H. Assignment #9 Problem 9-3 = Exercise 11.8.3

In[1551]:=
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Show[Import["9-3.png", "png"], ImageSize → 640]
```

Out[1551]=

(11.211) and (11.214) may also be employed for any other complete particle trajectory. This problem gives an idea of the frequency distribution of radiation expected when the trajectory is neither impulsive, as in the sample at the end of Section 11.3, nor completely periodic. Consider particles approaching a circle of radius R with a quarter turn, half turn and full turn around the circle. Let $\omega_0 = v/R \Rightarrow R/c = \beta/\omega_0$ and the observation direction be $\hat{n} = (0, 1, 0)$:

(a) Quarter turn trajectory:

$$\vec{v}(t) = \begin{cases} v(-1, 0, 0), & t > \frac{\pi}{2\omega_0}, \\ v(-\sin(\omega_0 t), \cos(\omega_0 t), 0), & 0 < t < \frac{\pi}{2\omega_0}, \\ v(0, 1, 0), & t < 0. \end{cases}$$

(b) Half turn trajectory:

$$\vec{v}(t) = \begin{cases} v(0, -1, 0), & t > \frac{\pi}{\omega_0}, \\ v(-\sin(\omega_0 t), \cos(\omega_0 t), 0), & 0 < t < \frac{\pi}{\omega_0}, \\ v(0, 1, 0), & t < 0. \end{cases}$$

(c) Full turn trajectory:

$$\vec{v}(t) = \begin{cases} v(0, 1, 0), & t > \frac{2\pi}{\omega_0}, \\ v(-\sin(\omega_0 t), \cos(\omega_0 t), 0), & 0 < t < \frac{2\pi}{\omega_0}, \\ v(0, 1, 0), & t < 0. \end{cases}$$

For convenience in your evaluations choose a time and distance scale such that $\omega_0 = 1$, $c = 1$. In each case make qualitative and quantitative observations on the frequency spectrum. Make sure to check the low velocity ($\beta \ll 1$) and high velocity ($\beta \approx 1$) cases. Can you see a periodic spectrum begin to emerge?

In[2009]:=
```
Remove["Global`*"]
```

In[2089]:=
```
nq = 4;
ϕF = nq * Pi / 2;
RXC[β_, η_] := NIntegrate[Sin[ϕ] * Cos[η * ϕ - η * β * Sin[ϕ]], {ϕ, 0, ϕF}]
IXC[β_, η_] := NIntegrate[Sin[ϕ] * Sin[η * ϕ - η * β * Sin[ϕ]], {ϕ, 0, ϕF}]
```

In[2093]:=
```
λ1[β_] := Sin[ϕF] / (1 - β * Cos[ϕF]);
λ2[β_] := -β * Sin[ϕF] - 2 * β * ϕF * Cos[ϕF] + ϕF;
RY[β_, η_] := (1 / η) * λ1[β] * Cos[η * λ2[β]];
IY[β_, η_] := (1 / η) * λ1[β] * Sin[η * λ2[β]];
```

In[2097]:=
```
dP[β_, η_] := (RXC[β, η] + RY[β, η]) ^ 2 + (IXC[β, η] + IY[β, η]) ^ 2
```

In[2100]:=
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Do[
 bc = 0.1 * ic;
 fig[ic] = LogPlot[dP[bc, η], {η, 0, 10},
   PlotRange → {All, {1*^-3, 1*^5}}];
 Print[ic], {ic, 1, 9}]
ans[nq] = Show[Table[fig[ic], {ic, 1, 9}]]
```
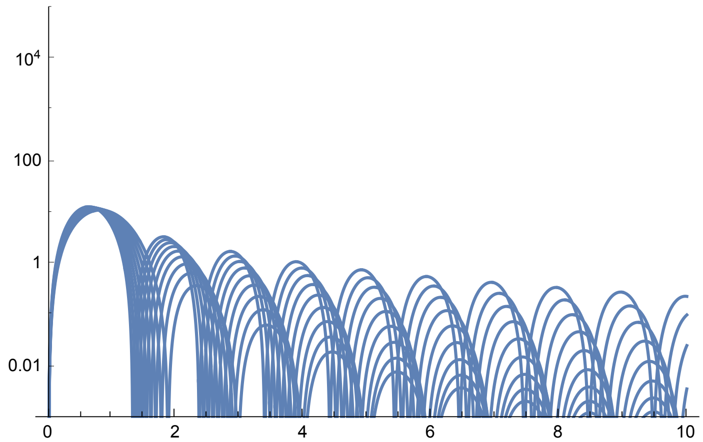
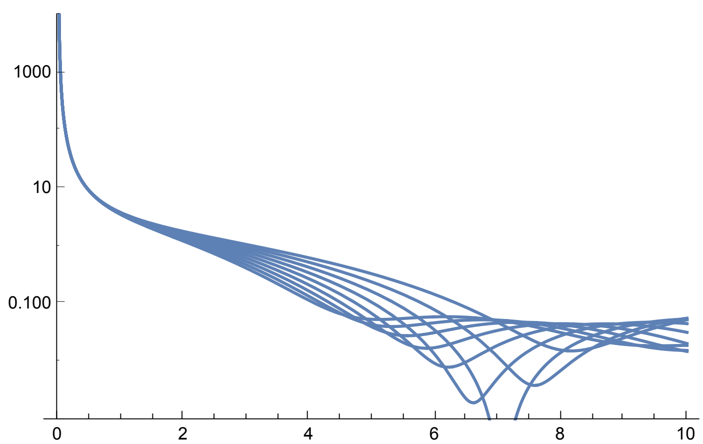

Out[2101]=



Part (a) : quarter turn

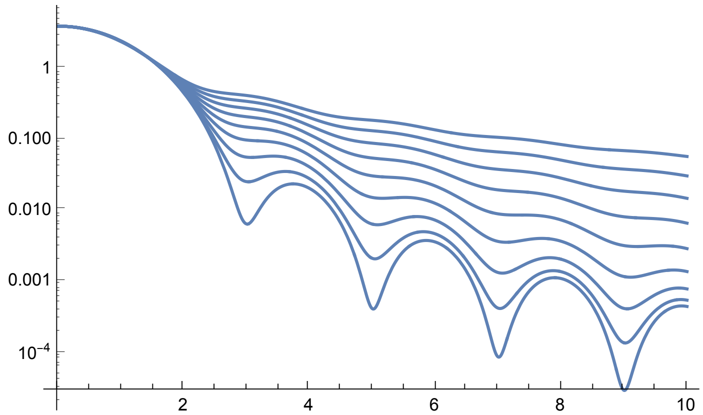
In[2073]:=
```
ans[1]
```

Out[2073]=



Part (b) : half turn

In[2088]:= ans[2]

Out[2088]=


Part (c) : full turn

In[2102]:= ans[4]

Out[2102]=