enter

How do Matter and Light Interact?

■ In classical electrodynamics

Propagation

- In vacuum the speed of light is

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \quad \text{(independent of } \omega\text{)}.$$

- In simple materials,

$$v_{phase} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{n} \;;$$

$$n = \sqrt{\mu\epsilon} = \text{index of refraction} \;.$$

Reflection and Refraction

- At an interface,

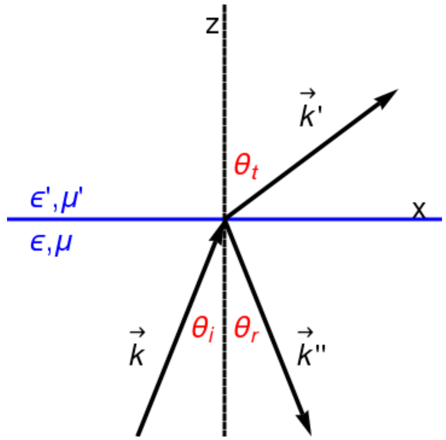
$$\theta_R = \theta_I \quad \text{and} \quad n_I \sin(\theta_I) = n_T \sin(\theta_T)$$

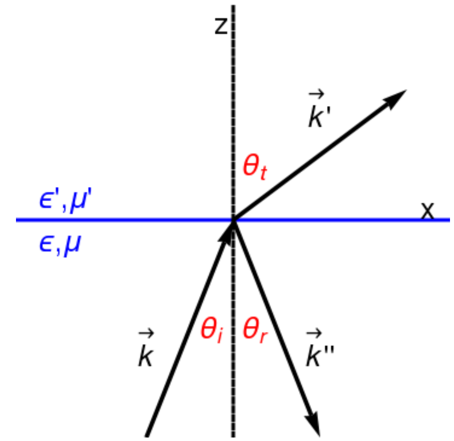
FRESNEL'S EQUATIONS (Augustin-Jean Fresnel, 1821)

Jackson Section 7.3

Light, or some other electromagnetic wave, propagating in one medium strikes a planar interface that bounds a second medium. Taking the media to be linear, isotropic and lossless, they have parameters $\{\epsilon, \mu\}$ and $\{\epsilon', \mu'\}$. There will be reflection and refraction at the interface.

Figure 7.5 defines the geometry of the problem.





- The *interface* is the xy–plane; i.e., z = 0.
- The normal to the interface is $\hat{n} = \hat{e}_z$.
- The *plane of incidence* is the xz–plane = the plane spanned by the normal to the interface ($\hat{n} = \hat{e}_z$) and the incident wave vector $\vec{k} = k_x\, \hat{e}_x + k_z\, \hat{e}_x$.

We already know the solutions of the field equations, from the last lecture.

INCIDENT WAVE; x < 0

$\vec{E}(\vec{x},t) = \vec{E}_0\, e^{i(\vec{k}\cdot\vec{x}-\omega t)}$

$\vec{B}(\vec{x},t) = \sqrt{\mu\epsilon}\; \hat{k} \times \vec{E}(\vec{x},t)$

..... the real part is the physical field

The law of reflection (≡ the law of equal angles) and the law of refraction (≡ Snell's law)

The boundary conditions must hold for all t;

∴ the frequencies must be equal

The boundary conditions must hold for all values of x on the interface (i.e., with z = 0)

$\therefore\ k_x = k_x' = k_x''$

$k \sin(\theta_i) = k' \sin(\theta_t) = k'' \sin(\theta_r)$

$k'' = k = \frac{\omega}{c}\, n$ and $k' = \frac{\omega}{c}\, n'$

$\theta_{refl} = \theta_{inc}$ and $n \sin(\theta_{inc}) = n' \sin(\theta_{trans})$

Boundary Conditions :: Solve for $\vec{E}_0'$ and $\vec{E}_0''$

- $E_{\text{tangential}}$ and B_{normal} are continuous at z = 0;
- there are no free charges or currents, so

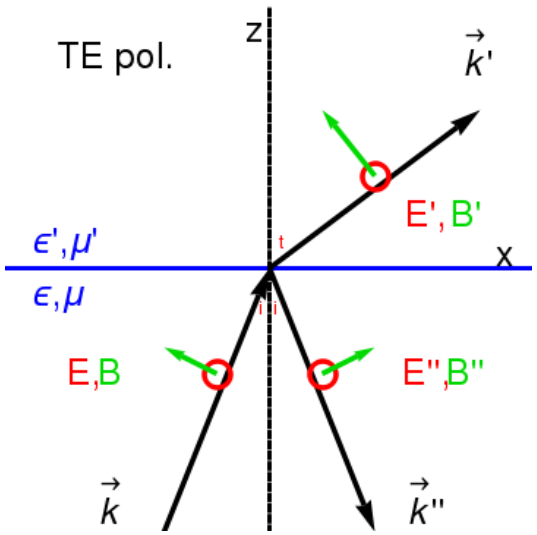
D_{normal} and $H_{\text{tangential}}$ are continuous at z = 0.

$\Longrightarrow$ four boundary conditions

D_n	$\epsilon(\vec{E}_0 + \vec{E}_0'') \cdot \hat{e}_z = \epsilon' \vec{E}_0' \cdot \hat{e}_z$
E_t	$(\vec{E}_0 + \vec{E}_0'') \times \hat{e}_z = \vec{E}_0' \times \hat{e}_z$
B_n	$(\vec{k} \times \vec{E}_0 + \vec{k}'' \times \vec{E}_0'') \cdot \hat{e}_z = (\vec{k}' \times \vec{E}_0') \cdot \hat{e}_z$
H_t	$\frac{1}{\mu}(\vec{k} \times \vec{E}_0 + \vec{k}'' \times \vec{E}_0'') \times \hat{e}_z = \frac{1}{\mu'}(\vec{k}' \times \vec{E}_0') \times \hat{e}_z$

Now given $\vec{E}_0$, solve for $\vec{E}_0'$ and $\vec{E}_0''$.
This should be easy because it is only linear algebra. But it is not so easy because it is *vectors !*
We must separate the problem into two parts, called “Transverse Electric polarization” and “Transverse Magnetic polarization”, which refer to two different polarizations of the incident waves.

TE pol	$\vec{E}$ is perpendicular to the plane of incidence
	$\vec{E} = E_y \hat{e}_y$
	$\vec{B} = B_x \hat{e}_x + B_z \hat{e}_z$



TE polarization	
$D_{norm} = D_z$	$0 = 0$
$E_{tang} = E_y$	$E_0 + E_0'' = E_0'$
$B_{norm} = B_z$	$k_x E_0 + k_x'' E_0'' = k_x' E_0'$; $E_0 + E_0'' = E_0'$
$H_{tang} = H_x$	$\frac{1}{\mu\omega}\,\hat{e}_x(-k_z E_0 - k_z'' E_0'')$ $= \frac{1}{\mu'\omega}\,\hat{e}_x(-k_z' E_0')$

The last equation $H_x + H_x''$ (z=0-) = H_x' (z=0+)

$$\vec{H} = \frac{1}{\mu}\vec{B} = \frac{1}{\mu}\frac{\vec{k}\times\vec{E}}{\omega} \text{ with } E = E_y\,\hat{e}_y$$

$$H_x \Longrightarrow \frac{1}{\mu\omega}(-k_z E_0 - k_z'' E_0'') = \frac{1}{\mu'\omega}(-k_z' E_0')$$

$$k_z = \frac{n\omega}{c}\cos\{i\} = -k_z'' \quad \text{and} \quad k_z' = \frac{n'\omega}{c}\cos\{t\}$$

$$\frac{n}{\mu}(E_0 - E_0'')\cos\{i\} = \frac{n'}{\mu'}E_0'\cos\{t\}$$

Now solve for E_0'/E_0 and E_0''/E_0.
(Later do the same for TM polarization.)

Fresnel's equations, ***assuming*** $\boldsymbol{\mu' = \mu = \mu_0}$ **.**

... the assumption is appropriate for dielectrics,
which generally have very small magnetic susceptibilities.

Eqs (7.39) and (7.41)

TE pol	$\vec{E}$ is perpendicular to the plane of incidence
TE pol	$\frac{E_0'}{E_0} = \frac{2n\cos(\theta_{inc})}{n\cos(\theta_{inc}) + n'\cos(\theta_{trans})}$
TE pol	$\frac{E_0''}{E_0} = \frac{n\cos(\theta_{inc}) - n'\cos(\theta_{trans})}{n\cos(\theta_{inc}) + n'\cos(\theta_{trans})}$
TM pol	$\vec{E}$ is parallel to the plane of incidence
TM pol	$\frac{E_0'}{E_0} = \frac{2n\cos(\theta_{inc}),}{n'\cos(\theta_{inc}) + n\cos(\theta_{trans})}$
TM pol	$\frac{E_0''}{E_0} = \frac{n'\cos(\theta_{inc}) - n\cos(\theta_{trans})}{n'\cos(\theta_{inc}) + n\cos(\theta_{trrans})}$

Example: suppose n = 1 (air) and n’ = 1.5 (glass)

TE polarization

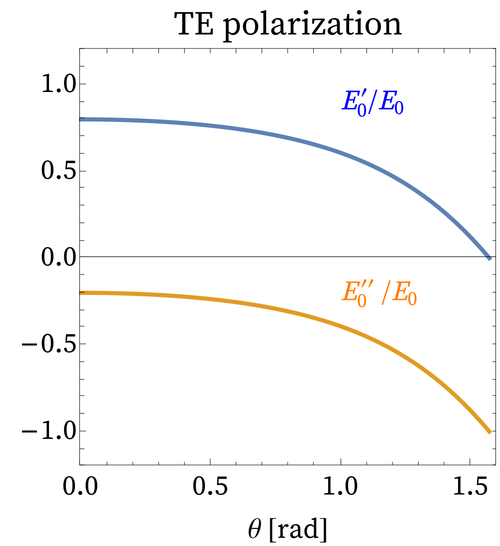
```
{n, np} = {1, 1.5};
(* Snells' law ; θ = inc, θp = transm *)
θp = ArcSin[n / np * Sin[θ]];
Etransm[θ_] = 2 * n * Cos[θ] / (n * Cos[θ] + np * Cos[θp]);
Erefl[θ_] = (n * Cos[θ] - np * Cos[θp]) /
    (n * Cos[θ] + np * Cos[θp]);
```

```
l1 = {Blue, Text["E0′/E0", {1.0, 0.9}, {-1, 0}]};
l2 = {Orange, Text["E0″ /E0", {1.0, -0.2}, {-1, 0}]};
```

```
sh = Show[
    Plot[{Etransm[θ], Erefl[θ]}, {θ, 0, Pi / 2},
     PlotRange → {{0, 1.6}, {-1.2, 1.2}},
     Frame → True, ImageSize → 450,
     FrameLabel → {"θ [rad]", ""},
     PlotLabel → "TE polarization",
     PlotStyle → Thickness[0.01], AspectRatio → 1,
     BaseStyle → {FontFamily → "Source Serif Pro", 24}],
    Graphics[{l1, l2}]];
```



```
sh
```



Note : ∃ phase change upon reflection (E_0''<0) because $n' > 1$. (▪▪I must have had a sign error last time.▪▪)

But the interesting question is, what are the *intensities* of the waves?

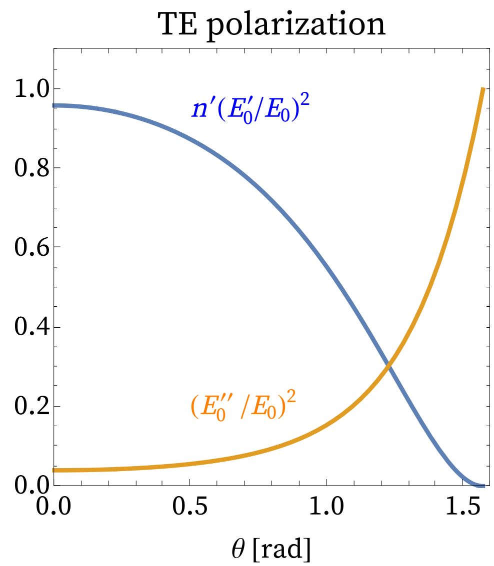
$$\langle \vec{S} \rangle = \frac{1}{2}\, \vec{E} \times \vec{H}*$$

$$\langle \vec{S} \rangle = \frac{1}{2}\, \vec{E} \times \frac{1}{\mu\omega} \left(\vec{k} \times \vec{E}* \right) = \frac{\vec{k}}{2\,\mu\omega}\, E_0^2$$

$$S_{in} = \frac{n}{2\,\mu c}\, E_0^2 \;\; ; \;\; S_{refl} = \frac{n}{2\,\mu c}\, (E_0'')^2 \;\; ; \;\; S_{transm} = \frac{n'}{2\,\mu' c}\, (E_0')^2$$


```
In[•]:= l1R = {Blue, Text["n′(E′0/E0)^2", {0.5, 0.95}, {-1, 0}]};
l2R = {Orange, Text["(E″0 /E0)^2", {0.5, 0.2}, {-1, 0}]};
sh2 = Show[
   Plot[{
     np * Etransm[θ]^2, Erefl[θ]^2}, {θ, 0, Pi / 2},
    PlotRange → {{0, 1.6}, {0, 1.1}}, Frame → True, ImageSize → 450,
    FrameLabel → {"θ [rad]", ""}, PlotLabel → "TE polarization",
    PlotStyle → Thickness[0.01], AspectRatio → 1,
    BaseStyle → {FontFamily → "Source Serif Pro", 24}],
   Graphics[{l1R, l2R}]];
```

In[•]:= sh2



Normal Incidence ($\theta = 0$) $\longleftrightarrow$ 96% transmitted, 4% reflected

Grazing Incidence ($\theta = \pi/2$) $\longleftrightarrow$ 100% reflected

TM polarization

In[•]:=

```
{n, np} = {1, 1.5};
(* Snells' law ; θ = inc, θp = transm *)
θp = ArcSin[n / np * Sin[θ]];
Etransm[θ_] = 2 * n * Cos[θ] / (np * Cos[θ] + n * Cos[θp]);
Erefl[θ_] = (np * Cos[θ] - n * Cos[θp]) /
    (np * Cos[θ] + n * Cos[θp]);
```

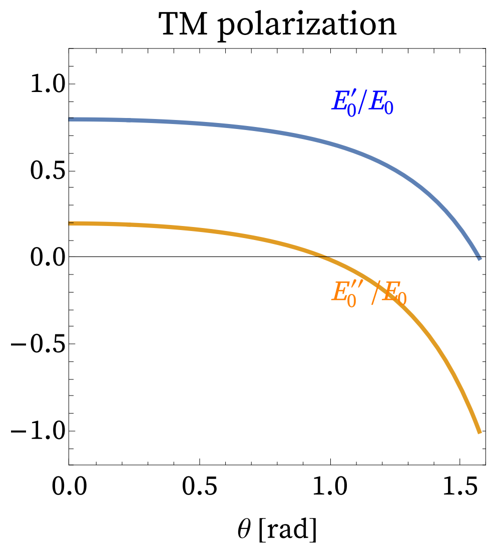
In[•]:=

```
l1 = {Blue, Text["E₀′/E₀", {1.0, 0.9}, {-1, 0}]};
l2 = {Orange, Text["E₀″ /E₀", {1.0, -0.2}, {-1, 0}]};
```

In[•]:=

```
sh3 = Show[
   Plot[{Etransm[θ], Erefl[θ]}, {θ, 0, Pi / 2},
    PlotRange → {{0, 1.6}, {-1.2, 1.2}},
    Frame → True, ImageSize → 450,
    FrameLabel → {"θ [rad]", ""},
    PlotLabel → "TM polarization",
    PlotStyle → Thickness[0.01], AspectRatio → 1,
    BaseStyle → {FontFamily → "Source Serif Pro", 24}],
   Graphics[{l1, l2}]];
```

In[»]:= sh3

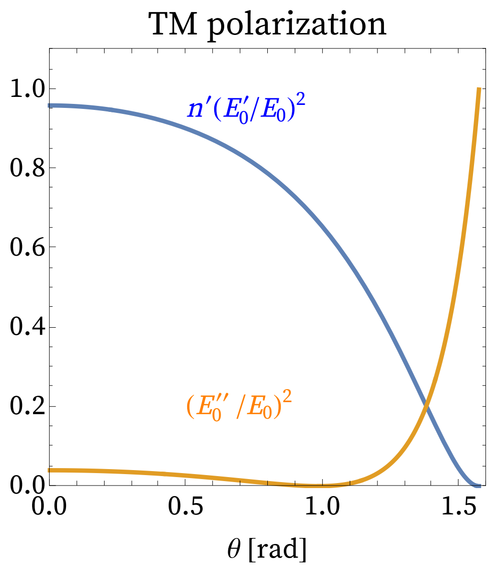


Interesting : ∃ phase change for $\theta > \theta_B$ but no phase change for $\theta < \theta_B$; Brewster's angle.

In[»]:=
```
l1R = {Blue, Text["n′ (E′₀/E₀)²", {0.6, 0.95}, {-1, 0}]};
l2R = {Orange, Text["(E″₀ /E₀)²", {0.5, 0.1}, {-1, 0}]};
```

In[»]:=
```
sh4 = Show[
   Plot[{np * (Etransm[θ]) ^2, (Erefl[θ]) ^2}, {θ, 0, Pi / 2},
    PlotRange → {{0, 1.6}, {0, 1.1}},
    Frame → True, ImageSize → 450,
    FrameLabel → {"θ [rad]", ""},
    PlotLabel → "TM polarization",
    PlotStyle → Thickness[0.01], AspectRatio → 1,
    BaseStyle → {FontFamily → "Source Serif Pro", 24}],
   Graphics[{l1R, l2R}]];
```

In[•]:= sh4



normal incidence...
grazing incidence...
Brewster’s angle.