Homework Assignment #5 = quasi-static magnetic fields
due Friday Oct 4

To aid in grading, for each problem draw a box around your final answers using red pencil.

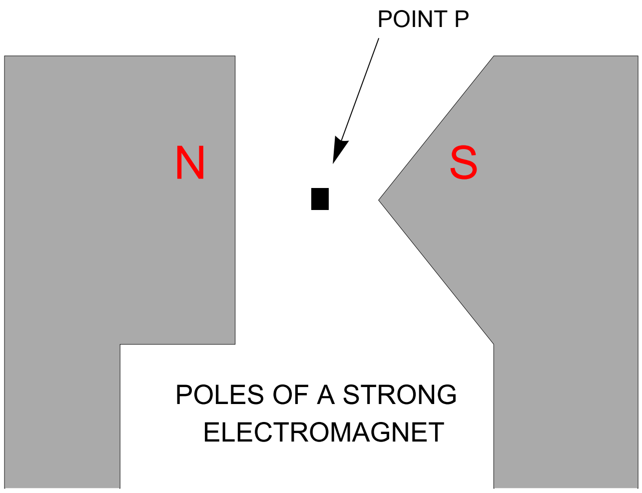
5-1. What is mu-metal, and what is it used for?

∎ a nickel iron alloy; 0.77 Ni, 0.16 Fe, 0.05 Cu, 0.02 Cr

∎ it has a high permeability, so it can be used for magnetic shielding

(2 points)

5-2. At point P, a paramagnetic object experiences a force toward the pointed pole; a diamagnetic object experience a force away from the pointed pole. Explain why, with basic equations.



The energy density of a linear magnetic material in a magnetic field is

$$u(\vec{x}) = \frac{1}{2}\vec{H}(\vec{x}) \bullet \vec{B}(\vec{x}) .$$

If the point P is occupied by a paramagnetic material then $B = \mu H$
where $\mu = \mu_0 (1 + \chi_M)$ and χ_M is positive. So the change of energy that results from putting the material in the gap is

$$\Delta U = \int \left(\frac{B^2}{2\mu} - \frac{B^2}{2\mu_0} \right) d^3x = -\frac{\chi_M}{1+\chi_M} \int_{\text{sample volume}} \frac{B^2}{2\mu_0} d^3x$$

ΔU is negative, and decreases if the sample is moved closer to the pointed pole (*where the magnetic field is stronger*). Therefore, the force on the sample is toward the pointed pole.

If the point P is occupied by a diamagnetic material then the susceptibility χ_M is negative. By the same calculation, ΔU is positive and decreases if the sample is moved closer to the flat pole. The force is away from the pointed pole.

(5 points)

5-3. In one paragraph with a figure, describe and explain how Faraday discovered electromagnetic induction.

The first experiment used an iron ring with two coils of wire wrapped around the ring. See the figure.

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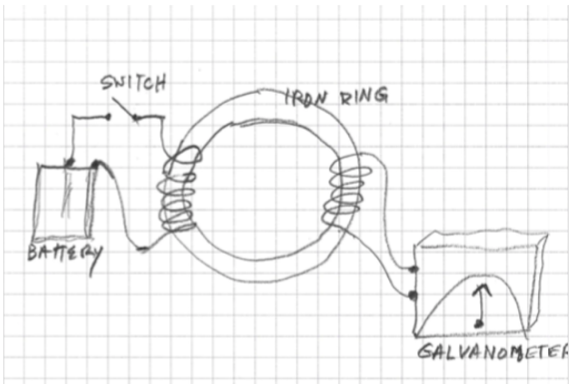

He observed that when the switch is closed, or opened, a temporary voltage is observed in the galvanometer.

His interpretation of this result was that closing the switch creates a temporary magnetic field; the magnetic field circles around the iron ring; then there is a temporary magnetic flux through the second coil—the one attached to the galvanometer; the galvanometer indicates current, which he attributed to an electromotive force. So,in Faradays language, ***$\partial\Phi/\partial t \propto$ emf.***

(3 points)

5-4. In a few equations prove this theorem:

The work that must be supplied to a magnetic system to change the vector potential by $\vec{A} \to \vec{A} + \delta\vec{A}$ is

$$\delta W = \int \delta \vec{A} \bullet \vec{J} \, d^3 x \, .$$

We can start with Jackson's equation (5.147) : the work done in assembling a magnetic system is

$$\delta W = \int \vec{H} \bullet \delta\vec{B} \, d^3 x$$

Now, $\vec{B} = \nabla\times \vec{A}$ and $\delta \vec{B} = \nabla\times \delta \vec{A}$, so

$$\delta W = \int \vec{H} \bullet (\nabla\times \delta\vec{A}) \, d^3 x = \int \epsilon_{ijk} \, H_i \, \partial_j \, (\delta A_k) \, d^3 x$$

$$= \int \{ -\nabla\bullet (\vec{H} \times \delta \vec{A}) + \delta\vec{A}\bullet \nabla\times \vec{H} \} \, d^3 x$$

$$= \int \delta\vec{A}\bullet \vec{J} \, d^3 x \, ,$$ which proves the theorem.

(The other term is zero by Gauss's theorem because H δA → 0 at infinity.)

(2 points)

A better proof: Use Faraday's law.
See Jackson's derivation on pages 212-213 ⇒ Equation 5.144.

5-5. In one paragraph with two equations, describe and explain the phenomenon of ***magnetic diffusion.***

When an magnetic field moves in a conducting material, an electric field is induced; by ***Faraday's law,***

$$\nabla \times \vec{E} = - \partial(\mu \vec{H}) / \partial t \,.$$

If the material is a conductor , then the electric field drives a current $\vec{J} = \sigma \vec{E}$ where σ is the conductivity (Ohm's law). Also, $\vec{J}$ is related to $\vec{H}$ by Ampere's law; $\nabla\times \vec{H} = \vec{J}$. Combining these effects we find that $\vec{H}(\vec{x},t)$ obeys a ***diffusion equation,***

$$\nabla\times (\nabla\times \vec{H}) = - \mu\sigma \frac{\partial \vec{H}}{\partial t} \,.$$

(3 points)