direction of outgoing radiation $\hat{n}$. Suppose $\overrightarrow{\Delta\beta}$ is perpendicular to $\vec{\beta}_1$. Use Figure 15.2 below to define coordinate axes x,y,z.

Out[•]=



We may define two orthogonal polarization vectors, $\vec{\epsilon}_{par}$ and $\vec{\epsilon}_{perp}$; these are parallel

and perpendicular to the plane P = the xz-plane.

$\hat{e}_{perp} \bullet \hat{n} \times (\hat{n} \times \hat{\Delta}) = - \sin\phi$

$\hat{e}_{par} \bullet \hat{n} \times (\hat{n} \times \hat{\Delta}) = \cos\theta \cos\phi$

In[1784]:=

```
(* calculations *)
nh = {Sin[θ], 0, Cos[θ]};
eperp = {0, 1, 0};
epar = Cross[nh, eperp] // TrigExpand;
Delta = {Cos[ϕ], Sin[ϕ], 0};
Dot[eperp, Cross[nh, Cross[nh, Delta]]] //
 TrigExpand
Dot[epar, Cross[nh, Cross[nh, Delta]]] // TrigExpand
Dot[nh, Delta] // TrigExpand
Print["cosΘ = ", %]
```

Out[1788]= -Sin[ϕ]

Out[1789]= Cos[θ] Cos[ϕ]

Out[1790]= Cos[ϕ] Sin[θ]

cosΘ = Cos[ϕ] Sin[θ]

Square and average over $\phi \Longrightarrow$

In[1792]:= `eq["15.9.png"]`

Out[1792]=

$$\left.\begin{aligned}\frac{dI_{\parallel}(\omega)}{d\Omega} &= \frac{e^2}{8\pi^2 c}|\Delta\boldsymbol{\beta}|^2\cos^2\theta \\ \frac{dI_{\perp}(\omega)}{d\Omega} &= \frac{e^2}{8\pi^2 c}|\Delta\boldsymbol{\beta}|^2\end{aligned}\right\}$$

15.9

- Eqs. (15.9) “have been verified for X-rays produced by electrons with kinetic energies in the kilovolt range”
- The sum of the two polarizations is Eq. 15.5;

$1+\cos^2\theta = 2 - \sin^2\theta = 2 - 2\cos^2\Theta = 2\sin^2\Theta$.