Homework Assignment 11
due Friday November 15

References : all answers are based on information obtained from Wikipedia.

1. 11-1. In physics, what is ether? And what is ethernet?

In physics, ether (or aether) is the fictitious medium that supports light waves.
Ethernet:
Ethernet is a family of computer networking technologies used in local area networks (Wikipedia)
Information is transmitted to and from a computer on the network, using a coaxial cable.

3 points

2. 11-2. What are the frequencies used by your cell phone?

Cell phones
My cell phone is an iPhone 8 + using an AT & T network.The frequency band (PCS - 1900) is
1850 - 1910 MHz (mobile to base), and
1930 - 1990 MHz (base to mobile)

2 points

3. 11-3. What are the frequencies used for WiFi communication?

WiFi communication

WLAN (Wireless Local Area Network) channels are frequently accessed using IEEE 802.11 protocols and are sold mostly under the trademark Wi-Fi.
The 802.11 standard provides several distinct radio frequency ranges for use in Wi-Fi communications :
900 MHz 2.4 GHz, 3.6 GHz, 4.9 GHz, 5 GHz, 5.9 GHz and 60 GHz bands.
Channels used in the 2.4 GHz frequency range , in MHz :

channel	center	range
1	2412	2401–2423
2	2417	2406–2428
3	2422	2411–2433

4	2427	2416-2438
5	2432	2421-2443
6	2437	2426-2448
7	2442	2431-2453
8	2447	2436-2458
9	2452	2441-2463
10	2457	2446-2468
11	2462	2451-2473

2 points

4. 11-4. Jackson Problem 8.2

We did the ideal field calculations for this example in the lecture Monday November 4.

4 points

5. 11-5. What is the impedance of free space? ***Explain.***

The impedance of free space

- The impedance of free space Z_0 is a physical constant relating the magnitudes of the electric and magnetic fields of electromagnetic radiation travelling through free space.That is,

$$Z_0 = \frac{|E|}{|H|}$$

where |E| is the electric field strength and |H| is the magnetic field strength.

- The impedance of free space (for a plane wave in free space) is equal to the $\mu_0 c$.

- The presently accepted value is

Z_0 = 376.730313668 (57) ohms.

A good approximation is $Z_0 \approx 120\,\pi$ ohms.

```
(*verify unit:*)
(* curl H = J = σ E; also, R = ρ L/A; *)
(* [H]/[m] = [σ][E]; [R]= 1/[σ] [m]/[m^2] = [σ]^-1[m]^-1;*)
(* So ... [E]/[H] = [σ]^-1 [m]^-1 = [R] = ohm *)

(* verify: μ0*c= (4Pi*^-7)*(3*/*^8) = 120 Pi *)
```

3 points

The next problems concern a rectangular waveguide with δx = a = 5 cm and δy = b = 2.5 cm;
also, set $\mu = \mu_0$ and $\epsilon = \epsilon_0$.

6. 11-6. Calculate the cutoff frequency and the corresponding wavelength for the TE_{10} mode.

The TE_{10} mode

The dispersion relation for the wave guide is given by $\gamma^2 = \mu\epsilon\omega^2 - k^2$.
The TE_{10} mode has $\gamma_{10} = \pi/a$, and the cutoff frequency is $\omega_{10} = \frac{1}{\sqrt{\mu\epsilon}}\gamma_{10} = c\,\gamma_{10}$.

Thus $\omega_{10} = c\pi/a = 1.88 \times 10^{10}\ s^{-1}$.

```
In[43]:= 3.0*^8 * Pi / 0.05
         % / (2 * Pi)
Out[43]= 1.88496 × 10^10
Out[44]= 3. × 10^9
```

The corresponding wavelength is $\lambda_{10} = \frac{2\pi c}{\omega_{10}}$ = 2a = 10 cm.

$\omega_{10} = 1.88 \times 10^{10}$ /s or f_{10} = 3.0 GHz ; λ = 10 cm

2 points

7. 11-7. Calculate the energy flux of the TE_{10} mode.
[Hint: the fields are the Real Parts of the complex functions in (8.46).
Take the real parts before you calculate the Poynting vector.]

The energy flux of the TE_{10} mode

The energy flux is $\vec{S} = \vec{E} \times \vec{H}$ where the fields are

$H_z = H_0 \cos(\pi x/a) \cos(kz-\omega t)$

$H_x = (ka/\pi)\, H0 \sin(\pi x/a) \sin(kz-\omega t)$

$E_y = (-\omega a\mu/\pi)\, H0 \sin(\pi x/a) \sin(kz-\omega t)$

$$S = \begin{pmatrix} i & j & k \\ 0 & Ey & 0 \\ Hx & 0 & Hz \end{pmatrix} = ex\ Ey\ Hz - ez\ Ey\ Hx$$

$Sx = H_0^2 (-\omega a\mu/\pi) \sin(\pi x/a) \sin(kz-\omega t) \cos(\pi x/a)\cos(kz-\omega t)$

$Sx = (-1/4)\, H_0^2(\omega a\mu/\pi) \sin(2\pi x/a) \sin[2(kz-\omega t)]$

$Sz = -H_0^2(-\omega a\mu/\pi) \sin(\pi x/a) \sin(kz-\omega t)\, (ka/\pi) \sin(\pi x/a)\sin(kz-\omega t)$

$Sz = H_0^2(\omega a\mu/\pi)\, (ka/\pi) \sin^2(\pi x/a) \sin^2(kz-\omega t)$

Averaging over the time, $\mathbf{S} = \hat{e}_z \langle S_z \rangle$ where $Sz = H_0^2 \left(\frac{\mu\omega k a^2}{2\pi^2}\right) \sin^2\left(\frac{\pi x}{a}\right)$

$$\langle S_z \rangle = H_0^2 \left(\frac{\mu\,\omega k\,a^2}{2\,\pi^2}\right) \sin^2\!\left(\frac{\pi x}{a}\right)$$

2 points

8. 11-8. Calculate the cutoff frequencies for the TM modes. Hand in a Table like the table below (8.46); the elements of the table should be $\omega_{cutoff}(TM_{mn})$ / $\omega_{cutoff}(TE_{10})$.

TM modes of the rectangular waveguide

For the TM modes, both m and n ∈ {1 2 3 ... }.

Recall $\gamma^2 = \mu\epsilon\,\omega^2 - k^2$. Therefore the cutoff frequencies are

$\omega_{mn} = \gamma_{mn}/\sqrt{\mu\epsilon} = \frac{1}{\sqrt{\mu\epsilon}}\,[\,(m\pi/a)^2 + (n\pi/b)^2\,]^{1/2}$

In[45]:=
```
Do[Do[tbl[m, n] = SetPrecision[
      Sqrt[(m*Pi)^2+4*(n*Pi)^2]/Pi, 3],
    {m, 1, 3}], {n, 1, 3}];
row[0] = {"", "n=1", "n=2", "n=3"};
col[0] = {"", "m=1", "m=2", "m=3"};
Do[
  row[i] = {col[0][[i+1]], tbl[i, 1], tbl[i, 2], tbl[i, 3]}, {i, 1, 3}];
Style[ TableForm[Join[{row[0]}, {row[1]}, {row[2]}, {row[3]}]] , {24, Purple, Bold}]
```

Out[49]=

	n=1	n=2	n=3
m=1	2.24	4.12	6.08
m=2	2.83	4.47	6.32
m=3	3.61	5.00	6.7

3 points

9. 11-9. For the waveguide mode TE_{32} ...
hand in a sketch (better: a computer graphic) of the effective surface current density $\boldsymbol{K}$(x,y) at the wall of the waveguide with y = 0, for $\omega = 2\,\omega_{32}$.

The TE_{32} waveguide mode

The goal is to calculate the surface current density **K**(x,y) at y = 0.

Recall the boundary condition: $\Delta H_{tangential} = K \times \hat{n}$.

For a perfect conductor, **H** = 0 inside the conductor; so the boundary condition is $\boldsymbol{H}_{tangential} = \mathbf{K} \times \hat{\boldsymbol{n}}$. For the wall at y = 0, the normal direction is $-\hat{\boldsymbol{e}}_y$, and the tangential directions are $\hat{\boldsymbol{e}}_x$ and $\hat{\boldsymbol{e}}_z$.

To calculate: $\mathbf{K} = \hat{\boldsymbol{n}} \times (\,\hat{\boldsymbol{e}}_x H_x + \hat{\boldsymbol{e}}_z H_z\,) = \hat{\boldsymbol{e}}_z H_x - \hat{\boldsymbol{e}}_x H_z$ (evaluated at y = 0)

Fields of the TE_{32} mode:

$$H_z = H_0 \cos\!\left(\frac{3\pi x}{a}\right) \cos\!\left(\frac{2\pi y}{a}\right) \cos(kz-\omega t) = H_0 \cos\!\left(\frac{3\pi x}{a}\right)\cos(kz-\omega t) \text{ @y=0}$$

$$H_x = \frac{kH_0}{\ } \frac{3\pi}{\ } \sin\!\left(\frac{3\pi x}{a}\right) \cos\!\left(\frac{2\pi y}{a}\right) \sin(kz-\omega t) = \frac{kH_0}{\ } \frac{3\pi}{\ } \sin\!\left(\frac{3\pi x}{a}\right)\sin(kz-\omega t) \text{ @y=0}$$

$$H_y = \frac{kH_0}{\gamma^2}\ \frac{2\pi}{b}\ \cos\left(\frac{3\pi x}{a}\right)\sin\left(\frac{2\pi y}{a}\right)\sin(kz-\omega t) = 0 \text{ @y=0}$$

The current density on the wall at y=0

$$K_x = -\ H_z = H_0\ \cos\left(\frac{3\pi x}{a}\right)\ \cos(kz-\omega t)$$

$$K_z = \ H_x = H_0\ \frac{3\pi k}{a\gamma^2}\sin\left(\frac{3\pi x}{a}\right)\ \sin(kz-\omega t);$$

here, $\gamma^2 = (3\pi/a)^2 + (2\pi/b)^2$ and $\gamma^2 = \mu\epsilon\,\omega^2 - k^2$.

Numerical parameters

a = 5 cm; b = 2.5 cm; $\mu = \mu_0$; $\epsilon = \epsilon_0$

$\gamma = \pi\ \text{cm}^{-1}$; $\omega = 2\ \gamma/c = 2\pi/c\ \text{cm}^{-1}$; $k = \sqrt{3}\,\pi\ \text{cm}^{-1}$

$K_x = H_0 \cos[3\pi x/(5\text{ cm})]\cos(kz-\omega t)$

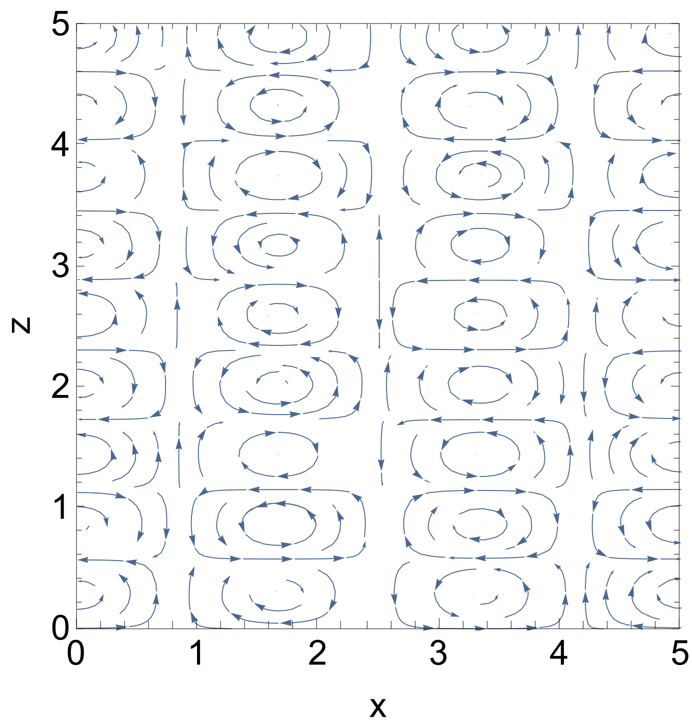
$K_z = H_0\left(\frac{3\sqrt{3}}{5}\right)\sin[3\pi x/(5\text{ cm})]\sin(kz-\omega t)$

In[∘]:=
```
{Sqrt[(3 * Pi / 5) ^2 + (2 * Pi / 2.5) ^2], 3^2 + 4^2, Sqrt[4 * γ^2 - γ^2] /. {γ → Pi}}
```

Out[∘]= $\{3.14159,\ 25,\ \sqrt{3}\,\pi\}$

In[∘]:=
```
Kx[x_, z_] = Cos[3 * Pi * x / 5] * Cos[Sqrt[3] * Pi * z];
Kz[x_, z_] = (3 * Sqrt[3] / 5) * Sin[3 * Pi * x / 5] * Sin[Sqrt[3] * Pi * z];
StreamPlot[{Kx[x, z], Kz[x, z]}, {x, 0, 5}, {z, 0, 5},
 PlotRange → {{0, 5}, {0, 5}}, AspectRatio → 1, StreamPoints → Automatic,
 Frame → True, FrameLabel → {"x", "z"}, BaseStyle → 18]
```

Out[∘]=



5 points

10. 11-10. Consider the TE_{mn} wave with $\omega > \omega_{mn}$.
(a) Calculate the phase velocity and show that it is is greater than c.
(b) Calculate the group velocity and show that $v_{group} = c^2/v_{phase}$.

Phase velocity and group velocity

Assume $\mu = \mu_0$ and $\epsilon = \epsilon_0$.

Recall $\gamma^2 = \mu_0 \epsilon_0 \omega^2 - k^2 = \mu_0 \epsilon_0 \omega_{mn}^2$.

Therefore, $\omega^2 = \omega_{mn}^2 + c^2 k^2$.

(A) The phase velocity is $v_{phase} = \frac{\omega}{k} = \frac{c\,\omega}{\sqrt{\omega^2 - \omega_{mn}^2}} > c$

(B) The group velocity is $v_{group} = \frac{d\omega}{dk} = \frac{c^2 k}{\omega} = \frac{c\sqrt{\omega^2-\omega_{mn}^2}}{\omega} = \frac{c^2}{v_{phase}}$.

(A) ω/k AND (B) $c^2 k/\omega$

4 points

11. 11-11. Why is a waveguide sometimes better than a coaxial cable? Explain, and define "better".

Both waveguides and coaxial cables can be used to carry energy and information in microwaves.

When is a waveguide better than a coaxial cable? From a Google search:

- The waveguide has no inner conductor, so it is easier to manufacture.
- The waveguide has less energy loss than a coaxial line:
  - no power is lost through radiation
  - no dielectric loss
- The waveguide can handle higher power than a coaxial cable:

The interior of the waveguide is air, and the breakdown voltage is 30 kV/cm; whereas there is a dielectric material in the coaxial cable with a lower breakdown voltage.

These are advantages of waveguide but there are also disadvantages,
so sometimes the coaxial cable is better.

4 points