enter

Lecture 2-5 { Mon , Sept 23 }
Paramagnetism and Diamagnetism

You won't find this in Jackson!

To Jackson this is just an experimental fact:

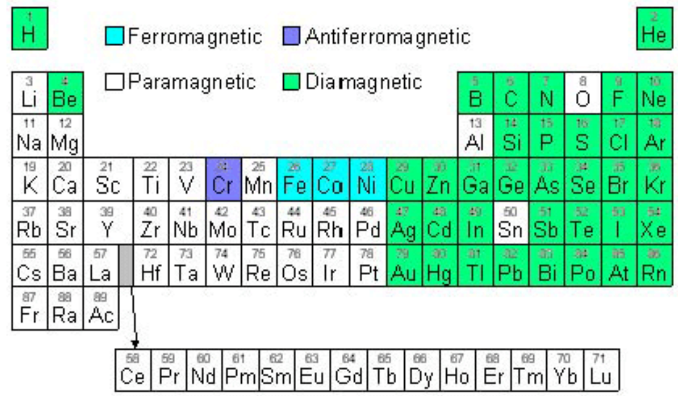
Linear media have $\vec{B} = \mu \vec{H}$

$\mu > \mu_0 \iff$ paramagnetism

$\mu < \mu_0 \iff$ diamagnetism

But ***why?*** The answer depends on atomic and molecular physics. It is not a topic in classical electrodynamics, so Jackson doesn't cover it.

Magnetic properties of the elements
Taken from some website …



Faraday did not discover diamagnetism but he studied it in depth — he identified the magnetic properties of many elements.

How can the magnetic property of a material — diamagnetic or paramagnetic — be determined?

From the Feynman Lectures on Physics

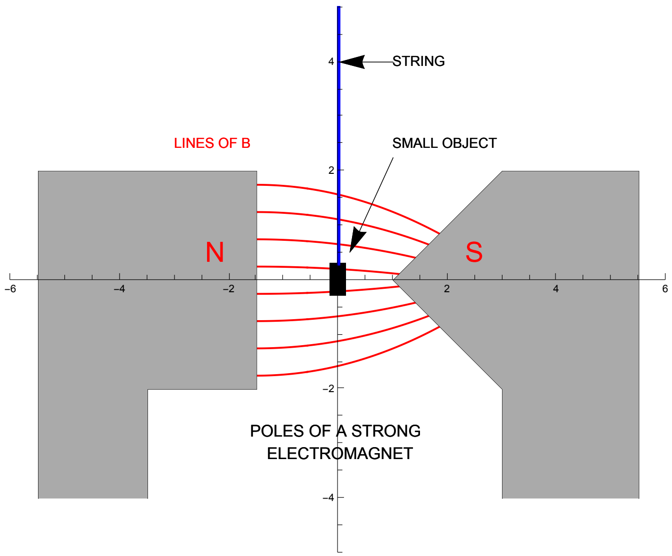


Figure 34-1. A small cylinder of bismuth is weakly repelled by the sharp pole; a piece of aluminum is attracted.

Quantitative measurements

MAGNETIC SUSCEPTIBILITIES (room temp)

In[•]:= tχm

Aluminum	+2.1 E−5	H2 (1 atm)	−0.22 E−8
Bismuth	−1.64 E−5	N2 (1 atm)	−0.67 E−8
Copper	−0.98 E−5	O2 (1 atm)	+193.5 E−8
Diamond	−2.2 E−5	GdCl3	+603 E−5
Gold	−3.5 E−5	–	–
Sodium	+0.84 E−5	–	–

We'll try to explain paramagnetism and diamagnetism using semi-classical models.

The Bohr— van Leewen theorem

"*At any finite temperature, and in all finite applied electrical or magnetical fields, the net magnetization of a collection of electrons in thermal equilibrium vanishes identically.*" (van Vleck, 1932)

History - from Wikipedia

- "What is today known as the Bohr–van Leeuwen theorem was discovered by Niels Bohr in 1911 in his doctoral dissertation, and was later rediscovered by Hendrika Johanna van Leeuwen in her doctoral thesis in 1919."

- "The significance of this discovery is that *classical physics does not allow for such things as paramagnetism, diamagnetism and ferromagnetism* and thus quantum physics are needed to explain the magnetic events."

<u>Preliminaries</u>

Thinking classically, an electron in a circular orbit has charge –e, orbital angular momentum L = mvr $\hat{n}$, and dipole moment

$$\vec{\mu} = \mathrm{I\ A}\ \hat{n} = \frac{-e}{\tau}\,\pi r^2\ \hat{n} = \frac{-e}{2\pi r/v}\,\pi r^2\ \hat{n}$$

$$\vec{\mu} = \frac{-e}{2m}\,\vec{L}$$

(For spin, $\vec{\mu} = \frac{-e}{m}\,\vec{S}$.)

PARAMAGNETISM

We can use quantum mechanics to get the right idea.

Matter is atomic or molecular. Suppose the ground state of a molecule has angular momentum quantum number = J.

The magnetic moment quantum operator will be $\vec{m} = -\lambda\,\vec{J}$ where λ is some constant; we expect

$$\lambda\,\hbar \propto \frac{e\hbar}{2\,mc} \equiv \text{the Bohr magneton}$$

Now turn on a weak magnetic field, $\vec{B} = \mathrm{B}\;\hat{e}_z$.

$$H_{int} = -\,\vec{m} \bullet \vec{B} = \lambda\,B\,J_z$$

(prefers negative j_z)

The ground state will split into 2J+1 energy levels, with $J_z = \hbar\,j_z$;
$j_z \in \{\,-J\,,\,-J+1,\,-J+2,\,\ldots\,,\,J-2,\,J-1,\,J$

At temperature T, the distribution of energy levels will be

$$P = C\,\exp[\,-\,\lambda\hbar B\;j_z\,]\,/\,kT$$

and the thermal average of the magnetic dipole moment will be

$$\langle\vec{m}\rangle = \hat{e}_z\,\frac{\Sigma\;(-\lambda\hbar\,j_z)\,\exp\,(-a j_z)}{\Sigma\,\exp\,(-a j_z)}$$

$$\text{where } a = \frac{\lambda\hbar B}{kT}$$

The thermal average

$$\langle m_z\rangle = \lambda\hbar\,\frac{\partial}{\partial a}\,\ln\,(\,\sum_{j=-J}^{J}\exp(a j_z)$$

$$= \lambda\hbar\,\frac{\partial}{\partial a}\,\ln\,\left[\,\frac{(e^{-aJ} - e^{a\,(J+1)})}{(1-e^{a})}\,\right]$$

$$= \lambda\hbar\,\frac{1}{1-e^{a}} + J + \frac{1+2\,J}{-1+e^{a\,(2\,J+1)}}$$

$$\approx \lambda\hbar\,\frac{a}{3}\,J\,(J+1) \qquad \text{for small a}$$

Result

For a weak field $\vec{B}$, $a = \lambda\hbar B/kT$ is small, and

$$\langle\vec{m}\rangle = \frac{(\lambda\hbar)^2}{3\,kT}\,J(J+1)\,\vec{B}$$

Magnetization, susceptibility and permeability

$$\vec{M} = N\,\langle\vec{m}\rangle = \chi\,\vec{H}$$

$$\chi_M = N\,\frac{(\lambda\,\hbar)^2}{3\,\mu_0\,kT}\,J(J+1)$$

$$\mu = (1 + \chi_M)\;\mu_0 \;\; > \mu_0$$

So there is a reasonable theory of paramagnetism.

INTERESTING FEATURE:
THE TEMPERATURE DEPENDENCE

DIAMAGNETISM

-- the Langevin theory of diamagnetism (1905, Peter Langevin).

- The theory is pre-quantum, and even pre-Bohr model. So how can it work in light of the Bohr-van Leewen theorem?
- Langevin started with a semi-classical assumption: the electrons in atoms (or molecules) move on circular orbits.

(definitely not a classical picture!)

<u>Feynman Lectures on Physics</u>
Chapter 34 Section 4 Diamagnetism
“Diamagnetism from the classical point of view can be worked out in several ways but one of the nice ways is the following… "

Turn on a magnetic field in the vicinity of an atom. An *electric field* is generated by magnetic induction (Faraday's law; Lenz's law).

(should think more about the vectors!)

$$E \cdot 2\pi r = -\frac{d}{dt}(B\pi r^2)$$

$$E = -\frac{r}{2}\frac{dB}{dt} \quad \text{i.e., } E_\phi = -\frac{r}{2}\dot{B}_z$$

$\Longrightarrow$ torque on an electron $= -e\,E\,r$

$$\therefore \frac{dJ}{dt} = \frac{e\,r^2}{2}\frac{dB}{dt}$$

$$\Delta J = \frac{e\,r^2}{2}B$$

So, in the presence of $\vec{B}$, there is an ***extra*** magnetic dipole moment

$$\Delta\vec{\mu} = \frac{-e}{2\,m\,r}\Delta\vec{J} = \frac{-e^2}{4\,m}r^2\vec{B}$$

Improved formula — not all electron orbits are in the same plane and having the same radius. The improved formula is

$$\Delta\vec{\mu} = -\frac{e^2}{6\,m}\langle r^2\rangle_{avg}\vec{B}$$

(Note: $\langle\ \rangle_{avg}$ does not mean a thermal average.)

The Langevin theory implies an induced dipole moment proportional to B, but in the opposite direction.

That explains diamagnetism, for atoms or molecules with J = 0; for example, think of the noble gases (He, Ne, Ar, Kr, ...).

The Langevin theory ...

```
Style["Susceptibility χM = (-e² ⟨r²⟩avg ne N μ0)/(6 m)
   where ne = # of electrons,
   and N = molecular density.", ff]
```

$$\chi_M = \frac{-e^2 \langle r^2 \rangle_{avg}\, n_e\, N\, \mu_0}{6\, m}$$

These were only *semi-classical* calculations.

Quoting the Feynman Lectures ...
" ... classical physics can give us some useful guesses as to what might happen — even though the really honest way to study the subject would be to understand the magnetism in terms of quantum mechanics."

"Section 34-4 : Classical physics gives neither diamagnetism nor paramagnetism"