Experiment 1: Thin Converging Lens PHY 431

This experiment is a classic exercise in geometric optics. The goal is to measure the radius of curvature and focal length of a single *converging* lens from which you can calculate the index of refraction *n*. We shall explicitly consider the errors that accompany any measurement and how errors are analyzed to yield a quantitative estimate of uncertainty. This includes quantities derived from measurements, such as the index of refraction here.

In the procedures for this lab, you are explicitly reminded to estimate the uncertainty several times. (In future labs these reminders will not necessarily be included.) Please see Appendix (i) and (ii) for reference material and relevant equations. The questions, labeled **Q1**, **Q2**, … should be directly addressed in your write-up in the Analysis & Discussion section.

Procedure:

A. Choose a converging lens for the experiment. By definition, the focal length *f* of a lens is the image distance from the lens center, for an infinitely distant object. To obtain a rough estimate of *f*, project an image of some distant object, e.g. table or plant, in the atrium outside the lab onto the white paper. ***Q1.*** *Why does the object appear upside down?*
B. Use a spherometer to measure the radius of curvature of both surfaces of your lens. See Appendix (i). You begin by finding the "zero" position, x_0, using a scratch-free spot on your bench (which is a good approximation to a flat surface). Then perform the measurement with your lens in place, x_1; the distance *h* is then $|x_0-x_1|$. (Always be sure to include the uncertainty. In this case you can repeat the measurements a few times to obtain an estimate for the spherometer's precision, σ_{x0}.) Consult the manual for the spherometer. You can use the tables there. ***Q2.*** *Having estimated measurement uncertainties* σ_{x0} *and* σ_{x1}, *write an expression for* σ_h *and evaluate it using your data.*
C. Arrange an object (the T on the lamp window) and screen on the optical rail, with a separation greater than 4*f*. Locate the lens position which gives a sharp image on the screen. Record the object and image distances measured from the center of the lens. (Be sure to estimate the uncertainty for these distances.) Use the thin lens equation to calculate *f*. (Also calculate σ_f.) Repeat this for 4 positions of the screen increasing the object-screen separation in increments of about 2 cm. Find your best value for the focal length using the equation by the end of Appendix (ii) (the mean). ***Q3.*** *How does the best value for f compare to your original rough estimate?*
D. Insert a variable iris before/after the lens. Observe the image as the aperture size is changed. Specifically note how the aperture affects your ability to focus the image. ***Q4.*** *What is the meaning of the term "depth of field" in this context?*
E. Place the light source a distance less than *f* from the lens. Try to position the screen to bring the object into focus. ***Q5.*** *Any difficulties? What is going on here?*
F. Calculate the index of refraction (including uncertainty) for the glass of your lens using the lensmaker's equation. (Remember that your write-up should include comments as to whether or not your value is reasonable).

Appendix (i): Miscellaneous Equations

Thin Lens Equation:

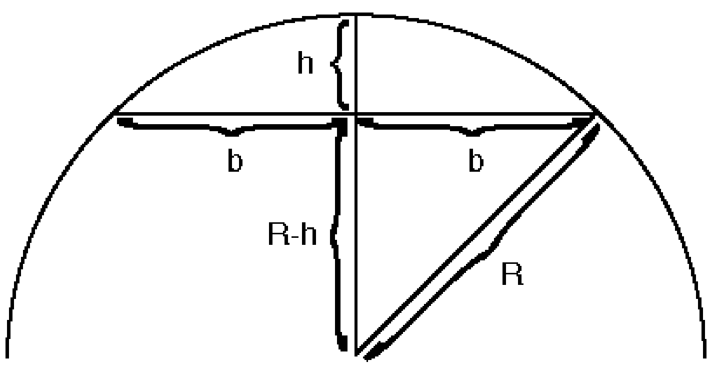
$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

Lensmaker's Equation:

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$, assuming that the index of refraction for the surrounding medium (air) is 1.000.

Spherometer Equation:

$$R=\frac{b^2}{2h}+\frac{h}{2}$$



From Pythagoras' Theorem: $R^2 = (R-h)^2 + b^2$

$$R^2 = R^2 - 2Rh + h^2 + b^2$$
$$R=\frac{b^2}{2h}+\frac{h}{2}$$

Appendix (ii): Elements of Error Analysis

Random fluctuations in the measurement process lead to a Gaussian distribution about the true value. This distribution gives us a parameter, σ, called the "standard deviation". (Systematic errors lead to a non-Gaussian distribution.) Essentially, if many measurements are taken, 68% of the data points lie within $x_0 \pm \sigma_x$, where x_0 is true value.

Now, suppose an arbitrary function f(x,y) depends on the variables x and y, assumed to be independent of each other. How do we compute the uncertainty in f, σ_f, given σ_y and σ_x? Under the assumption that the uncertainties are small compared to the range over which f significantly varies, the following expression works:

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \sigma_x{}^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y{}^2}$$

For exemplary specific functions this yields:

$$f = ax + by$$
$$\sigma_f = \sqrt{a^2\sigma_x^2 + b^2\sigma_y^2}$$

$$f = cxy$$
$$\frac{\sigma_f}{f} = \sqrt{\left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2}$$

$$f = cx^a y^b$$
$$\frac{\sigma_f}{f} = \sqrt{\left(\frac{a\sigma_x}{x}\right)^2 + \left(\frac{b\sigma_y}{y}\right)^2}$$

$$f = ce^{bx}$$
$$\frac{\sigma_f}{f} = b\sigma_x$$

$$f = ca^{bx}$$
$$\frac{\sigma_f}{f} = (b\ln a)\sigma_x$$

Lastly, we address the situation where we make N measurements of the same quantity x, each with an uncertainty of σ_x. Intuitively, we expect that combination of a number of measurements will yield uncertainty smaller than σ_x. In fact, if the fluctuations of measurements around the true value are uncorrelated, the estimated uncertainty in the average over measurements is reduced by $1/\sqrt{N}$ compared to individual measurements, when N is large:

$$\bar{x} = \frac{(x_1 + x_2 + \cdots + x_N)}{N} \rightarrow \sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}}$$

If σ_x is unknown, it can be estimated from the spread in measurements and formulaically from

$$\sigma_x^2 \approx \frac{\overline{x^2} - \bar{x}^2}{N - 1}$$

for large N, with the approximation improving as $N \rightarrow \infty$