PHY 842 Quiz NAME Solutions for Grading

Friday October 18

Instructions:

- Closed book, closed notes, closed internet, do not discuss the questions with other students.
- For each question write your answer in the available space only (use a sharp pencil).
- Answers may include equations, text, pictures and calculations.

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(1) For a plane electromagnetic wave, prove $\omega \vec{B} = \vec{k} \times \vec{E}$.

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow i\vec{k} \times \vec{E} = i\omega \vec{B}$ for $e^{i(\vec{k}\cdot\vec{x}-\omega t)}$

$$\omega \vec{B} = \vec{k} \times \vec{E}$$

(2)

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(2) In free space $\vec{E} = E_0\, \hat{e}_x \sin(kz-\omega t)$. What is the magnetic field?

$$\vec{B} = \frac{k}{\omega} E_0\, \hat{e}_y \sin(kz-\omega t)$$

(2)

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(3) The index of refraction for water is 1.33. Then what is the electric permittivity?

$$v_{phase} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{n}$$

(2)

$\mu \cong \mu_0$ and $n = 1.33$ and $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

$$\therefore \quad \sqrt{\frac{\epsilon_0}{\epsilon}} = \frac{1}{1.33}$$

$$\epsilon = n^2 \epsilon_0 = 1.77\, \epsilon_0 \quad \text{or} \quad 1.57 \times 10^{-11}\ \text{F/m}$$

(4) In a simple linear dielectric, the Poynting vector is $\vec{S} = \vec{E} \times \vec{H}$; calculate $\nabla \cdot \vec{S}$.

$$\nabla \cdot \vec{S} = \nabla \cdot (\vec{E} \times \vec{H}) = \epsilon_{ijk}\, \partial_i (E_j H_k)$$

$$= \epsilon_{ijk} \left\{ (\partial_i E_j) H_k + E_j (\partial_i H_k) \right\}$$

$$= \vec{H} \cdot (\nabla \times \vec{E}) + - \vec{E} \cdot (\nabla \times \vec{H})$$

$$= -\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} - \vec{E} \cdot \frac{\partial \vec{D}}{\partial t}$$

(2)

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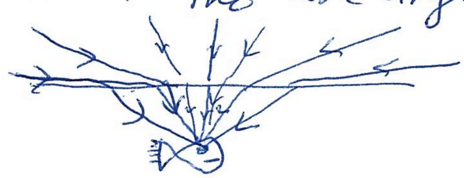
(5) Explain the term `fish-eye lens'.

A very wide angle lens; so called because a fish sees all 2π sterradians of the sky compressed into a cone by refraction. The cone angle = $\theta_{critical}$ for T. I. R. (2)



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(6) Calculate Brewster's angle for the interface between air and water (n = 1.33)

From first principles, $\theta = \theta_B$ when $E_0'' = 0$ for TM polarization

$n' \cos\theta_B = n \cos\theta'$ (Fresnel) and $n \sin\theta_B = n' \sin\theta'$ (Snell)

Solve for $\theta_B \Rightarrow \cos\theta_B = \dfrac{1}{\sqrt{(n'/n)^2 + 1}}$

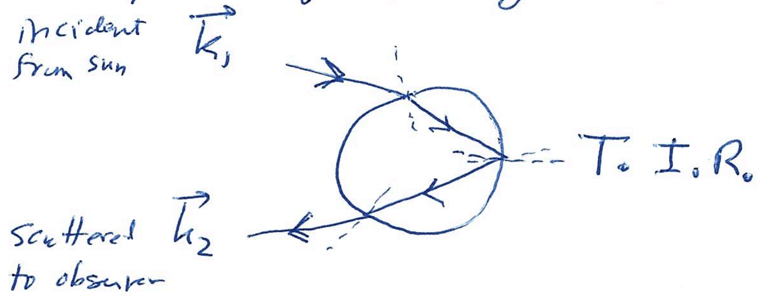
For $n = 1$ and $n' = 1.33$, $\cos\theta_B = \dfrac{1}{\sqrt{(1.33)^2 + 1}} = 0.601$

$\theta_B = 0.926$ radians or 53.1 degrees

(2)

(7) **What is the role of total internal reflection in a rainbow?**

Reflection and refraction of light by a sphere of ~~water~~ water (raindrop)



(2)

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(8) **In a metal, $\vec{J} = g\,\vec{E}$. Derive the wave equation for $\vec{E}(\vec{x},t)$. What is the effect of g?**

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\times(\nabla\times\vec{E}) = \nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E} = -\nabla^2\vec{E} = -\frac{\partial}{\partial t}(\nabla\times\vec{B}) = -\mu\frac{\partial}{\partial t}(\nabla\times\vec{H})$$

$$-\nabla^2\vec{E} = -\mu\frac{\partial}{\partial t}\left(g\vec{E} + \epsilon\frac{\partial\vec{E}}{\partial t}\right) = -\mu g\frac{\partial\vec{E}}{\partial t} - \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

(2)

$$\mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} - \nabla^2\vec{E} + \mu g\frac{\partial\vec{E}}{\partial t} = 0$$

The effect of g is damping.

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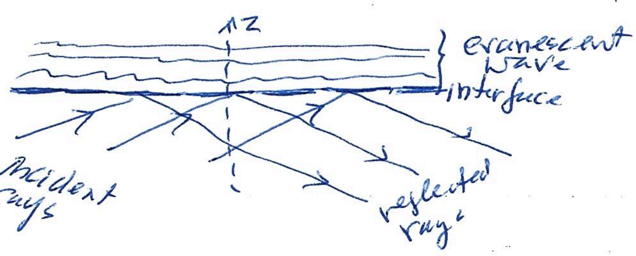
(9) **Explain the evanescent wave in total internal reflection.**



Snell's law (glass) (air) $n\sin\theta = \sin\theta'$

$\theta = \theta_{cr} = \arcsin\left(\frac{1}{n}\right) \Rightarrow \theta' = \pi/2$ (2)

If $\theta > \theta_{cr}$ then the field in air $(z>0) \propto e^{ik_x x}\, e^{-z/\delta}$; that is

the evanescent wave

$k_z' = k'\cos\theta' = k'\sqrt{1-\sin^2\theta'} = k'\sqrt{1-n^2\sin^2\theta}$ imaginary

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(10) **For linearly polarized light incident normally from air onto a smooth surface of water, calculate the amplitude of the reflected wave.**

$$\frac{E_0''}{E_0} = \pm\left(\frac{n-n'}{n+n'}\right) = \mp\frac{1}{7} \quad \text{for} \begin{cases} \text{TE polarization} \\ \text{TM polarization} \end{cases}$$

(2)