Homework Assignment #1 (16 points total)

(1-1) N_e = # of electrons in a $\underbrace{\text{liter of water}}_{\text{Volume} = 1L}$

density $\rho = 10^3$ g/L

mol. weight $W = 18$ g/mol

(mol $= 6.02 \times 10^{23}$)

each HOH molecule has 10 electrons

$$N_e = \frac{\rho}{W} \times 10 \times V = 3.34 \times 10^{26}$$

(2)

(1-2) Exact values

obsolete

$\mu_0 = 4\pi \times 10^{-7}$ N/A² (until May 20, 2019)

$$\mu_0 = 1.256\ 637\ 062\ 12(19) \times 10^{-6} \text{ N/A}^2$$

$$= 2\alpha \frac{h}{e^2 c}$$ current (after May 20, 2019)

$$\epsilon_0 = \frac{1}{\mu_0 c^2}$$

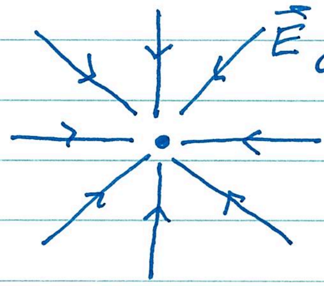
Approx. values $\mu_0 \approx 1.256\ 637 \times 10^{-6}$ N/A²

$\epsilon_0 \approx 8.854 \times 10^{-12}$ A² s⁴ kg⁻¹ m⁻³

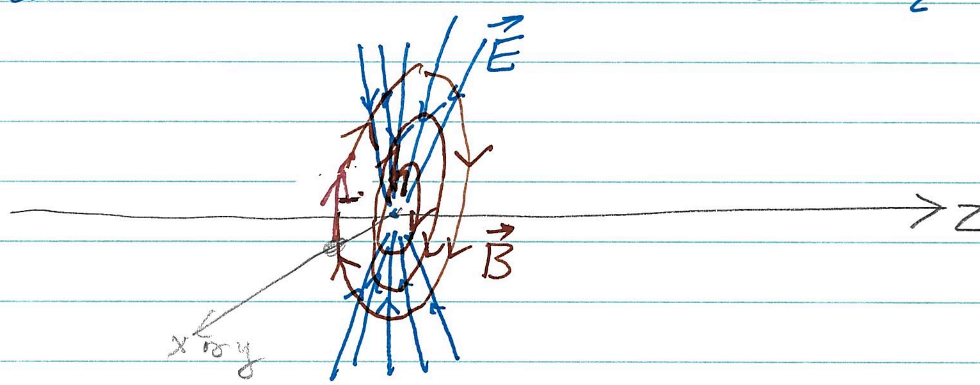
with SI units. (4)

(1-3)

(a) an electron at rest



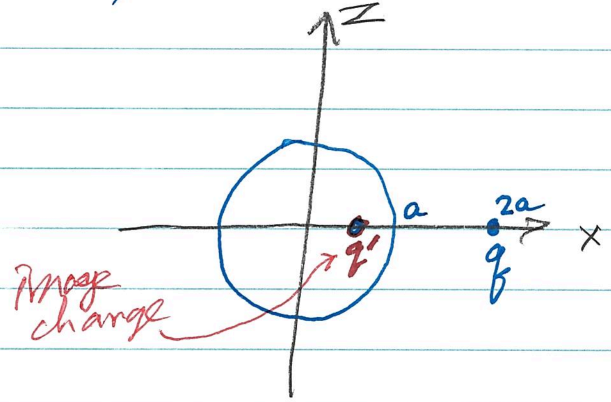
(b) an electron with $\vec{v} = 0.9c\ \hat{e}_z$



(See Figure 11.9.)

(4)

(1-4)



Charge q is at $(x_0, y_0, z_0) = (2a, 0, 0)$

Use the method of images. To calculate the field outside the sphere, imagine an image charge q' at $(x' y' z') = \left(\frac{a^2}{r}, 0, 0\right)$; $q' = \frac{-a}{r} q$

$$\Phi(x, y, z) = \frac{q}{4\pi\epsilon_0}\left\{\frac{1}{\sqrt{(x-2a)^2+y^2+z^2}} - \frac{a/r}{\sqrt{(x-a^2/r)^2+y^2+z^2}}\right\}$$

Then $\vec{E} = -\nabla\Phi = \frac{q}{4\pi\epsilon_0 a^2}(f_x, f_y, f_z)$

(a) At $(x, y, z) = (0, 0, 2a)$

$$(f_x, f_y, f_z) = \left(\frac{-1}{8\sqrt{2}} + \frac{2}{17\sqrt{17}}, 0, \frac{1}{8\sqrt{2}} - \frac{8}{17\sqrt{17}}\right)$$

$$= (-0.0599, 0, -0.0257)$$

(b) At $(x, y, z) = (-2a, 0, 0)$

$$(f_x, f_y, f_z) = \left(\frac{7}{400}, 0, 0\right)$$

$$= (0.0175, 0, 0)$$

(4)

1-5

In[*]:=
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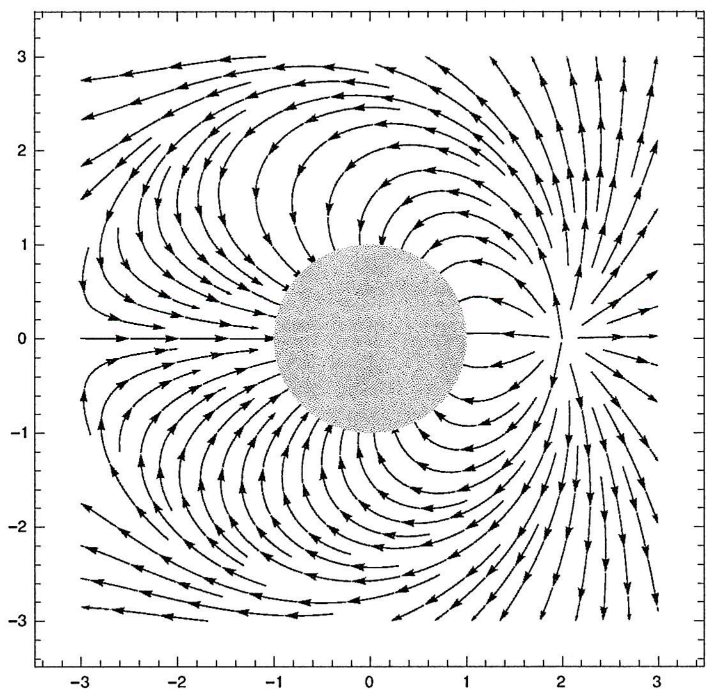
(*Image charge*)
a = 1; q = 1;
{x0, y0, z0} = {2 a, 0, 0}; r = Sqrt[x0^2 + y0^2 + z0^2];
qp = -a / r * q;
{xp, yp, zp} = a^2 / r * {1, 0, 0};
Φ[x_, y_, z_] = q / Sqrt[(x - x0)^2 + (y - y0)^2 + (z - z0)^2] +
   qp / Sqrt[(x - xp)^2 + (y - yp)^2 + (z - zp)^2];
Ex[x_, y_, z_] = -D[Φ[x, y, z], x];
Ey[x_, y_, z_] = -D[Φ[x, y, z], y];
Ez[x_, y_, z_] = -D[Φ[x, y, z], z];
Show[
 StreamPlot[{Ex[x, 0, z], Ez[x, 0, z]}, {x, -3, 3}, {z, -3, 3}],
 Graphics[{LightGray, Disk[{0, 0}, 1]}]]
```

Out[*]=



(1-5)

Given $\vec{J}(\vec{x})$

Then $\vec{B} = \nabla \times \vec{A}$ and $\nabla \times \vec{B} = \mu_0 \vec{J}$

$$\nabla \times (\nabla \times \vec{A}) = \nabla(\underbrace{\nabla \cdot \vec{A}}_{=0 \text{ in Coulomb gauge}}) - \nabla^2 \vec{A}$$

$$-\nabla^2 \vec{A} = \mu_0 \vec{J}$$

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{x}')}{\|\vec{x} - \vec{x}'\|} d^3x'$$

(2)