

## LESSON 1 = INTRODUCTION      week of January 7

Reading: Napolitano Chapter 1

Topics:

- 1- get Mathematica
  - 2- Notebooks
  - 3- Numbers and operations
  - 4- Exercises
  - 5- Summary
- 

### (1) MATHEMATICA

You will be able to use Wolfram Alpha for some simple problems (most of Lesson 1) but you will need Mathematica on your laptop for more difficult problems.

#### ■ ■ [To use Wolfram Alpha, set up a Wolfram ID...](#)

- go to      URL : [www.wolframalpha.com](http://www.wolframalpha.com)
- create a Wolfram ID
- create a small file test.nb
- sign out
- go to your e-mail message "Wolfram Account" and follow the directions
- go to      URL: [www.wolframcloud.com](http://www.wolframcloud.com)
- click on Cloud Files

#### ■ ■ [Instructions for downloading a free Mathematica license and software onto your laptop...](#)

- Go to the web page at <https://cstore.msu.edu>, and follow the directions.
- Click Software
  - Student Use → search software
  - Browse → Wolfram → Mathematica
  - Wolfram Mathematica License → 2019 to August 2020 → add to cart
  - Go to cart → Checkout
  - E-mail address ; use your MSU email address

- Sign in
- Now fill out the long form, and continue...

It may take a few days to get the license. Watch your email for a message from Wolfram. Reply. Then watch your email for a message from the MSU computer store. Reply to that.

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## What is available in Mathematica?

Mathematica is a unified software package that can do many tasks in mathematics:

- numerical calculations  $\Rightarrow$  it is like a super calculator
- algebra  $\Rightarrow$  symbolic calculations
- plotting functions  $\Rightarrow$  graphical analysis of functions
- calculus  $\Rightarrow$  derivatives, integrals, differential equations
- statistics and data analysis

These tasks are often encountered in physics and astrophysics, because physics is a mathematical science.

So Mathematica is a powerful tool for students and research scientists in physics.

There exist other computer software packages that have some of the same capabilities as Mathematica. Examples are MATLAB, Maple, Python. However, for most practicing physicists, Mathematica is the preferred software. The reason is because Mathematica can do many varied tasks in one unified package.

Mathematica was originally developed by Steve Wolfram — a theoretical physicist. So physics has always been a central feature of the software. Just about anything that physicists need is included in Mathematica.

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## (2) Using the Mathematica Notebook Interface

Unlike a programming language, Mathematica is interactive. The interface between the user and the computer is called a “Notebook”.

Please understand the interactive nature of Mathematica, compared to a programming language:

Suppose you wanted to do a calculation using FORTRAN or C++ or Python (programming languages). Then you would need to go through many steps:

- open a text editor and type in the program;
- compile the program, i.e., convert the readable source file into machine language;
- run the program, which will produce output in one or more output files;
- then open the output file in a text editor to read the results;
- interpret the results using some other software;
- write a report with a word processor.

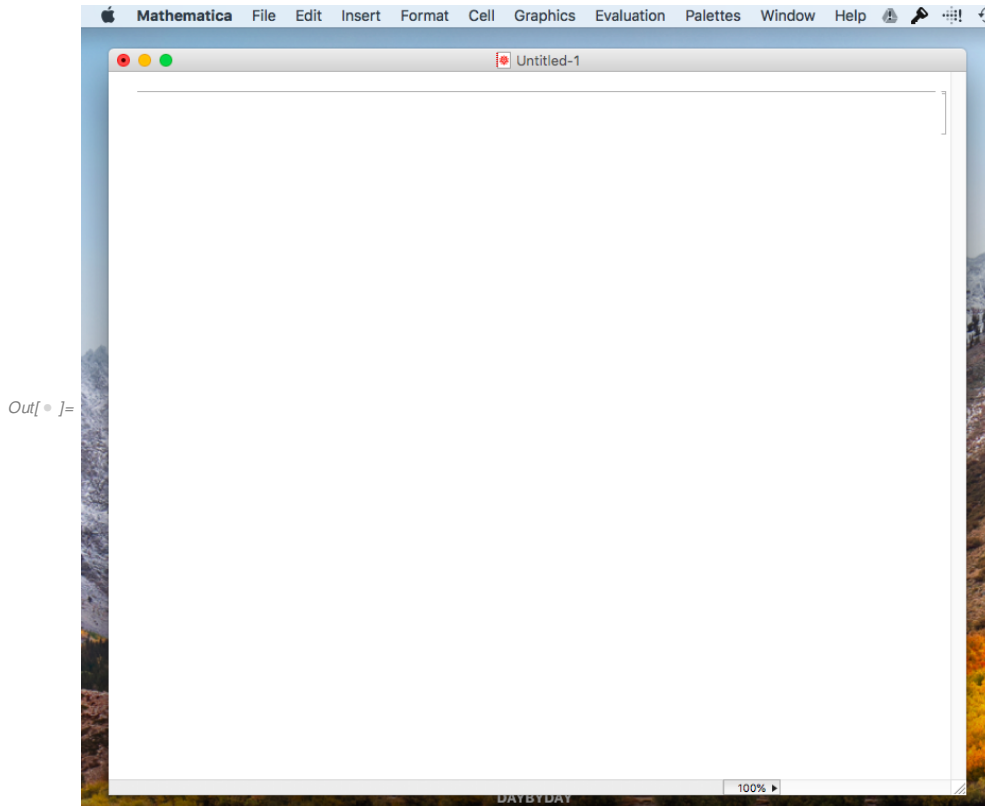
In contrast, Mathematica is interactive.

You can do all of these steps in one Notebook.

OK, now let's get started.

- Open a new Mathematica Notebook.

What do you get?



A blank window with a TOOLBAR at the top.

Place the cursor in the blank space; type  $2 + 2$  and then hit Shift-Enter. Now you should have two cells named In[1] and Out[1]. In[1] is an Input Cell, and it contains what you typed. Out[1] is an Output Cell and it contains the output from Mathematica.

Now place the cursor below Out[1], and type `Plot[Sin[x], {x, 0, 3*Pi}]`; make sure you type the square brackets and curly brackets exactly right; hit Shift-Enter to execute this new Input Cell.

Now you should have 4 cells, named In[1] .. Out[1] .. In[2] .. Out[2]. The input cells contain the commands you typed. The output cells contain the Mathematica responses.

Place the cursor below Out[2]. Type three lines (but do not hit Shift-Enter yet !)

:

```
a = 5  
b = 3  
Plot[a + b*x, {x, -5, 5}]
```

After you have typed all three lines, hit Shift-Enter.

What did you get ?

There should be a new Input Cell In[3] with three lines; and three Output Cells with responses. Each Output Cell contains the Mathematica response to a command in In[3].

OK, now you should have the general idea about how to use the Notebook Interface :

- In an Input Cell you type one or more commands.
- When you are ready, hit Shift-Enter.
- Mathematica will execute the commands that you typed and put the results into the Notebook as Output Cells. It does not delete the Input Cells; so as you work along you accumulate a series of calculations and their results.

To suppress output, end the command line with a semicolon (;).



Suppose you have a Notebook with commands and results that you want to save.

Click on the button 'File' in the TOOLBAR. You'll get a pull-down menu. To save the Notebook you are working on, click 'Save As'. You will be prompted for a name for the file. Save the Notebook to a file on your flash drive (or on a disk drive). Give it a name such as 'MyNotebook.nb'; or whatever name you want with the ending '.nb' .

Now when you close Mathematica, the notebook that you saved is stored on your flash drive (or disk drive) . If you want to work on it again later, you can open it when you restart Mathematica.

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## The Notebook Interface

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### ◆ The TOOLBAR

The TOOLBAR has 10 buttons; click a button; get a pull down menu; click on the thing you want to do in the pull down menu. Here are some important actions :

TOOLBAR COMMANDS	purpose
File > Save As	save a Notebook with new name
File > Save	save a notebook with the current name
File > Open	open an existing notebook
File > Print	print or print to PDF
Out[ ] = Edit > Cut,Copy,Paste	cut and paste when editing
Format > Style	select a style for a cell
Cell > Divide Cell	break a large cell into 2 parts
Cell > Merge Cell	merge 2 cells into 1
Evaluate > Evaluate Notebook	evaluate all input cells
Evaluate > Abort Evaluation	stop all running evaluations

Many other tasks can be done from the TOOLBAR, but those listed above are the most common.

#### **A comment about saving your work :**

If you are working on a Notebook, **SAVE YOUR WORK EVERY SO OFTEN!** If the computer crashes, you will lose anything that has not been saved. Do not assume that the computer will not crash! Every few minutes, or each time you complete something in a Notebook, save the Notebook.

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### ◆ The CELL BRACKETS

Note that every cell has a long bracket ( ] ) on the right side of the window.

When you click on the bracket, you "select" that cell.

Then you can do various things with the selection :

- delete the cell ⇒ select the cell, and hit the delete button
- merge two cells ⇒ select the two cells; merge them from the Cell TOOL

- cut or copy and paste a cell ⇒ select the cell; cut, paste, copy from the Edit TOOL

### (3) Numbers and Operations

For numerical calculations, Mathematica is a super calculator.

Mathematica can do numerical calculations, such as you could do with a calculator, but much more powerfully. By default, Mathematica does **exact** calculations, whereas your calculator only does approximate calculations to 9 or 10 significant figures. The calculator is accurate enough for most purposes, but Mathematica is easier to use, and more flexible and more powerful.

Here is an example. Find the solution to this equation:  $3x^2 - 5x - 1 = 0$ .

First solve the problem with your calculator, and write down the answer.

Now do the calculation using Mathematica. In an Input Cell, type

```
In[ ]:= (5 ± Sqrt[5 ^ 2 - 4 * 3 * (-1)]) / (2 * 3)
(* and now type Shift-Enter *)
```

Note that Mathematica gives the exact answer.

If you want a decimal approximation to 15 place accuracy, then type

```
In[ ]:= exact = (5 + Sqrt[5 ^ 2 - 4 * 3 * (-1)]) / (2 * 3)
approx = N[solution, 15](* and now type Shift-Enter *)
```

The result should agree with the result from your calculator.

#### **Example**

Solve this equation  $7x^2 - 5x + 1 = 0$ . Get both the exact answer and a decimal approximation to 6 place precision.

```
In[ ]:= {a, b, c} = {7, -5, +1}
sol = (-b + Sqrt[b^2 - 4*a*c]) / (2*a)
N[sol, 6]
```

Do you see why Mathematica is superior to a calculator?



Mathematica does exact mathematical calculations.

Mathematica can handle all kinds of numbers:

integers, rationals, irrational roots, transcendentals, complex numbers .

Mathematica can deal with them all.

Your calculator is very limited; it can only handle decimal approximations to reals.

### **Example**

Use Mathematica. Type these two numbers on two lines of an Input Cell:

`Sqrt[2]` and `Sqrt[2.0]` ; then hit Shift-Enter.

You should get two different results. **Explain why.**

```
In[ ]:= Sqrt[2]
        Sqrt[2.0]
```

Sometimes we need exact mathematics, but sometimes we only need decimal approximations. Type and execute the Mathematica examples below to get different kinds of information.

### **Example**

The number  $\pi$  is a transcendental number.

Use Mathematica to determine 100 digits of  $\pi$ .

(You can use either `N[Pi, 100]` or `SetPrecision[Pi, 100]` .)

```
In[ ]:= N[Pi, 100]
```

Can you do that with your calculator?

If Mathematica encounters a number that can be simplified, it will simplify it.

### **Example**

Use Mathematica to evaluate the square root of 72.

(Type and execute `r = Sqrt[72]`) . Also, verify that the result is correct by calculating `r*r`.

```

In[ ]:= r=Sqrt[72]
        r*r

```

### Example

Calculate the square root of 42,069,746,374,656.

Then apply the command **FactorInteger** to reduce the result to integer factors.

(First execute **?FactorInteger** in an Input Cell to get help about this command.)

```

In[ ]:= big = 42069746374656
        sqrt = Sqrt[big]
        {sqrt[[1]], sqrt[[2]]}
        FactorInteger[ 120736 ]
        FactorInteger[ 2886 ]

```

The answer is ...

$$\begin{aligned}
 & \sqrt{42,069,746,374,656} \\
 & = 2^5 \cdot 7^3 \cdot 11 \cdot \sqrt{2 \cdot 3 \cdot 13 \cdot 37}
 \end{aligned}$$

## (4) Exercises

Now do the exercises below to get some practice using Mathematica commands.

By doing these exercises, you will begin to learn how to use Mathematica as a super calculator.

Remember : save your work in case of a computer crash.

- **Exercise 1.** Calculate the square root of 13.  
Type `Sqrt[13]`, hit Shift-Enter.
- **Exercise 2.** Calculate the decimal approximation of the square root of 13, accurate to 10 significant figures.
- **Exercise 3.** Calculate the cosine of 1 radian.  
Type `Cos[1]`, hit Shift-Enter.
- **Exercise 4.** Calculate the decimal approximation of the cosine of 1 radian, accurate to 10 significant figures.
- **Exercise 5.** Calculate the two solutions of the equation  $11x^2 + 13x + 17 = 0$ .  
Type `Solve[11*x^2 + 13*x + 17 == 0, x]`.
- **Exercise 6.** Calculate the two solutions (for  $x$ ) of the equation  $Ax^2 + Bx + C = 0$  where  $A, B, C$  are unspecified constants.
- **Exercise 7.** Calculate the prime factors of 579.

The next three questions are examples from physics.

- **Exercise 8.** Imagine that you drop a small mass ( $m = 0.27$  kg) from a tower 10 meters high, in Earth's gravity. Calculate the time when the mass will hit the ground, neglecting the effect of air resistance.
- **Exercise 9.** One particle is moving on the  $x$  axis with constant velocity  $v_{1x} = 5$  m/s, and is decelerating with  $a_{1x} = -1$  m/s<sup>2</sup>. A second particle is moving parallel to the  $x$  axis, with  $y = 0.5$  m. (It might help to sketch a picture.)

At time  $t = 0$ :

the two particles are both at  $x = 0$ ;

the second particle has velocity  $v_{2x} = 0$ ;

and after  $t = 0$  the second particle accelerates with  $a_{2x} = + 2 \text{ m/s}^2$ .

Find the position  $x_{\text{pass}}$  where the second particle will pass the first particle.

Hint: Use the command **Solve**. First write expressions for  $x_1$  and  $x_2$  and then execute `Solve[x1==x2, t]`.

- **Exercise 10.** Calculate the mean speed of the Earth as it revolves around the sun. Express the answer in km/s and use appropriate significant figures.

Hints: The mean radius of the Earth's orbit is  $149.6 \times 10^6$  km. You can enter the number into Mathematica as either `149.6*^6` or `149.6*10^6`. The period of revolution is 365.25 days.

## (5) Summary of Lesson 1

Now you should understand:

- Exact numbers and decimal approximations
- The function **Sqrt[x]**
- The function **Cos[x]**; x is in radians
- The commands **N[ r ,d ]** and **SetPrecision[ r ,d ]**
- Every Mathematica command starts with a capital letter.
- Every built-in Mathematica function starts with a capital letter.
- For every Mathematica function, the independent variable is enclosed in square brackets.
- To get help, **?Command**

In[ ]:=

```
? Sqrt  
? Cos  
? N
```

Out[ ]:=

Symbol i

Sqrt[z] or  $\sqrt{z}$  gives the square root of  $z$ .

▼

Out[ ]:=

Symbol i

Cos[z] gives the cosine of  $z$ .

▼

Out[ ]:=

Symbol i

N[*expr*] gives the numerical value of *expr*.  
N[*expr*, *n*] attempts to give a result with *n*-digit precision.

▼