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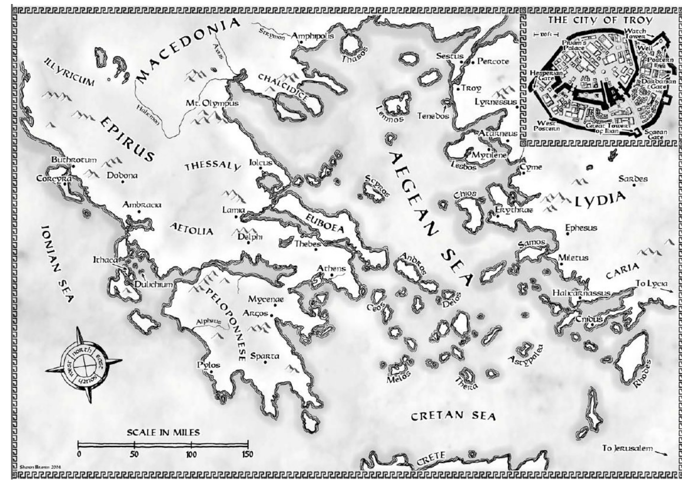
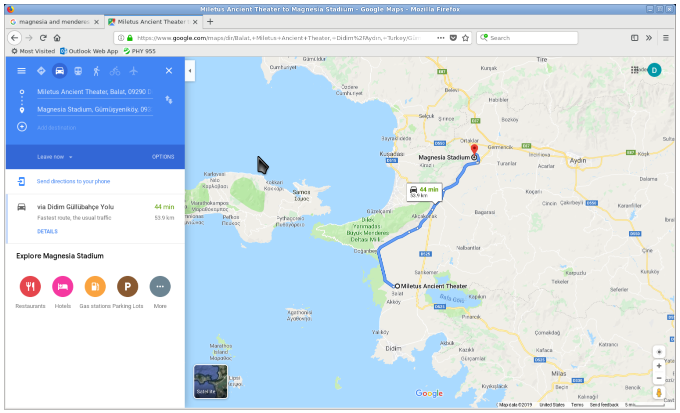
Some ancient history

Who discovered magnetism?

- The discovery of magnetism is attributed to *Thales of Miletus*.
- Thales (624 – 546 BC) was the first pre-Socratic philosopher of Ancient Greece.
- Thales was the first scientist in the history of Western Civilization—remembered for discoveries in mathematics, astronomy, electricity and magnetism.
- “A lodestone has a soul because it can cause movement of iron.”
- Origin of the word “magnet”.





Lecture 2-3 {Wed , Sept 18}
Methods of solving boundary-value problems in magnetostatics

Jackson Section 5.9

The field equations are

$$\nabla \cdot \vec{B} = 0 \quad \text{and} \quad \nabla \times \vec{H} = \vec{J}$$

where $\vec{J}$ means $\vec{J}_{\text{free}}$. $\vec{B}$ and $\vec{H}$ are related by some constitutive relation. But that's only part of the problem. The other part consists of the *boundary conditions*.

Jackson gives three methods.

A. The vector potential

We always have $\nabla \cdot \vec{B} = 0$; then we can write

$$\vec{B} = \nabla \times \vec{A}$$

and now solve for $\vec{A}(\vec{x})$.

If there is a linear constitutive equation, $\vec{B} = \mu \vec{H}$, then

$$\nabla\times(\nabla\times\vec{A}) = \mu \vec{J}$$

which is analogous to *Poisson's equation.*

B. The scalar potential (requires $\vec{J}_{free} = 0$)

This method can be used in any region of space where $\vec{J} = 0$. Then $\nabla \times \vec{H} = 0$ so we can write

$$\vec{H} = -\nabla \Phi_M$$

For a linear constitutive equation, $\vec{B} = \mu \vec{H}$, the other field equation gives

$$\nabla \cdot (\mu \nabla \Phi_M) = 0$$

which is analogous to *Laplace's equation*.

C. "Hard Ferromagnets": $\vec{M}$ is given and $\vec{J}_{\text{free}} = 0$.

Although $\vec{J}_{\text{free}} = 0$, there are *bound molecular currents* in (or on the surface) of the matter (iron or ...) producing a magnetic field.

C (a) Using a scalar potential ...

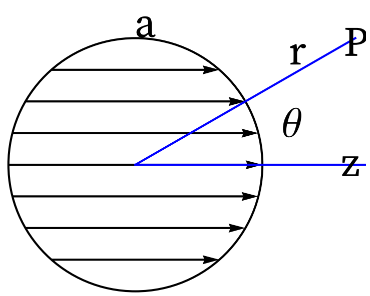
Since $\vec{J} = 0$, write $\vec{H} = -\,\text{grad}\,\Phi_M$.
Now div $\vec{B} = 0 = \mu_0$ div (-grad $\Phi_M + \vec{M}$);
$\therefore \nabla^2 \Phi_M = -\rho_M$ where $\rho_M = -\,\text{div}\,\vec{M}$.
The problem reduces to Poisson's equation.

C (b) Using a vector potential ...

We can always write $\vec{B} = \text{curl}\,\vec{A}$.
Then curl $\vec{H} = \vec{J} = 0$ implies
curl [$\vec{B} / \mu_0 - \vec{M}$] = 0
Or, $\nabla^2 \vec{A} = -\mu_0 \vec{J}_M$ where $J_M = \nabla \times \vec{M}$.
Again, this is Poisson's equation.

Example — a Uniformly Magnetized Sphere

Jackson Section 5.10



$\vec{M} = M_0\, \hat{e}_z$ inside, i.e. for r < a.

What are $\vec{B}$ and $\vec{H}$ both inside and outside?

Solution by method C (a).

Write $\vec{H} = -\nabla \Phi_M$ because $\vec{J}_{\text{free}} = 0$.

For r < a, $\Phi_M(r,\theta) = -\, c_1\, r \cos\theta = -\, c_1\, z$;

For r > a, $\Phi_M(r,\theta) = c_2\, \frac{\cos\theta}{r^2}$;

... solutions of Laplace's equation.

Boundary conditions at r = a ...

B_r is continuous at r = a

$B_r = \mu_0 \{ H_r + M_0 \cos\theta \} = \mu_0 H_r$

$c_1 + M_0 = 2\, c_2 / a^3$

H_t is continuous at r = a

$H_\theta(r=a-) = H_\theta(r=a+)$

$-\, c_1 = c_2 / a^3$

Solution: $c_1 = -\, \frac{M_0}{3}$ and $c_2 = \frac{M_0\, a^3}{3}$

$$\vec{H}(r,\theta) = \begin{cases} c_1\, \hat{e}_z = -\frac{M_0}{3}\, \hat{e}_z & \text{for } r < a \\ c_2 \left[\hat{r}\, \frac{2\cos\theta}{r^3} + \hat{\theta}\, \frac{\sin\theta}{r^3} \right] & \text{for } r > a \end{cases}$$

$$\vec{B}(r,\theta) = \begin{cases} \mu_0\, \frac{2 M_0}{3} & \text{for } r < a \\ \mu_0\, \vec{H}(r,\theta) & \text{for } r > a \end{cases}$$

Solution by method C (b).

Write $\vec{B}$ = curl $\vec{A}$.

There is a bound surface current

$$\vec{K}_M(\vec{x}') = M_0\, \hat{e}_z \times \hat{r}' = M_0 \sin\theta'\, \hat{e}_\phi'$$

The vector potential $\vec{A}(\vec{x}) = A_\phi\, \hat{e}_\phi$ can be calculated from the Green's function integral $\Longrightarrow$

$$A_\phi(\vec{x}) = \frac{\mu_0}{3} M_0\, a^2\, \frac{r_<}{r_>^2} \sin\theta$$

where $r_<$ = min{r,a} and $r_>$ = max{r,a}.

This gives the same constant $\vec{B}$ inside the sphere, and the same dipole field $\vec{B}$ outside the sphere.

Figure 5.11 shows the field lines inside and outside the magnetized sphere.