Lecture 0 (Wed Aug 28)

Review of Microscopic Electrodynamics

In PHY 841 you studied "microscopic electrodynamics" — that is, electric and magnetic fields with isolated charges and currents in empty space.

(Of course the space is not really empty because there are fields and charges and currents present.)

What we mean by "empty space" is that there are *no macroscopic materials* present.

Macroscopic materials:

• solids • liquids • gases • plasmas

↦ materials with huge numbers of molecules, atoms, ions, nuclei, …

How huge is huge?

Exercise: How many electrons are in a liter of water?

We cannot deal with all the individual particles, so we average over "small" volumes that contain "large" numbers of particles. (How small is "small"? How large is "large"?)

EXAMPLE. The *microscopic charge density* is

$$\rho_{\mathrm{pt}}\,(\vec{x}) = \sum_i q_i\, \delta^3(\,\vec{x} - \vec{x}_i)\;.$$

What would a graph of $\rho_{\mathrm{pt}}(\vec{x})$ look like?

We'll replace $\rho_{\mathrm{pt}}(\vec{x})$ by the *macroscopic charge density*

$$\rho(\vec{x}) \equiv \frac{1}{\Delta V} \int_{\Delta V} \rho_{\mathrm{pt}}\,(\vec{x})\; d^3x = \frac{1}{\Delta V} \sum_{i\,\in \Delta V} q_i\;.$$

$\rho(\vec{x})$ is a continuous function. (What would a graph look like?)

This is the charge density of *macro-scopic electrodynamics* — the subject of PHY 842.

Review of Microscopic Electrodynamics

Microscopic electrodynamics means the theory of charges, currents, fields in “empty space”.

■ Field equations

microscopic electrodynamics	
$\nabla \cdot \vec{E} = \rho/\epsilon_0$	$\nabla \times \vec{E} = -\,\partial \vec{B}/\partial t$
$\nabla \cdot \vec{B} = 0$	$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0\, \epsilon_0\, \partial \vec{E}/\partial t$

In SI units we need two parameters, vacuum permeability μ_0 and permittivity ϵ_0. (These are not necessary in Gaussian units, which is why Gaussian units are superior.) Homework: what are the exact values and units?

The continuity equation for charge is

$$\nabla \cdot \vec{J} = -\partial\rho/\partial t\,.$$

This equation says that *<u>charge is locally conserved.</u>*

The forces on a test charge q

$$\vec{F} = q\,(\vec{E} + \vec{v} \times \vec{B})$$

i.e.,

$$\vec{F}(\vec{x},t) = q\,[\,\vec{E}(\vec{x},t) + \vec{v}(t) \times \vec{B}(\vec{x},t)\,]$$

where $\vec{x}$ is the position of the charge q at time t, and $\vec{v} = d\vec{x}/dt$.

These forces acting on a test charge q, may be taken to <u>define</u> the fields $\vec{\boldsymbol{E}}$ *and* $\vec{\boldsymbol{B}}$.

Energy and Momentum

You already know these quantities from PHY 841.

- The field energy in a volume Ω is

$$U(t) = \int_\Omega u(\vec{x},t)\; d^3x$$

$$u(\vec{x},t) = \left\{ \frac{\epsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2 \right\} \quad (\vec{x},t)$$

- The work per unit time per unit volume is $\vec{J} \cdot \vec{E}\,(\vec{x}, t)$;

this is the work (*per unit time per unit volume*) done on the charge by the fields.

β

- The energy flux is

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

= the Poynting vector (units are W m^{-2})

- Poynting's theorem, for microscopic electrodynamics,

$$\nabla \bullet \vec{S} = -\partial u / \partial t - \vec{J} \bullet \vec{E}$$

Note that *Poynting's theorem is the continuity equation for energy*. It says that *energy is locally conserved.*

THREE SPECIAL CASES

1. Electrostatics
2. Magnetostatics
3. E.M. waves in vacuum

1-ELECTROSTATICS

$\Longleftrightarrow$ a static system with

$$\rho(\vec{x}) \text{ and } \vec{E}(\vec{x}) \text{ (independent of time)}$$

$$\vec{J}(\vec{x}) = 0 \text{ and } \vec{B}(\vec{x}) = 0$$

The field equations are

$$\nabla \cdot \vec{E} = \rho / \epsilon_0 \quad \text{and} \quad \nabla \times \vec{E} = 0$$

Because curl $\vec{E} = 0$, we may write

$$\vec{E} = -\nabla\Phi \quad \text{with} \quad -\nabla^2\Phi = \rho/\epsilon_0$$

$\Longrightarrow$ we'll encounter Poisson' s equation and Laplace's equation;

an important issue will be, *what are the boundary conditions?*

2-MAGNETOSTATICS

⟺ a static system with

$$\rho = 0 \quad \text{and} \quad \vec{E} = 0$$

$$\vec{J}(x) \quad \text{and} \quad \vec{B}(x)$$

$$\nabla \bullet \vec{J} = 0$$

The field equations are

$$\nabla \bullet \vec{B} = 0 \quad \text{and} \quad \nabla \times \vec{B} = \mu_0 \vec{J}$$

Because $\nabla \times \vec{B} = 0$ we may write

$$\vec{B} = \nabla \times \vec{A}$$

Also, by gauge invariance we may require $\nabla \bullet \vec{A} = 0$ (the "Coulomb gauge condition"). However, this is not necessary.

homework

3-ELECTROMAGNETIC WAVES

$$\vec{E}(\vec{x},t) = \vec{E}_0 \cos(\vec{k} \bullet \vec{x} - \omega t)$$

$$\vec{B}(\vec{x},t) = \vec{B}_0 \cos(\vec{k} \bullet \vec{x} - \omega t)$$

with $\omega = ck$ and $\vec{k} \times \vec{E}_0 = \omega \vec{B}_0$.

Also ...

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

All the equations so far are for *micro-scopic electrodynamics.*

You studied these equations in PHY 841 and I'll assume that you are familiar with them.

Reading Assignment : Chapter 4