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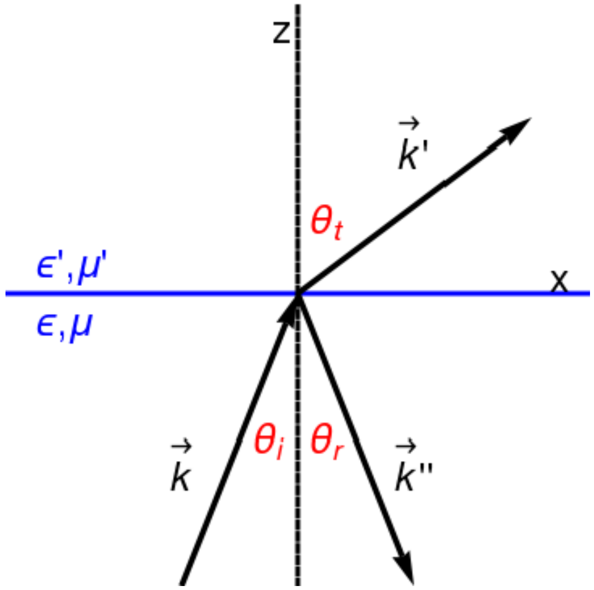
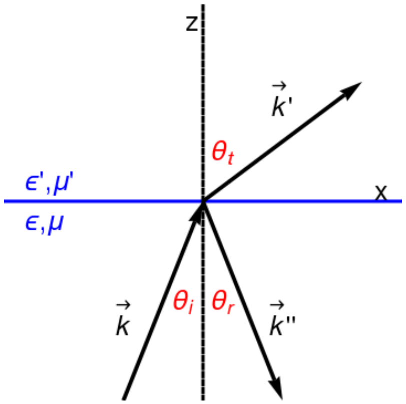
REFLECTION AND REFRACTION
at a plane interface

Jackson Section 7.3

Light, or some other electromagnetic wave, propagating in one medium strikes a planar interface that bounds a second medium. Taking the media to be linear, isotropic and lossless, they have parameters $\{\epsilon, \mu\}$ and $\{\epsilon', \mu'\}$. There will be reflection and refraction at the interface.

Figure 7.5 defines the geometry of the problem.





∎ The interface is the xy–plane; i.e., z = 0.

∎ The *plane of incidence* is the xz–plane = the plane spanned by the normal to the interface ($\hat{n} = \hat{e}_z$) and the incident wave vector $\vec{k}$.

We already know the solutions of the field equations, from the last lecture.

INCIDENT WAVE; x < 0

$$\vec{E}(\vec{x},t) = \vec{E}_0\, e^{i\left(\vec{k}\cdot\vec{x} - \omega t\right)}$$

$$c\,\vec{B}(\vec{x},t) = \sqrt{\mu\epsilon}\;\hat{k} \times \vec{E}(\vec{x},t)$$

..... the real part is the physical field

REFRACTED WAVE; x > 0

$$\vec{E}'(\vec{x},t) = \vec{E}_0{}' \, e^{i\left(\vec{k}'\cdot\vec{x} - \omega t\right)}$$

$$c\,\vec{B}'(\vec{x},t) = \sqrt{\mu'\epsilon'}\;\hat{k}' \times \vec{E}'(\vec{x},t)$$

REFLECTED WAVE; x < 0

$$\vec{E}''(\vec{x},t) = \vec{E}_0{}'' \, e^{i\left(\vec{k}''\cdot\vec{x} - \omega t\right)}$$

$$c\,\vec{B}''(\vec{x},t) = \sqrt{\mu\epsilon}\;\hat{k}'' \times \vec{E}''(\vec{x},t)$$

Note: the frequencies of all three waves are the same. The reason is because there are boundary conditions at $x = 0$, which must be satisfied for all t; ∴ the frequencies of oscillation must be the same.

Also : the wave vector magnitudes are

$$ck = ck'' = \omega\sqrt{\mu\epsilon} \quad \text{and} \quad ck' = \omega\sqrt{\mu'\epsilon'}$$

The law of reflection (≡ the law of equal angles) and the law of refraction (≡ Snell’s law)

The boundary conditions must hold for all t;

$$e^{-i\omega t} = e^{-i\omega' t} = e^{-i\omega'' t}$$

∴ the frequencies must be equal

The boundary conditions must hold for all points on the interface (x and y with $z = 0$)

$e^{i\vec{k}\cdot\vec{x}} = e^{i\vec{k}'\cdot\vec{x}} = e^{i\vec{k}''\cdot\vec{x}}$ with z = 0

- the three waves vector must lie in a plane

= the plane of incidence (y =0)

- $k_x = k_x' = k_x''$
- $k \sin(\theta_i) = k' \sin(\theta_t) = k'' \sin(\theta_r)$

Note: k" = k and k' / k = $\sqrt{\mu' \epsilon' / \mu\epsilon}$.

Results

$\theta_r = \theta_i$ {law of equal angles}

$n \sin(\theta_i) = n' \sin(\theta_t)$ {Snell's law}

The *index of refraction* (n and n') for a transparent medium is defined by

$$n = \sqrt{\mu\epsilon} = \frac{c}{v_{phase}}$$

$$n' = \sqrt{\mu' \epsilon'} = \frac{c}{v'_{phase}}$$

The equations so far determine the *directions* of the reflected and refracted waves. Next, *what are their intensities?* I.e., what is the *energy flux* for each of the three waves?

Recall:

$\vec{S} = \vec{E} \times \vec{H}$ (using real fields)

$\langle \vec{S} \rangle = \frac{1}{2} \vec{E}(\omega) \times \vec{H}(\omega)^*$ (for harmonic fields)

Fresnel’s equations

Wikipedia —
The Fresnel equations (or Fresnel coefficients) describe the reflection and transmission of light (or electromagnetic radiation in general) when incident on an interface between different optical media

They were deduced by Augustin-Jean Fresnel, who was the first to understand that light is a transverse wave, even though no one realized that the "vibrations" of the wave were electric and magnetic fields.

For the first time, polarization could be understood quantitatively, as Fresnel' s equations correctly predicted the differing behaviour of waves of the s and p polarizations incident upon a material interface.

Jackson refers to these issues as the “Dynamic Properties” of reflection and refraction:
(a) Intensities; (b) phase changes and polarization.

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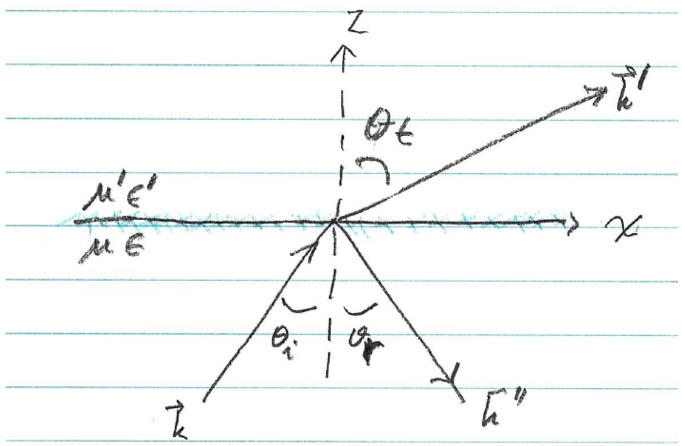
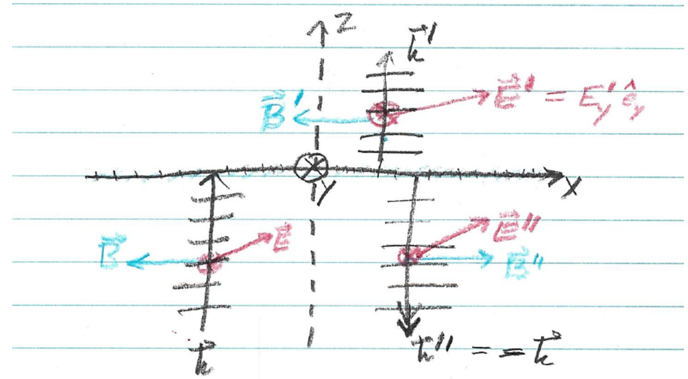


Normal Incidence



$$\theta_{inc} = \theta_{refl} = \theta_{trans} = 0$$

$$\omega \vec{B} = \vec{k} \times \vec{E}$$

$$\vec{E} = E_y \, \hat{e}_y \quad \text{and} \quad \vec{B} = - \, \hat{e}_x \frac{E_y}{v_{phase}}$$

$$v_{phase} = \sqrt{\mu \epsilon} = \frac{c}{n}$$

Reflection and refraction at normal incidence: $\theta_{Inc} = \theta_{Trans} = \theta_{Refl} = 0$.
{ set $c = 1$ }

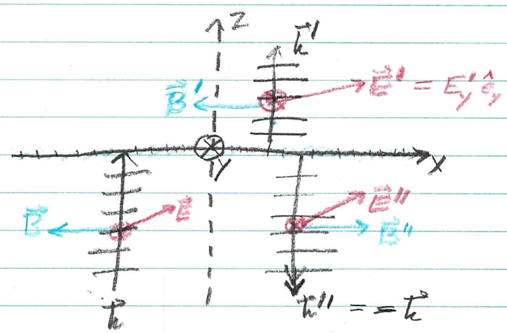
Incident :

$$\vec{K} = k_z\,\hat{e}_z \quad ; \quad k_z = \frac{\omega}{\sqrt{\mu\epsilon}}$$

$$\vec{E} = E_0\, e^{i(k_z z - \omega t)}\,\hat{e}_y \quad (TE)$$

$$\omega\,\vec{B} = \vec{K} \times \vec{E}$$

$$\vec{B} = \frac{-E_0}{\sqrt{\mu\epsilon}}\, e^{i(k_z z - \omega t)}\,\hat{e}_x \quad (TM)$$



$\theta_{Inc} = \theta_{refl} = \theta_{trans} = 0$

$\omega\vec{B} = \vec{k}\times\vec{E}$

$\vec{E} = E_y\,\hat{e}_y$ and $\vec{B} = -\,\hat{e}_x\,\frac{E_y}{\upsilon_{phase}}$

$\upsilon_{phase} = \sqrt{\mu\epsilon} = \frac{c}{n}$

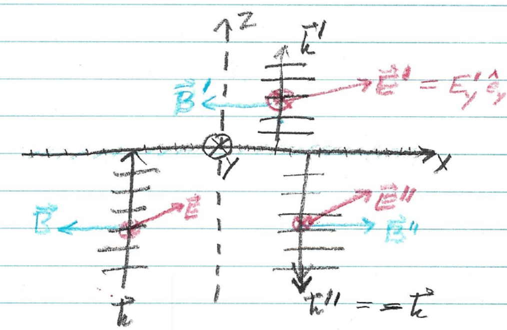
Transmitted :

$$\vec{K}' = k_z'\,\hat{e}_z \quad ; \quad k_z' = \frac{\omega}{\sqrt{\mu'\epsilon'}}$$

$$\vec{E}' = E_0'\, e^{i(k_z' z - \omega t)}\,\hat{e}_y \quad (TE)$$

$$\omega\,\vec{B}' = \vec{K}' \times \vec{E}'$$

$$\vec{B}' = \frac{-E_0'}{\sqrt{\mu'\epsilon'}}\, e^{i(k_z' z - \omega t)}\,\hat{e}_x \quad (TM)$$



$\theta_{Inc} = \theta_{refl} = \theta_{trans} = 0$

$\omega\vec{B} = \vec{k}\times\vec{E}$

$\vec{E} = E_y\,\hat{e}_y$ and $\vec{B} = -\,\hat{e}_x\,\frac{E_y}{\upsilon_{phase}}$

$\upsilon_{phase} = \sqrt{\mu\epsilon} = \frac{c}{n}$

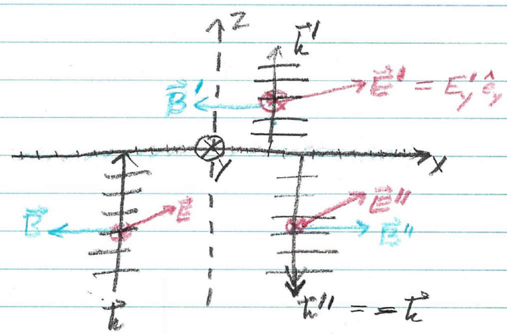
Reflected :

$$\vec{K}'' = k_z''\,\hat{e}_z \quad ; \quad k_z'' = \frac{-\omega}{\sqrt{\mu\epsilon}}$$

$$\vec{E}'' = E_0''\, e^{i(k_z'' z - \omega t)}\,\hat{e}_y \quad (TE)$$

$$\omega\,\vec{B}'' = \vec{K}'' \times \vec{E}''$$

$$\vec{B}'' = \frac{+E_0''}{\sqrt{\mu\epsilon}}\, e^{i(k_z'' z - \omega t)}\,\hat{e}_x \quad (TM)$$



$\theta_{inc} = \theta_{refl} = \theta_{trans} = 0$

$\omega\vec{B} = \vec{k}\times\vec{E}$

$\vec{E} = E_y\,\hat{e}_y$ and $\vec{B} = -\,\hat{e}_x\,\dfrac{E_y}{v_{phase}}$

$v_{phase} = \sqrt{\mu\epsilon} = \dfrac{c}{n}$

Boundary Conditions

- $E_{tang.}(z=0) = E_y\,(z=0)$

$$= E_0 + E_0'' = E_0'$$

- $H_{tang.}(z=0) = H_x\,(z=0)$

$$= \frac{-E_0 + E_0''}{\mu n} = \frac{-E_0'}{\mu' n'}$$

$\Longrightarrow$

$$E_0' = \frac{2n'\mu'}{n\mu + n'\mu'}\,E_0 \quad \text{and} \quad E_0'' = \frac{-n\mu + n'\mu'}{n\mu + n'\mu'}\,E_0$$

Example : air and glass

$\mu = \mu' = \mu_0$; all factors of μ_0 cancel;

$n=1.0$ and $n' = 1.5$

$E_0' = 1.2\ E_0$ and $E_0'' = 0.2\ E_0$

Intensities: $\vec{S} = \vec{E} \times \vec{H}$ $\{$set $\mu_0 = 1\}$

$S_{inc} = E_0 \frac{E_0}{\mu n} = E_0^2$

$S_{trans} = E_0' \frac{E_0'}{\mu' n'} = 0.96\ E_0^2$

$S_{refl} = E_0'' \frac{E_0''}{\mu n} = 0.04\ E_0^2$