Topic #4

# Plane Electromagnetic Waves and Wave Propagation

Chapter 7



The subject has two parts

- *propagation*

How EM waves move through a medium; wave motion without sources.

- *radiation*

How EM waves are created; what are the sources?

## Lecture 4-1 { Mon , Oct 7 }

## Plane Waves in a Nonconducting Medium

Section 7.1

Maxwell's equations for E.M. fields in an *infinite medium*, and *no sources*, are

| | |
|---|---|
| $\nabla \cdot \vec{B} = 0$ | $\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$ |
| $\nabla \cdot \vec{D} = 0$ | $\nabla \times \vec{H} - \frac{\partial \vec{D}}{\partial t} = 0$ |
| $\vec{D} = \epsilon \vec{E}$ | $\vec{B} = \mu \vec{H}$ |



Solutions with a definite frequency

Write

| | |
|---|---|
| $\vec{E}(\vec{x},t) = \vec{E}(\vec{x})\, e^{-i\omega t}$ | $\vec{B}(\vec{x},t) = \vec{B}(\vec{x})\, e^{-i\omega t}$ |
| $\nabla \cdot \vec{B} = 0$ | $\nabla \times \vec{E} - i\omega \vec{B} = 0$ |
| $\nabla \cdot \vec{D} = 0$ | $\nabla \times \vec{H} + i\omega \vec{D} = 0$ |

At first we'll assume that $\epsilon$ and $\mu$ are real and positive $\Longleftrightarrow \exists$ no absorption!

## Comment on the use of complex functions

$\vec{E}(\vec{x})\, e^{-i\omega t}$ is a complex mathematical function, but of course the electric field must be real (# volts per meter). So when we write

$$\vec{E}(\vec{x},t) = \vec{E}(\vec{x})\, e^{-i\omega t},$$

what we really mean is

$$\vec{E}(\vec{x},t) = \mathrm{Re}\, \vec{E}(\vec{x})\, e^{-i\omega t};$$

the real part (Re) is *understood*.

What about the imaginary part? That's just another solution.



## Plane waves

A plane wave is a mathematical idealization. For a plane wave,

$$\vec{E}(\vec{x},t) = \vec{E}_0\, e^{i\left(\vec{k}\cdot\vec{x} - \omega t\right)}$$

$$\vec{B}(\vec{x},t) = \vec{B}_0\, e^{i\left(\vec{k}\cdot\vec{x} - \omega t\right)}$$

In words, *the wave fronts are infinite planes.*

Of course this is not physically possible, but the plane wave is important for two reasons:

- it can approximate a wave with large (but finite) coherence length;
- the plane waves are ***complete;*** you know what is meant by "completeness" from quantum mechanics.

# Solving Maxwell's equations for plane waves

| | |
|---|---|
| $\vec{E}(\vec{x},t) = \vec{E}_0\, e^{i(\vec{k}\cdot\vec{x} - \omega t)}$ | $\vec{B}(\vec{x},t) = \vec{B}_0\, e^{i(\vec{k}\cdot\vec{x} - \omega t)}$ |
| 1) $i\,\vec{k}\cdot\vec{B}_0 = 0$ | 2) $i\,\vec{k}\times\vec{E}_0 - i\omega\,\vec{B}_0 = 0$ |
| 3) $i\,\vec{k}\cdot(\epsilon\,\vec{E}_0) = 0$ | 4) $i\,\vec{k}\times(\vec{B}_0/\mu) + i\omega(\epsilon\vec{E}_0) = 0$ |

We have 4 linear equations to solve.

▪▪ Equations (1) through (3) imply

$\{\vec{k}, \vec{E}_0, \vec{B}_0\}$ form a right-handed orthogonal triad of vectors.

Prove it.



▪▪ Now Equation (2) implies

$$k|\vec{E}_0| = \omega\,|\vec{B}_0|$$

or, $$B_0 = \frac{1}{v_{phase}}\,E_0$$

where $v_{phase} = \omega/k$ = the phase velocity.

Understand *phase velocity*

$$\text{fields} \propto \exp\{\, i\,(\vec{k}\cdot\vec{x} - \omega\, t)\,\}$$

∴ constant phase means

$$k\,\delta x - \omega\,\delta t = 0$$

$$\frac{\delta x}{\delta t} = \frac{\omega}{k} = v_{phase}.$$

## Equation (4)

```
In[•]:= Grid[{
  {Style["k⃗ × B⃗₀ = - μϵ ω E⃗₀", ff]},
  {Style["k B₀  =  μϵ ω E₀", ff]},
  {Style["k E₀ / v_phase  = μϵ  k v_phase  E₀", ff]},
  {Style["v_phase = 1/√(μϵ)", ff, Red]}}, Alignment → Left, Spacings → {0, 0.75}]
```

$$\vec{k} \times \vec{B}_0 = -\,\mu\epsilon\,\omega\,\vec{E}_0$$

$$k\,B_0 \;=\; \mu\epsilon\,\omega\,E_0$$

$$k\,E_0\,/\,v_{phase} \;=\; \mu\epsilon\;\;k\,v_{phase}\;\;E_0$$

$$v_{phase} = \frac{1}{\sqrt{\mu\epsilon}}$$

In vacuum all plane EM waves have the same phase velocity,

$$c = \frac{1}{\sqrt{\mu_0\,\epsilon_0}} = 2.998 \times 10^8 \;\; \text{m/s.}$$

That is how Maxwell discovered that light is an electromagnetic wave.



In an ideal medium,

$$v_{\text{phase}} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{n}$$

where the index of refraction is

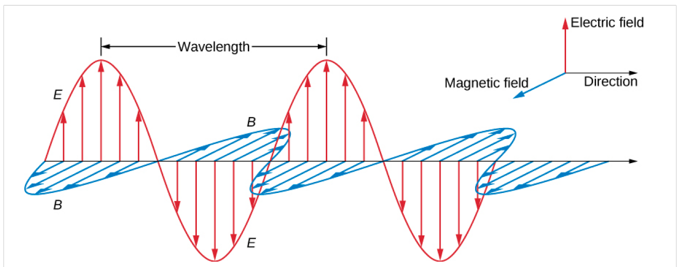
$$n = \sqrt{\mu\epsilon} \;/\; \sqrt{\mu_0\,\epsilon_0}$$

Because $\mu \approx \mu_0$ and $\epsilon > \epsilon_0$ any medium will have n > 1; i.e., $v_{\text{phase}} < c$.

## Pictures of an E.M. plane wave

The usual picture of an electromagnetic plane wave, which you'll see in many books and web sites, looks like this

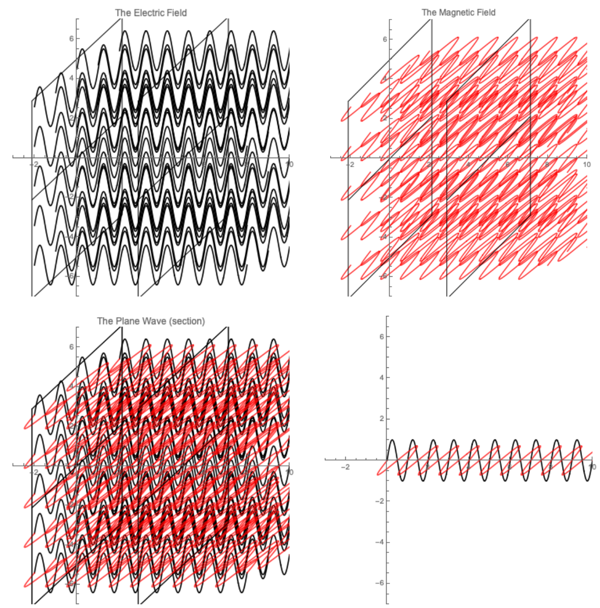
```
In[◦]:= emw
```




But that is quite misleading, because it only shows one ray. A better picture would somehow illustrate that the plane wave fills an infinite volume.



```
In[◦]:= GraphicsGrid[{
         {sh1, sh2}, {sh3, sh4}}]
```



## Animation

Of course the field vectors are varying in time, so the static pictures are only a ***snapshot*** of the wave. To understand propagation of an EM wave we need an animation.