Assignment #5; part 1

Problem 5-1. (2 points)

For what wavelengths of light are metals transparent? (page 451)
Calculate a quantitative answer.

Solution 5-1.

Consider an example: silver.
Let ω_p be the plasma frequency. Electromagnetic waves propagate through the material if $\omega < \omega_p$.

In[9]:=
```
(* Calculations *)
{e, m} = { 4.8*^-10, 9.1*^-28 };
{ρ, A, u} = { 10.5, 107.9, 1.66*^-24 };
n = ρ / (A * u);
ωp = Sqrt[4 π e^2 n / m]
```

Out[12]= 1.3657×10^{16}

The plasma frequency is 13.7 PHz.
The corresponding wavelength in the electromagnetic spectrum is λ.

In[13]:=
```
f = ωp / (2 π); c = 3.0*^10; λ = c / f;
λ = λ * Ucm * (10 ^ 9 * Unm) / (10 ^ 2 * Ucm)
```

Out[14]= 138.021 Unm

λ = 138 nm, in the ultraviolet part of the electromagnetic spectrum.
Final answer: Silver is transparent for wavelengths < 138 nm.

Problem 5-2. (6 points)

(a) How do you know that AM radio waves reflect from the ionosphere?

(b) From this, calculate a bound on the electron density in the ionosphere.

(c) Explain, in one clearly written and legible paragraph, the ionosonde.

Solution 5-2.

(a) We can receive AM radio signals from very distant antennas, especially at night, because the AM waves reflect from the ionosphere.

(b) Reflection of EM waves by a plasma occurs if $\omega < \omega_p$ = the plasma frequency $= \{4\pi e^2 n / m\}^{1/2}$. The frequencies of AM radio waves are from 535 to 1605 kHz. Thus the electron density must have

$$n > m\,\omega / (4\pi e^2) \sim m\,(2\pi \times 1605\text{ kHz})^2 / (4\pi e^2).$$

In[15]:=
```
(* calculations *)
(9.1*^-28) * (2 * Pi * 1605*^3)^2 / (4 * Pi * (4.8*^-10)^2)
(9.1*^-28) * (2 * Pi * 535*^3)^2 / (4 * Pi * (4.8*^-10)^2)
```

Out[15]= 31 963.8

Out[16]= 3551.54

Thus the estimated electron density is $n \sim 3 \times 10^4\ \text{cm}^{-3}$.

(c) The ionosonde is a device that is used to measure the electron density in the ionosphere, as a function of height, by observing reflection of pulses of radio waves.

Problem 5-3. (4 points)

The density of matter in interstellar space is approximately one electron and one proton per cubic volume. What is the limit on the frequency of electromagnetic radiation that can propagate through such a medium without attenuation? Explain the result physically.

Solution 5-3.

$n = 1\ \text{cm}^{-3}$

Calculate the plasma frequency.

```
In[17]:= {e, m, n} = {4.8*^-10, 9.1*^-28, 1}
ωp = Sqrt[4 * Pi * e^2 * n / m]
fp = ωp / (2 π)
λp = 3*^10 / fp
Out[17]= {4.8 × 10^-10, 9.1 × 10^-28, 1}
Out[18]= 56406.
Out[19]= 8977.3
Out[20]= 3.34176 × 10^6
```

The plasma frequency is 9×10^3 Hz, and this is the lower limit on the frequency of electromagnetic radiation that can propagate through interstellar space without attenuation.

The corresponding *wavelengths* of EM radiation are $\lambda > 3.3 \times 10^6$ cm.
These are extremely long waves—much longer than the radio waves observed in radio astronomy. For example, the cosmic microwave background peaks at about 0.3 cm.