PHY 842 SUMMARY

The purpose of a course like PHY 842 is to learn...

- first principles
- math methods
- examples

The course had many subjects:

CH9 Electromagnetic waves & optics

+ Frequency dependence of polarization; $\epsilon(\omega)$

CH11 Radiation by systems; $\vec{J}(x,t) \longrightarrow dP/d\Omega$

+ Radiation by charges; $\vec{r}(t) \longrightarrow \vec{E}$ and $\vec{B} \longrightarrow dP/d\Omega$

- → general results
- → Larmor's formula
- → synchrotron radiation
- → Cherenkov radiation
- → Bremsstrahlung

CH12 Scattering of light

+ Diffraction of light

FIRST PRINCIPLES FOR EVERYTHING

Microscopic Maxwell's equations

$\nabla \cdot \vec{E} = 4\pi \rho$ and $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{c\, \partial t}$

$\nabla \cdot \vec{B} = 0$ and $\nabla \times \vec{B} = \frac{4\pi}{c} \vec{J} + \frac{\partial \vec{E}}{c\, \partial t}$

gaussian units

First Principles for continuous media

Macroscopic Maxwell's equations

$\nabla \cdot \vec{D} = 4\pi \rho_{free}$ and $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{c\, \partial t}$

$\nabla \cdot \vec{B} = 0$ and $\nabla \times \vec{H} = \frac{4\pi}{c} \vec{J}_{free} + \frac{\partial \vec{D}}{c\, \partial t}$

,

BUT what are $\vec{D}$ and $\vec{H}$?

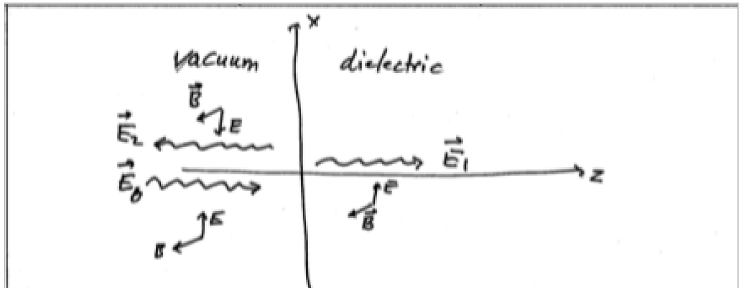
They come from $\vec{P}$ and $\vec{M}$ (how?)

& that leads to ϵ and μ (how?)

You'll need to know this for the final exam.

Optics

Light is normally incident, from air, on the planar surface of a dielectric medium. What fraction of the energy reflects from the surface?



$$\vec{E}_0 = E_0 \hat{e}_x e^{i(kz-\omega t)}$$

$$\vec{H}_0 = \vec{B}_0 = E_0 \hat{e}_y e^{i(kz-\omega t)}$$

$$\vec{E}_1 = E_1 \hat{e}_x e^{i(k'z-\omega t)}$$

$$\vec{H}_1 = \vec{B}_1 = B_1 \hat{e}_y e^{i(k'z-\omega t)}$$

$$\vec{E}_2 = -E_2 \hat{e}_x e^{i(-kz-\omega t)}$$

$$\vec{H}_2 = \vec{B}_2 = E_2 \hat{e}_y e^{i(-kz-\omega t)}$$

Boundary Conditions: $E_0 - E_2 = E_1$

$E_0 + E_2 = B_1$

Questions:

(1) What is B_1? (2) What is k'?

The classical electron theory $\Longrightarrow \epsilon(\omega)$

How did we do that?

recall...

$$m\,\ddot{x} = -\,\gamma\,\dot{x} - m\,\omega_0^2\,\vec{x} \;-\, e\,\vec{E}_0\,e^{-i\,\omega\,t}$$

Re implied

$$\vec{P}(t) = -\,e\,\vec{x}(t)\,n_b$$

$\vec{P} = \chi_e(\omega)\,\vec{E}_0\,e^{-i\omega t}$ and $\epsilon = 1 + 4\pi\,\chi_e$

$\Longrightarrow$

$$\epsilon(\omega) = 1 + \frac{4\,\pi\,e^2}{m}\;\frac{n_b^2}{\omega_0^2 - \omega^2 - i\,\omega\,\gamma}$$

complex and frequency dependent, which has interesting consequences.

Plasma frequency?

For a plasma, $\omega_0 = 0$.

$$v_x(t) = \dot{x}(t) = \frac{-eE_0}{m(\gamma - i\omega)}\,e^{-i\omega t}$$

$$J_x(t) = n_c\,(-e\,v_x(t)) = \sigma(\omega)\,E_x(t)$$

$$\therefore\ \sigma(\omega) = \frac{e^2}{m}\;\frac{n_c}{\gamma - i\omega}$$

$\Longrightarrow$ general dispersion relation is

$$c^2\,k^2 = \omega^2\,\mu\,\{\,\epsilon(\omega) + 4\pi\,i\,\sigma(\omega)/\omega\,\}$$

exercise 9.5.5

,

For high frequencies,

$\epsilon_{eff}(\omega) = 1 - \dfrac{\omega_p^2}{\omega^2}$ where ω_p= ???

The general theory of radiation

The general theory applies to radiation, and also to scattering of light by microscopic particles.

Exercise 12.1.1. Use the model of Section 9.5 and the harmonic results in (11.112) to discuss the scattering from free or bound electrons in a medium in electric dipole approximation. Using a linearly polarized incoming wave and integrating over angles, show that one obtains the cross section

$$\sigma(\omega) \equiv \int d\Omega \left(\frac{dP}{d\Omega}\right)_{\text{avg}} / |\vec{S}_0| = \frac{8\pi}{3} r_e^2 f(\omega),$$

where $r_e = e^2/(mc^2)$ is called the *classical electron radius* (encountered again in Chapter 14), and where

$$f(\omega) \equiv \frac{\omega^4}{[(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2]},$$

which gives a resonance in the cross section at $\omega \approx \omega_0$. The low and high energy forms of this cross section are

$$\omega \ll \omega_0, \frac{\omega_0^2}{\gamma}: \quad \sigma(\omega) \approx \frac{8\pi}{3} r_e^2 \left(\frac{\omega}{\omega_0}\right)^2,$$

$$\omega \gg \omega_0, \gamma: \quad \sigma(\omega) \approx \frac{8\pi}{3} r_e^2.$$

The first result gives rise to so-called *Rayleigh scattering* and the second is called the *Thomson cross section*. In particular, Rayleigh scattering implies that sunlight is preferentially scattered at higher frequencies, resulting in our blue sky and red sunsets!

Lecture of October 26
The classical electron model of light scattering

- An electromagnetic wave impinges upon a molecule

The incoming E.M. wave is a plane wave, moving in the z direction, and linearly polarized in the x direction,

$$\vec{E}(\vec{x},t) = \hat{e}_x E_0 \; e^{i(kz - \omega t)}$$

● Motion of the electron(s)

Using the classical electron model, the motion of the negative charge is described by

$$m \frac{d^2 \vec{r}}{dt^2} = - K \vec{r} - \gamma \frac{d \vec{r}}{dt} - e E_0 \hat{e}_x e^{-i \omega t}$$

i.e., a damped driven oscillator.

The steady−state solution: the electron oscillates in the x direction, with the driving frequency ω;

$$\vec{r}(t) = \hat{e}_x e^{-i\omega t} A$$

where

$$A = \frac{- e E_0}{m(\omega_0^2 - \omega^2) - i\gamma \omega}$$

$\omega_0 = \sqrt{K / m}$ = the “natural frequency”.

● The electron radiates

Larmor's formula; the average power integrated over all directions is

$$P_{avg} = \frac{2\,e^2\,a^2}{3\,c^3} \text{ where } a^2 = \left\langle (\ddot{x})^2 \right\rangle.$$

be careful about taking the real part of x(t)

We have $A = A_1 + i\,A_2$, and so
$x(t) = \text{Re}\,\{\,e^{-i\omega t}\,A\,\} = A_1 \cos(\omega\,t) + A_2 \sin(\omega$

$t)$

$$\left\langle \ddot{x}^{\,2} \right\rangle .= \frac{1}{2}\,\omega^4\,(\,A_1^2 + A_2^2\,)$$

$$P_{avg} = \frac{e^2}{3\,c^3}\,\omega^4\,(\,A_1^2 + A_2^2\,)$$

$$= \frac{e^4\,E0^2\,\omega^4}{3\,c^3\,\left(\gamma^2\,\omega^2 + m^2\,\left(\omega^2 - \omega0^2\right)^2\right)}$$

● Scattering and the cross section

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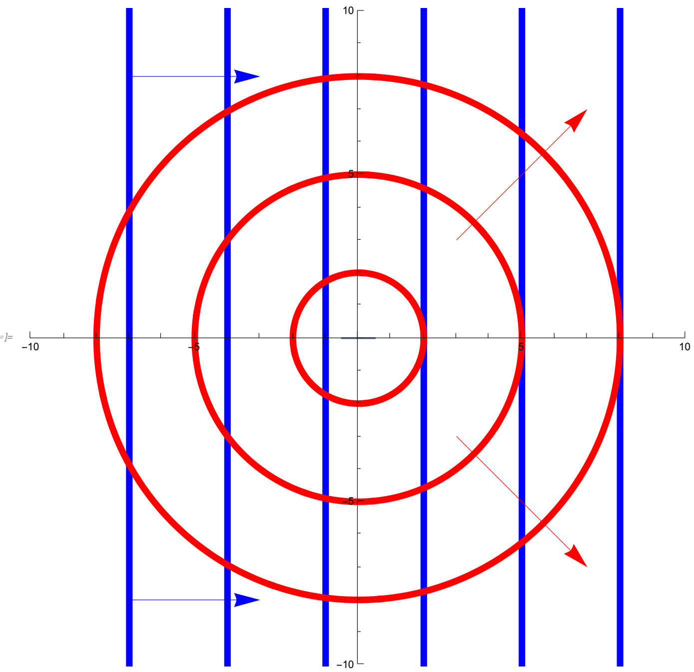

The definition of the scattering cross section is

$$\sigma = \frac{P}{S_{inc}}$$

where P = outgoing power averaged over time;

and S_{inc} = incoming intensity

≡ incident power per unit area averaged over time

$$S_{inc} = \frac{c}{4\pi} \langle \vec{E} \times \vec{B} \rangle = \frac{c}{8\pi} E_0^2$$

Thus,

$$\sigma = \frac{e^4 E_0^2 \omega^4}{3 c^3} \frac{1}{m^2(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2} \frac{8\pi}{c E_0^2}$$

$$\sigma = \frac{8\pi}{3} r_e^2 \frac{\omega^4}{(\omega_0^2 - \omega^2)^2 + (\gamma\omega/m)^2}$$

$r_e = \frac{e^2}{mc^2}$ = the "classical radius" of the electron

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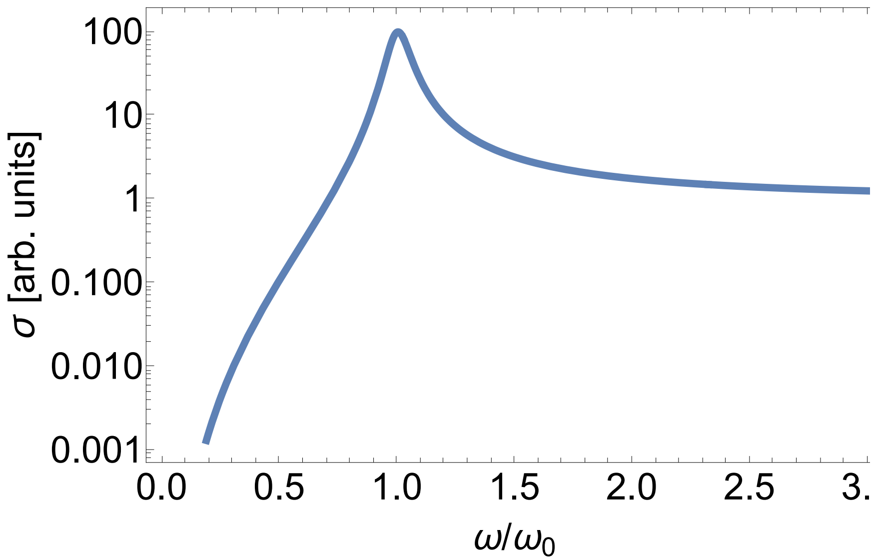

← Rayleigh →

← resonant →

← Thomson →