Homework Assignment 9 due FRIDAY November 1

To aid in grading, draw a box around your answer, using a red pencil.

9-1. Be quantitative.
(a) Which is larger—an atom or a wavelength of visible light?
(b) Is the difference in sizes a large or small difference?

(a) The wavelengths of visible light are from 400 to 700 nm.
Atomic radii are typically comparable to the Bohr radius,
$a_0 = 4\pi\epsilon_0 \hbar^2/(m e^2) = 5.3 \times 10^{-11}$ m = 0.053 nm.
The wavelength of visible light is larger. (2)

(b) It is a large difference: $\lambda/a_0 \sim 9000$. (2)

```
(* In SI units *)
{ϵ0, hbar, m, e} = {8.85*^-12, 1.055*^-34, 9.11*^-31, 1.602*^-19};
a0 = 4 * Pi * ϵ0 * hbar ^ 2 / (m * e ^ 2)
ratio = (500) / (0.053)
```

5.29437×10^{-11}

9433.96

9-2. Be quantitative.
(a) Explain why a microwave oven does not melt ice cubes effectively.
(b) Explain why a microwave oven does melt crushed ice effectively.

In[∘]:=
```
(* calculate photon energy *)
f = 2.45*^9 / second;
hbar = 6.6*^-16 * eV * second;
Eγ = 2 * Pi * hbar * f
```

Out[∘]= 0.0000101599 eV

(a) A microwave oven heats water by exciting rotational energy levels of the molecules.
In an ice cube the water molecules are arranged in a crystal lattice, so they are not free to rotate. So, an ice cube does not absorb microwaves effectively. (2)
(b) Molecules at the surface can undergo rotations and absorb microwaves
Crushed ice has a large surface to volume ratio compared to an ice cube.
Quantitatively? Do an experiment! (2)

If you want to use the concept of penetration depth, be accurate.

In[∘]:=
```
f92
```

Out[∘]=

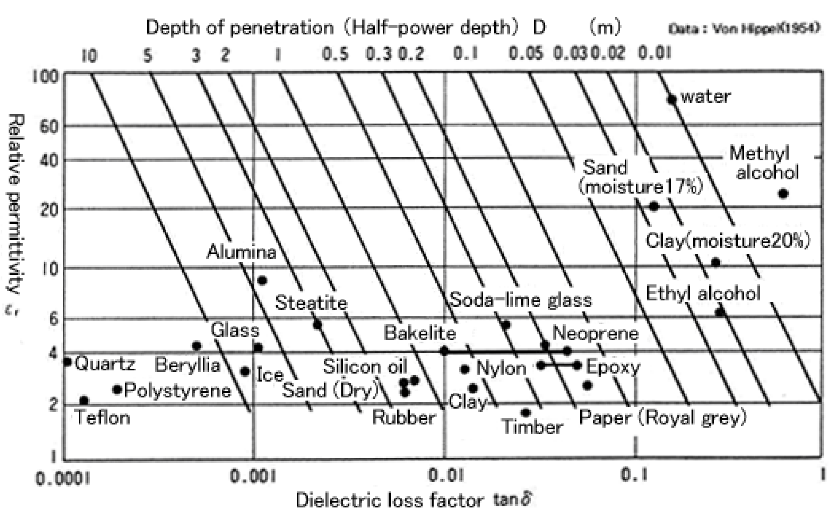
And, the formula for the penetration depth is given by **Equation 4**.

$$D = \frac{3.31\times10^7}{f\sqrt{\varepsilon_r}\cdot\tan\delta} \quad [\mathrm{m}] \qquad \textbf{Equation 4}$$

▶ (D) Dielectric properties of materials

We have explained that relative permittivity of the dielectric material ε r and dielectric loss angle of the dielectric material tanδ show characteristics of material (dielectric) at section (A).

Figure 8 is a characteristic diagram showing the relative permittivity ε r and the dielectric loss angle tanδ of various materials. As a result, roughly materials on up-right which has a large absorption of microwave's penetration depth is shallow, and materials on below-left which has a little absorption of microwave's penetration depth is deep.



In[∘]:=
```
(* penetration depth in meters *)
depth = 3.3*^7 / (2.45*^9) / Sqrt[80.0] / 0.1
```

Out[∘]= 0.0150592

9-3. Explain why a distant AM radio station can be received at night, but not during the day.

The frequencies of AM radio waves range from 530 to 1700 kHz. These frequencies are less than the plasma frequency of the ionosphere, which is approximately 9 MHz. Therefore AM radio waves reflect from the ionosphere. **(2)**

At night the only layer of the ionosphere with significant ionization is the F2 layer, at an altitude of approximately 300 km. Radio waves reflecting from this high altitude will have originated at large distances. **(2)**

9-4. Be quantitative.
Calculate the plasma frequency of silver.
Explain why silver looks white and shiny in sunlight.

▪▪ f_p(Ag) = 2.17×10^{15} Hz. See the calculation below. (2)

▪▪ If the frequency of light is less than the plasma frequency then the light waves are reflected at the surface, as for a plasma.
f_p(Ag) is in the ultraviolet range so visible light is reflected — the surface is white and shiny. (2)

(There is no color, as there would be for example in copper, because excitation energies for bound electrons are in the ultraviolet.)

```
(* calculation *)
(* ωp^2= N*Z*e^2/ (ϵ0*m)  in SI units *)
(* Z = number of conduction electrons per atom = 1 *)
Remove["Global`*"]
rho = 10.49*^3 * kg / meter ^ 3;
M = 107.9 * amu /. {amu → 1.66*^-27 * kg};
NAg = rho / M ;  Z = 1;
{e, m, ϵ0} = {1.6*^-19 * coulomb, 9.11*^-31 * kg, 8.85*^-12 * farad / meter};
ωp = Sqrt[NAg * Z * e ^ 2 / (ϵ0 * m)] /. {kg → joule * second ^ 2 / meter ^ 2};
ωp = ωp /. {joule → coulomb ^ 2 / farad} // PowerExpand
energy = hbar * ωp /. {hbar → 6.58*^-16 * eV * second}
fp = ωp / (2 * Pi)
```

$\frac{1.36368 \times 10^{16}}{\text{second}}$

8.97301 eV

$\frac{2.17036 \times 10^{15}}{\text{second}}$

```
(* verify that fp is in the ultraviolet *)
λp = (3.0*^8 * meter / second) / fp /. {meter → 1.0*^9 * nm}
```

138.226 nm

9-5. Be quantitative.
Explain why the navy uses ELF waves to communicate with submarines.

ELF waves are radio waves in the range from 3 to 30 Hz(1) (low freq radio waves)
They can penetrate seawater deeply(1), so they can be used for communication with a submerged submarine; i.e., the submarine does not need to come to the surface(1) for communication.
The reason is that seawater is a conductor(1), so it shields the depths from higher frequency waves.
See Figure 7.9 for quantitative information.

(4)

9-6. Here is a short extract from Jackson, Section 7.5.
Verify the statements that are made in the extract.

sh

window! In the very far ultraviolet the absorption has a peak value of $\alpha \simeq 1.1 \times 10^8\ \mathrm{m}^{-1}$ at $\nu \simeq 5 \times 10^{15}$ Hz (21 eV). This is exactly at the plasmon energy $\hbar\omega_p$, corresponding to a collective excitation of all the electrons in the molecule. The attenuation is given in order of magnitude by (7.62). At higher frequencies data

Given α = 1.1 x 10^8 m^{-1} at f = 5 x 10^{15} Hz.

(A)

First calculate $\hbar\omega_p$ = plasmon frequency (for all the electrons in the molecule)

$$\omega_p^2 = \frac{n e^2}{\epsilon_0 m}$$

In[∘]:=
```
(* calculation *)
rho = 1.0*^3 * kg / meter ^ 3;
Mwater = 18 * amu /. {amu → 1.66*^-27 * kg};
nw = rho / Mwater;
ne = nw * 10;
e = 1.602*^-19 * coulomb;
m = 9.11*^-31 * kg;
ϵ0 = 8.85*^-12 * farad / meter;
ωpsq = ne * e ^ 2 / ϵ0 / m /. {farad -> coulomb / volt};
ωpsq = ωpsq /. {coulomb → kg * meter ^ 2 / second ^ 2 / volt};
fp = Sqrt[ωpsq] / (2 * Pi) // PowerExpand (* the frequency *)
```

Out[∘]= $\frac{5.19471 \times 10^{15}}{\text{second}}$

which verifies that f = 5 X 10^{15} Hz agrees with f_{plasma} . (1 point)

(B)

In[∘]:=
```
(* the plasmon energy *)
Eplasmon = hbar * Sqrt[ωpsq] /. {hbar → 6.58*^-16 * eV * second}
Eplasmon // PowerExpand
```

Out[∘]= $21.4767\ \text{eV} \sqrt{\frac{1}{\text{second}^2}}\ \text{second}$

Out[∘]= 21.4767 eV

which verifies that $\hbar\, \omega_p$ = 21 eV. (1 point)

(C)

In[∘]:=
```
(* Calculate the attenuation from eq. 7.62 *)
αplasma = 2 * ωp / cl /. {cl → 3.0*^8 * meter / second}
```

Out[∘]= $\frac{9.09119 \times 10^{7}}{\text{meter}}$

The calculated αp = 0.9 x10^8 m^{-1} is in good agreement with the experimental value = 1.1 x 10^8 m^{-1}. (1 point)

9-7. An electromagnetic plane wave enters a medium in which the index of refraction is complex, n = n_0 – i κ.

(a) Write the form of $\vec{E}$(z,t); the wave propagates in the z direction and the frequency is ω.

$\vec{E}(\vec{x},t) = E_0 \exp[\,i\,(k\,z - \omega\,t\,)\,] \quad = E_0 \exp[i\,(\beta\,z - \omega\,t\,)]\exp[\,-\alpha z/2\,]$

oscillating wave + damped

(b) What is the phase velocity?

$$k^2 = \omega^2(\mu_0\,\epsilon) = \frac{\omega^2\,\epsilon}{c^2\,\epsilon_0} = \frac{\omega^2\,n^2}{c^2} = \frac{\omega^2}{c^2}\left(n_0^2 - \kappa^2 - 2\,i\,\kappa\,n_0\right)$$

$k = \beta + i\,\alpha/2$ where $k^2 = \left(\beta^2 - \alpha^2/4 + i\alpha\beta\right)$

$e^{ikz}\,e^{-i\omega t} = e^{i\beta z}\,e^{-\alpha z/2}\,e^{-i\omega t}$

The phase velocity is $v_{phase} = \omega/\beta$ where $\beta = \omega\,n_0/c$;

$$\Longrightarrow v_{phase} = \frac{c}{n_0}.$$

(c) What is the absorption length, i.e., such that the intensity is reduced by the factor 1/e.

The absorption length is $1/\alpha$ where $\frac{\alpha^2}{4} = \frac{\omega^2\,\kappa^2}{c^2}$ or $i\alpha\beta = \frac{-2\,i\,\omega^2\,\kappa\,n_0}{c^2}$ (sign error?)

Thus $\alpha = -\,2\omega\kappa/c$. (OK but κ should be negative.)

(d) What is the wavelength?

$$\lambda = \frac{2\,\pi\,v_{phase}}{\omega} = \frac{2\,\pi\,c}{\omega\,n_0}$$

(1+1+1+1 = 4 points)

These are the answers: E0 exp[i(kz-ωt)] ; c/n0 ; 2$\omega\kappa$/c ; 2πc/(ω n0)

9-8. What is the color of pure water? Explain using words and figures.

Blue.

We know that water is blue, because a picture of the Earth from space shows a blue planet.
The reason is because the absorption coefficient $\alpha(\omega)$ for visible light depends on frequency ω.
(see the figure of $\alpha(\omega)$ in the visible frequencies)
Absorption of the red end of the spectrum is greater than absorption of the blue part of the spectrum.
(see the figure of $\alpha(\omega)$ in the visible frequencies)

9-9. Why is snow white?

Snow is a powder of small ice crystals.
The ice crystals are large compared to a wavelength of visible light.
When sunlight hits an ice crystal, reflection and refraction occur,
from a dielectric surface with index of refraction
n that is approximately constant for wavelengths from 400 to 700 nm.
Multiple reflections and refractions occur if the snow is piled on the ground.
With no significant absorption, the combined reflection is white, like the incident sunlight. (4)

Problem 9-10 Free Electron Plasma

In terms of complex permittivity we have

$$\frac{\epsilon}{\epsilon_0} = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2 - i\omega\gamma} \quad \text{when } \omega_0 = 0$$

$$\frac{\epsilon}{\epsilon_0} = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)} = \frac{\omega^2 - \omega_p^2 + i\omega\gamma}{\omega(\omega + i\gamma)}$$

For $\omega = \omega_p$ and $\gamma = 0.01\,\omega_p$

$$\frac{\epsilon}{\epsilon_0} = \frac{0.01\,i}{1 + \underbrace{0.01\,i}_{\text{neglect}}} \approx 0.01\,i$$

The complex index of refraction is $n_c = \sqrt{\frac{\mu_0 \epsilon}{\mu_0 \epsilon_0}} = \sqrt{0.01\,i}$

We write n_c in terms of real and imaginary parts,

$$n_c = n + i\kappa.$$

Thus

$$n + i\kappa = \sqrt{0.01\,i} = 0.1\, e^{i\pi/4}$$

$$\boxed{n = \kappa = \frac{0.1}{\sqrt{2}}}$$