Polarization Tutorial

CVI Laser Optics

Polarization States

Four numbers are required to describe a single plane wave Fourier component traveling in the +z direction. These can be thought of as the amplitude and phase shift of the field along two orthogonal directions.

a. Cartesian Representation

$$E = (xE_x e^{i\phi x} + yE_y e^{i\phi y})\, e^{i(kz-\omega t)}$$

This is the simplest representation to think about. E_x, E_y, ϕ_x, and ϕ_y are four real numbers describing the magnitudes and phases of field components along two orthogonal unit vectors x and y. If the origin of time is irrelevant, only the relative phase shift

$$\phi = \phi_x - \phi_y$$

need be specified.

b. Circular Representation

In the circular representation, we resolve the field into circularly polarized components. The basic states are represented by the complex unit vectors

$$e_+ = (1/\sqrt{2})(x + iy)$$
$$e_- = (1/\sqrt{2})(x - iy)$$

$\mathbf{e}_+$ is the unit vector for *left circularly polarized* light; for *positive helicity* light; for light that *rotates counterclockwise* in a fixed plane as viewed facing into the light wave; and for light whose electric field rotation *obeys the right hand rule* with thumb pointing in the direction of propagation.

$\mathbf{e}_-$ is the unit vector for *right circularly polarized* light; for *negative helicity* light; for light that *rotates clockwise* in a fixed plane as viewed facing into the light wave; and for light whose electric field rotation *disobeys the right hand rule* with thumb pointing in the direction of propagation.

As in the case of the Cartesian representation, we write:

$$\mathbf{E} = (\mathbf{e}_+ E_+ e^{i\phi_+} + \mathbf{e}_- E_- e^{i\phi_-})\, e^{i(kz-\omega t)}$$

where E_+, E_-, ϕ_+, and ϕ_- are four real numbers describing the magnitudes and phases of the field components of the left and right circularly polarized components. Note that

$$E_+ = e_- \bullet E$$
$$E_- = e_+ \bullet E$$

c. Elliptical Representation

An arbitrary polarization state is generally elliptically polarized. This means that the tip of the electric field vector will describe an ellipse, rotating once per optical cycle.

Let a be the semimajor and b be the semiminor axis of the polarization ellipse. Let ψ be the angle that the semimajor axis makes with the X-axis. Let ξ and η be the axes of a right-handed coordinate system rotated by an angle $+\psi$ with respect to the X-axis and aligned with the polarization ellipse as shown in the diagram below.

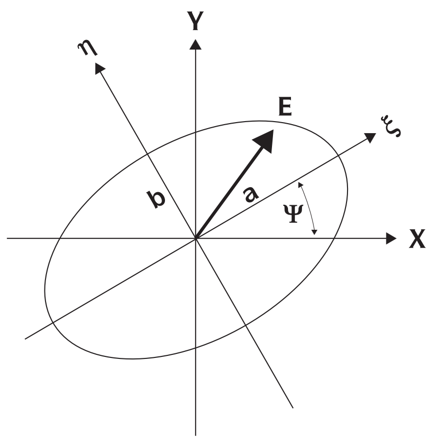

Figure 1. The polarization ellipse.

The elliptical representation is:

$$E = (a\hat{\xi} + b\hat{\eta})e^{i\delta_0}\, e^{i(kz-\omega t)}$$

Note that the phase shift δ_0 above is required to adjust the time origin, and the parameter ψ is implicit in the rotation of the ξ, η axes with respect to the X, Y axes.

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To summarize the three representations:

Representation	Complex Field Amplitude	Parameters Specifying Polarization State
Cartesian	$xE_x e^{i\phi_x} + yE_y e^{i\phi_y}$	E_x, ϕ_x, E_y, ϕ_y
Circular	$e_+ E_+ e^{i\phi_+} + e_- E_- e^{i\phi_-}$	E_+, ϕ_+, E_-, ϕ_-
Elliptical	$(a\hat{\xi} + b\hat{\eta})e^{i\delta_0}$	a, b, ψ, δ_o

Conversion Between Representations

For brevity, we will provide only the Cartesian to Circular and Cartesian to Elliptical transformations. The inverse transformations are straightforward. We define the following quantities:

$$\begin{aligned}
g_1 &= E_x \cos\phi_x - E_y \sin\phi_y \\
g_2 &= E_x \sin\phi_x + E_y \cos\phi_y \\
g_3 &= E_x \cos\phi_x + E_y \sin\phi_y \\
g_4 &= E_x \sin\phi_x - E_y \cos\phi_y \\
u &= +\left[(g_1)^2 + (g_2)^2\right]^{1/2} \\
v &= +\left[(g_3)^2 + (g_4)^2\right]^{1/2} \\
\phi_{12} &= \text{atan}\,(g_1, g_2) \\
\phi_{34} &= \text{atan}\,(g_3, g_4)
\end{aligned}$$

In the above, atan(x,y) is the four quadrant arc tangent function. This means that atan(x,y) = atan(y/x) with the provision that the quadrant of the angle returned by the function is controlled by the signs of both x and y, not just the sign of their quotient;for example, if $g_2 = g_1 = -1$, then ϕ_{12} above is $5\pi/4$ or $-3\pi/4$, not $\pi/4$.

a. Cartesian to Circular Transformation

$$\begin{aligned}
E_+ &= v/\sqrt{2} \\
E_- &= u/\sqrt{2} \\
\phi_+ &= \phi_{34} \\
\phi_- &= \phi_{12}
\end{aligned}$$

b. Cartesian to Elliptical Transformation

$$\begin{aligned}
a &= 1/2\,(u + v) \\
b &= 1/2\,(-u + v) \\
\psi &= 1/2\,(\phi_{12} - \phi_{34}) \\
\delta_0 &= 1/2\,(\phi_{12} + \phi_{34})
\end{aligned}$$

Linear Polarizers

A linear polarizer is a device that creates a linear polarization state from an arbitrary input. It does this by removing the component orthogonal to the selected state. Some polarizers reflect the rejected state, creating a new, usable beam. Examples are the CVI Glan Laser Polarizers, Thin Film Polarizers, and many types of polarizing beamsplitter cubes. Others may turn the rejected beam into heat. As do polaroid sheet and Polarcor™ polarizers. Still others may refract the two polarized beams at different angles, thereby separating them. Examples are Wollaston and Rochon prism polarizers.

Suppose the pass direction of the polarizer is determined by unit vector p. Then the transmitted field E_2, in terms of the incident field E_1, is:

$$E_2 = p(p \bullet E_1)$$

where the phase shift of the transmitted field has been ignored.

A real polarizer has a pass transmission, $T_{\parallel}$, less than 1. The transmission of the rejected beam, $T_{\perp}$, may not be 0. If **r** is a unit vector along the rejected direction, then

$$E_2 = (T_{\parallel})^{1/2}\, p(p \bullet E_1)e^{i\phi_{\parallel}} + (T_{\perp})^{1/2}\, r(r \bullet E_1)e^{i\phi_{\perp}}$$

In the above, the phase shifts along the two directions must be retained. Similar expressions could be arrived at for the rejected beam. If θ is the angle between the field E_1 and the polarizer pass direction p, the above equation predicts for the transmission:

$$T = T_{\parallel} \cos^2\theta + T_{\perp} \sin^2\theta$$

The above equation shows that when the polarizer is aligned so that $\theta = 0$, $T = T_{\parallel}$. When it is "crossed", $\theta = \pi/2$, and $T = T_{\perp}$. The extinction ratio is $\varepsilon = T_{\parallel} / T_{\perp}$. A polarizer with perfect extinction has $T_{\perp} = 0$, and thus $T = T_{\parallel} \cos^2\theta$, is a familiar result. Because $\cos^2\theta$ has a broad maximum as a function of orientation angle, setting a polarizer at a maximum of transmission is generally not very accurate. One has to either map the $\cos^2\theta$ with sufficient accuracy to find the $\theta = 0$ point, or do a null measurement at $\theta = \pm\,\pi/2$.

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Waveplates

Waveplates operate by imparting unequal phase shifts to orthogonally polarized field components of an incident wave. This causes the conversion of one polarization state into another.

There are two cases. With linear birefringence, the index of refraction and hence phase shift differs for two orthogonally polarized linear polarization states. This is the operation mode of standard waveplates.

With circular birefringence, the index of refraction and hence phase shift differs for left and right circularly polarized components. This is the operation mode of polarization rotators.

a. Standard Waveplates: Linear Birefringence

Suppose a waveplate made from a uniaxial material has light propagating perpendicular to the optic axis. This makes the field component parallel to the optic axis an extraordinary wave and the component perpendicular to the optic axis an ordinary wave. If the crystal is positive uniaxial, $n_e > n_o$, then the optic axis is called the slow axis, which is the case for crystal quartz. For negative uniaxial crystals, $n_e < n_o$, the optic axis is called the fast axis.

The equation for the transmitted field E_2, in terms of the incident field E_1 is:

$$E_2 = s(s \bullet E_1)e^{i\phi_s} + f(f \bullet E_1)e^{i\phi_f}$$

where s and f are unit vectors along the slow and fast axes. This equation shows explicity how the waveplate acts on the field. Reading from left to right, the waveplate takes the component of the input field along its slow axis and appends the slow axis phase shift to it. It does a similar operation to the fast component.

The slow and fast axis phase shifts are given by:

$$\phi_s = n_s(\omega)\omega t/c = 2\pi n_s(\lambda)t/\lambda$$
$$\phi_f = n_f(\omega)\omega t/c = 2\pi n_f(\lambda)t/\lambda$$

where n_s and n_f are, respectively, the indices of refraction along the slow and fast axes, and t is the thickness of the waveplate.

To further analyze the effect of a waveplate, we throw away a phase factor lost in measuring intensity, and assign the entire phase delay to the slow axis:

$$E_2 = s(s \bullet E_1)e^{i\phi} + f(f \bullet E_1)$$
$$\phi = \phi_s - \phi_f = 2\pi(n_s(\lambda) - n_f(\lambda))t/\lambda$$
$$= 2\pi\Delta n(\lambda)t/\lambda$$

In the above, $\Delta n(\lambda)$ is the birefringence $n_s(\lambda) - n_f(\lambda)$. The dispersion of the birefringence is very important in waveplate design; a quarter waveplate at a given wavelength is never exactly a half waveplate at half that wavelength.

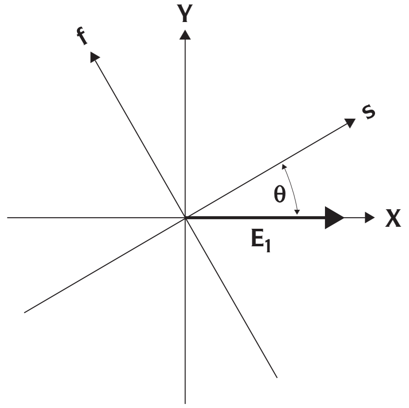

Figure 2. Orientation of the slow and fast axes of a waveplate with respect to an X-polarized input field.

Let E_1 be initially polarized along X, and let the waveplate slow axis make an angle θ with the X-axis. This orientation is shown in Figure 2. When the waveplate is placed between parallel and perpendicular polarizers the transmissions are given by:

$$T_{\parallel} \propto |E_{2x}|^2 = 1 - \sin^2 2\theta \sin^2 \phi/2$$
$$T_{\perp} \propto |E_{2y}|^2 = \sin^2 2\theta \sin^2 \phi/2$$

Note that θ is only a function of the waveplate orientation, and φ is only a function of the wavelength, the birefringence is a function of wavelength and the plate thickness.

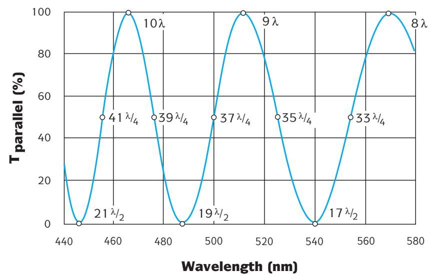

Figure 3. Transmission of a 0.5 mm thick crystal quartz waveplate between parallel polarizers.

For a *full waveplate*:
$\phi = 2m\pi$, $T_{\parallel} = 1$, and $T_{\perp} = 0$, regardless of waveplate orientation.

For a *half waveplate*:
$\phi = (2m + 1)\pi$, $T_{\parallel} = \cos^2 2\theta$, and $T_{\perp} = \sin^2 2\theta$. This transmission result is the same as if an initial linearly polarized wave were rotated through an angle 2θ. Thus, a half waveplate finds use as a polarization rotator.

For a *quarter waveplate*:
$\phi = (2m + 1)\pi/2$; ie. an odd multiple of $\pi/2$. To analyze this, we have to go back to the field equation. Assume that the slow and fast axis unit vectors s and f form a right handed coordinate system such that s x f = +z, the direction of propagation. To obtain circularly polarized light, linearly polarized light must be aligned midway between the slow and fast axes. There are four possibilities listed in the table below.

Phase Shift	Input Field Along $(\mathbf{s} + \mathbf{f})/\sqrt{2}$	Input Field Along $(\mathbf{s} - \mathbf{f})/\sqrt{2}$
$\phi = \pi/2 + 2m\pi$	RCP	LCP
$\phi = 3\pi/2 + 2m\pi$	LCP	RCP

Sometimes, waveplates described by the second line above are called 3/4 waveplates. For multiple order waveplates, CVI permits the use of either of the above classes of waveplates to satisfy the requirements of a quarter waveplate.

b. Multiple Order Waveplates

For the full, half, and quarter waveplate examples given in the preceeding section, the order of the waveplate is given by the integer m. For m > 0, the waveplate is termed a *multiple order waveplate*. For m = 0, we have a *zero order waveplate*.

The birefringence of crystal quartz near 500nm is approximately 0.00925. Consider a 0.5mm thick crystal quartz waveplate. A simple calculation shows that this is useful as a quarter waveplate for 500nm; in fact, it is a $37\lambda/4$ waveplate at 500nm with m = 18. Multiple order waveplates are inexpensive, high damage threshold retarders. Further analysis shows that this same 0.5mm plate is a $19\lambda/2$ half waveplate at 488.2nm and a 10λ full waveplate at 466.5nm. The transmission of this plate between parallel polarizers is shown in Figure 3 as a function of wavelength. The retardance of the plate at various key points is shown. Note how quickly the retardance changes with wavelength. Because of this, multiple order waveplates are generally useful only at their design wavelength.

c. Zero Order Waveplates

As discussed above, multiple order waveplates are not useful with tunable or broad bandwidth sources (example: femtosecond lasers). A zero order waveplate can greatly improve the useful bandwidth in a compact, high damage threshold device.

As an example, consider the design of a broadband half waveplate centered at 800nm. Maximum tuning range is obtained if the plate has a single π phase shift at 800nm. If made from a single plate of crystal quartz, the waveplate would be about 45µm thick, which is too thin for easy fabrication and handling. The solution is to take two crystal quartz plates differing in thickness by 45 µm and align them with the slow axis of one against the fast axis of the other. The net phase shift of this zero order waveplate is π. The two plates may be either air-spaced or optically contacted. The transmission of an 800nm zero order half waveplate between parallel polarizers is shown in Figure 4 using a 0-10% scale. Its extinction is better than 100:1 over a bandwidth of about 95 nm centered at 800nm.

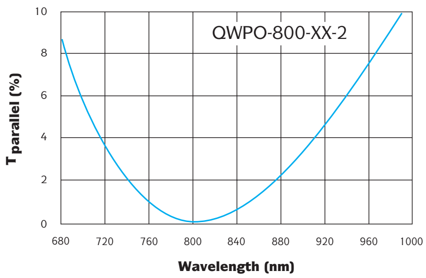

Figure 4. Zero order crystal quartz half waveplate for 800nm.

CVI produces multiple order and zero order crystal quartz waveplates at any wavelength between 213nm and 2020nm. Virtually all popular laser wavelengths are kept in stock, and custom wavelength parts are available with short delivery time.

CVI's line of MWPS Series Mica Waveplates are an inexpensive zero order waveplate solution. They are useful in low power applications and in detection schemes.

CVI's FR Series Fresnel Rhombs are available in quarter and half waveplate configurations. The retardance effect is produced by the differing phase shifts for S and P-polarized total internal reflection within a glass or fused silica rhomb.

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d. Achromatic Waveplates

At 500 nm, a crystal quartz zero order half waveplate has a retardation tolerance of λ/50 over a bandwidth of about 50nm. This increases to about 100nm at a center wavelength of 800nm. However, there is a waveplate design that can give an even larger bandwidth.

If two different materials are used to create a zero or low order waveplate, cancellation can occur between the dispersions of the two materials. This requires a judicious choice of thicknesses. Then, the net birefringent phase shift can be held constant over a much wider range than in waveplates made from one material. The ACWP-series crystal quartz and MgF_2 Achromatic Waveplates are used in high power, air-spaced designs.

Three wavelength ranges are available in both quarter and half wave retardances. Retardation tolerance is better than λ/100 over the entire wavelength range. We plot the intensity transmission and the actual phase shift for each design versus wavelength. For quarter waveplates, perfect retardance is a multiple of 0.25 waves, and transmission through a linear polarizer must be between 33% and 67%. (In all but the shortest wavelength design, quarter wave retardation tolerance is better than λ/100.) For half waveplates, perfect retardance is 0.5 waves, while perfect transmission through a linear polarizer parallel to the intitial polarization state should be zero. The curves on page ▶ **228** demonstrate the high degree of achromatization achievable by the dual material design. In addition, our use of thin plates of low dispersion material assure low group velocity dispersion in ultrashort pulse applications.

e. Dual Wavelength Waveplates

Dual wavelength waveplates have a number of applications. One common appliction is separation of different wavelengths with a polarization beam splitter by rotating the polarization of one wavelength by 90°, and leaving the other unchanged. This application frequently occurs in nonlinear doubling or tripling laser sources such as Nd:YAG (1064/532/355/266).

CVI Laser Corporation's QWPD line of dual wavelength waveplates achieves the multiple retardation specifications through judicious selection of a multiple order waveplate that meets both wavelength/retardation conditions. Unfortunately, this often results in a solution that corresponds to a relatively high order. This in turn causes the QWPD to have a narrow bandwidth, and makes it somewhat sensitive to temperature variation.

In single waveplate applications, a more robust approach is to combine two quartz waveplates with their optical axes orthogonal to one another. In this configuration, the temperature dependence is a function of the thickness difference between the waveplates, resulting in excellent temperature stability. The retardation of the compound waveplate is also a function of the thickness difference, so a zero order waveplate can be realized, resulting in a very wide bandwidth.

f. Low Order Dual Wavelength Waveplates

Low order dual wavelength waveplates offer the temperature and stability of a zero order waveplate while still meeting the specified retardation at two different wavelengths. These high energy air-spaced waveplates, made of magnesium flouride and crystal quartz, are perfect for OPOs, spectrophotometry, femtosecond pulses, and continuum generation. In addition, low order dual waveplates can rotate the polarization of a single wavelength while unaffecting the polarization of a second wavelength.

g. Savart Plates

A Savart plate is actually two calcite plates of equal thickness that are cut parallel to their natural cleavage faces, rotated, and cemented together at right angles. This creates an equal axis of separation of S and P-polarizations upon transmission, forming interferance fringes. Common applications for these plates include the Savart Polariscope and wavefront shearing interferometers. Savart plates are available by special request. Contact a CVI applications engineer for more information.

h. Soleil-Babinet Compensator

Soleil-Babinet compensators produce a phase-change consistent through its field. This is achieved using two opposed quartz wedges of equal angle. One wedge is translated along its length by a micrometer screw. Then, both wedges are cut leaving the fast directions along and perpendicular to the direction of motion, respectively. The phase-change consistency of the Soleil-Babinet compensation is most commonly used to measure retardation and to determine birefringence. To special order these compensators, contact a CVI applications engineer.