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How do Matter and Light Interact?

■ In classical electrodynamics

- In simple materials,

$$v_{phase} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{n} \;;$$

- At an interface,

$\theta_R = \theta_I$ and $n_I \sin(\theta_I) = n_T \sin(\theta_T)$

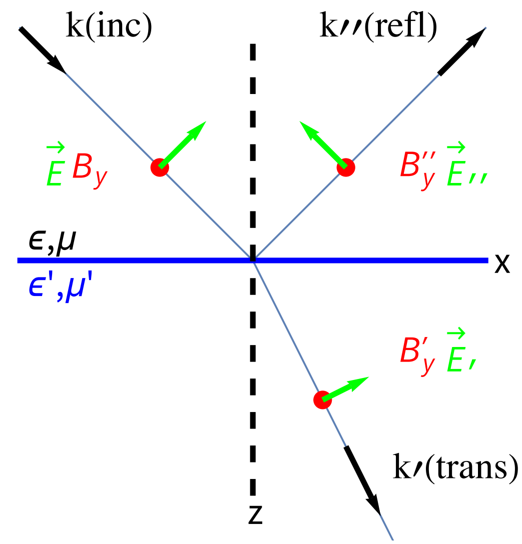
■ But in reality, *energy and light are quantized*. So does the classical theory work? Or, does it?

■ Many materials are transparent. Why is that?
E.g., gases, water, glass, crystals (salt, quartz, ...) etc

■ Almost everything we see is seen by reflection;
very few materials are self-luminous.

FRESNEL'S EQUATIONS FOR REFLECTION AND REFRACTION (Augustin-Jean Fresnel, 1821)

In[•]:= sh



E.g., the surface of a lake

Something remarkable occurs for plane waves with TM polarization—Brewster's angle

Boundary Conditions :: Solve for $\vec{E}_0'$ and $\vec{E}_0''$

- $E_{\text{tangential}}$ and B_{normal} are continuous at z = 0;
- there are no free charges or currents, so

D_{normal} and $H_{\text{tangential}}$ are continuous at z = 0.

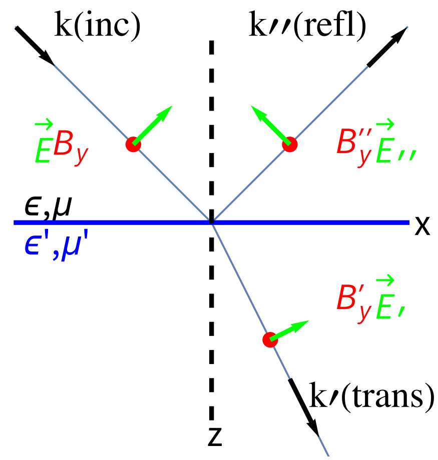
$\Longrightarrow$ $\exists$ four boundary conditions

$$\omega \vec{B} = \vec{k} \times \vec{E}; \qquad \text{or,} \quad \vec{H} = \frac{1}{\mu\omega} \vec{k} \times \vec{E}$$

D_n	$\epsilon(\vec{E}_0 + \vec{E}_0'') \cdot \hat{e}_z = \epsilon' \vec{E}_0' \cdot \hat{e}_z$
E_t	$(\vec{E}_0 + \vec{E}_0'') \times \hat{e}_z = \vec{E}_0' \times \hat{e}_z$
B_n	$(\vec{k} \times \vec{E}_0 + \vec{k}'' \times \vec{E}_0'') \cdot \hat{e}_z = (\vec{k}' \times \vec{E}_0') \cdot \hat{e}_z$
H_t	$\frac{1}{\mu}(\vec{k} \times \vec{E}_0 + \vec{k}'' \times \vec{E}_0'') \times \hat{e}_z = \frac{1}{\mu'} (\vec{k}' \times \vec{E}_0') \times \hat{e}_z$

Solving the boundary conditions for TM Polarization (note: +z points downward; +y points out of board)

In[•]:= sh



TM pol	$\vec{H}$ is perpendicular to the plane of incidence
	$\vec{H} = H_y \hat{e}_y$
	$\vec{E} = E_x \hat{e}_x + E_z \hat{e}_z$

TM polarization	
$D_{norm} = \epsilon E_z$	$\epsilon (E_z + E''_z) = \epsilon' E'_z$
$E_{tang} = E_x$	$E_x + E''_x = E'_x$
$B_{norm} = B_z$	$0 = 0$
$H_{tang} = H_y$	$H_y + H''_y = H'_y$

Fresnel's equations, ***assuming*** $\mu' = \mu = \mu_0$ **.**

... the assumption is appropriate for dielectrics, which generally have very small magnetic susceptibilities.

$\vec{E} = E_0\, \hat{\epsilon} \exp i(\vec{k} \cdot \vec{x} - \omega t)$; $\hat{\epsilon} = \hat{e}_x \cos\{i\} - \hat{e}_z \sin\{i\}$

$\vec{E}' = E_0'\, \hat{\epsilon}' \exp i(\vec{k}' \cdot \vec{x} - \omega t)$; $\hat{\epsilon}' = \hat{e}_x \cos\{t\} - \hat{e}_z \sin\{t\}$

$\vec{E}'' = E_0''\, \hat{\epsilon}'' \exp i(\vec{k}'' \cdot \vec{x} - \omega t)$; $\hat{\epsilon}'' = -\hat{e}_x \cos\{r\} - \hat{e}_z \sin\{r\}$

$\Longrightarrow$ Eq. (7.41)

TM pol	$\vec{H}$ is perpendicular to the plane of incidence
TM pol	$\frac{E_0'}{E_0} = \frac{2\, n \cos(\theta_{inc}),}{n' \cos(\theta_{inc}) + n \cos(\theta_{trans})}$
TM pol	$\frac{E_0''}{E_0} = \frac{n' \cos(\theta_{inc}) - n \cos(\theta_{trans})}{n' \cos(\theta_{inc}) + n \cos(\theta_{trrans})}$

I can verify that the boundary conditions are obeyed

```
In[•]:= Remove["Global`*"]

In[•]:= rpl1 = {E0p → E0 * 2 * n * Cos[θ] / (np * Cos[θ] + n * Cos[θp])};
        rpl2 = {E0pp → E0 *
             (np * Cos[θ] - n * Cos[θp]) /
              (np * Cos[θ] + n * Cos[θp])};

In[•]:= (* 1: Dnorm *)
        Ez = -E0 * Sin[θ];
        Ezp = -E0p * Sin[θp];
        Ezpp = -E0pp * Sin[θ];
        test1 = n ^ 2 * (Ez + Ezpp) - np ^ 2 * Ezp;
        test1 = test1 /. rpl1;
        test1 = test1 /. rpl2;
        test1 = test1 /. {E0 → 1};
        test1 = test1 // Expand // Simplify
        test1 /. {np →  n * Sin[θ] / Sin[θp]}
```

$$\frac{2\, n\, np\, \mathrm{Cos}[\theta]\, (-n\, \mathrm{Sin}[\theta] + np\, \mathrm{Sin}[\theta p])}{np\, \mathrm{Cos}[\theta] + n\, \mathrm{Cos}[\theta p]}$$

$$0$$


```
In[•]:= (* 2: Etang *)
        Ex = +E0 * Cos[θ];
        ExP = +E0p * Cos[θp];
        ExPP = -E0pp * Cos[θ];
        test2 = (Ex + ExPP) - ExP;
        test2 = test2 /. rpl1;
        test2 = test2 /. rpl2;
        test2 = test2 /. {E0 → 1}
        test2 = test2 // Expand // Simplify
```

$$\mathrm{Cos}[\theta] - \frac{2\, n\, \mathrm{Cos}[\theta]\, \mathrm{Cos}[\theta p]}{np\, \mathrm{Cos}[\theta] + n\, \mathrm{Cos}[\theta p]} - \frac{\mathrm{Cos}[\theta]\, (np\, \mathrm{Cos}[\theta] - n\, \mathrm{Cos}[\theta p])}{np\, \mathrm{Cos}[\theta] + n\, \mathrm{Cos}[\theta p]}$$

$$0$$

```
(* Htang *)
Remove[Ex, Ez, ExP, EzP, ExPP, EzPP]
kv = {kx, 0, kz}; kP = {kxP, 0, kzP};
kPP = {kxPP, 0, kzPP};
Ev = {Ex, 0, Ez}; EP = {ExP, 0, EzP};
EPP = {ExPP, 0, EzPP};
Hy = Dot[Cross[kv, Ev], {0, 1, 0}]
HyP = Dot[Cross[kP, EP], {0, 1, 0}]
HyPP = Dot[Cross[kPP, EPP], {0, 1, 0}]
```

-Ez kx + Ex kz

-EzP kxP + ExP kzP

-EzPP kxPP + ExPP kzPP

```
{kx, kxP, kxPP} = {n*Sin[θ], np*Sin[θp], n*Sin[θ]};
{kz, kzP, kzPP} = {n*Cos[θ], np*Cos[θp], -n*Cos[θ]};
{Ex, ExP, ExPP} = {E0*Cos[θ], E0p*Cos[θp], -E0pp*Cos[θ]};
{Ez, EzP, EzPP} = {-E0*Sin[θ], -E0p*Sin[θp], -E0pp*Sin[θ]};
```

```
test3 = (Hy + HyPP) - HyP
test3 = test3 /. rpl1;
test3 = test3 /. rpl2;
test3 // Simplify
```

$$E0\, n \cos[\theta]^2 + E0pp\, n \cos[\theta]^2 - E0p\, np \cos[\theta p]^2 + E0\, n \sin[\theta]^2 + E0pp\, n \sin[\theta]^2 - E0p\, np \sin[\theta p]^2$$

0

TM polarization

Amplitudes

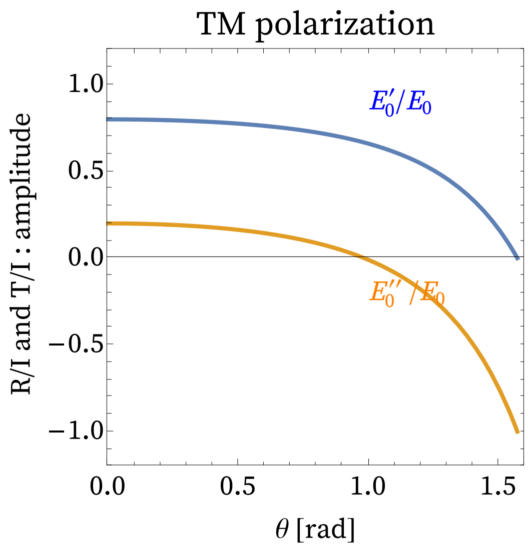
```
Remove["Global`*"]
```

```
{n, np} = {1, 1.5};
(* Snells' law ; θ = inc, θp = transm *)
θp = ArcSin[n / np * Sin[θ]];
Etransm[θ_] = 2 * n * Cos[θ] / (np * Cos[θ] + n * Cos[θp]);
Erefl[θ_] = (np * Cos[θ] - n * Cos[θp]) /
   (np * Cos[θ] + n * Cos[θp]);
```

```
l1 = {Blue, Text["E0'/E0", {1.0, 0.9}, {-1, 0}]};
l2 = {Orange, Text["E0'' /E0", {1.0, -0.2}, {-1, 0}]};
```

```
sh11 = Show[
   Plot[{Etransm[θ], Erefl[θ]}, {θ, 0, Pi / 2},
    PlotRange → {{0, 1.6}, {-1.2, 1.2}},
    Frame → True, ImageSize → 450,
    FrameLabel → {"θ [rad]", " R/I and T/I : amplitude"},
    PlotLabel → "TM polarization",
    PlotStyle → Thickness[0.01], AspectRatio → 1,
    BaseStyle → {FontFamily → "Source Serif Pro", 24}],
   Graphics[{l1, l2}]];
```

In[•]:= sh11



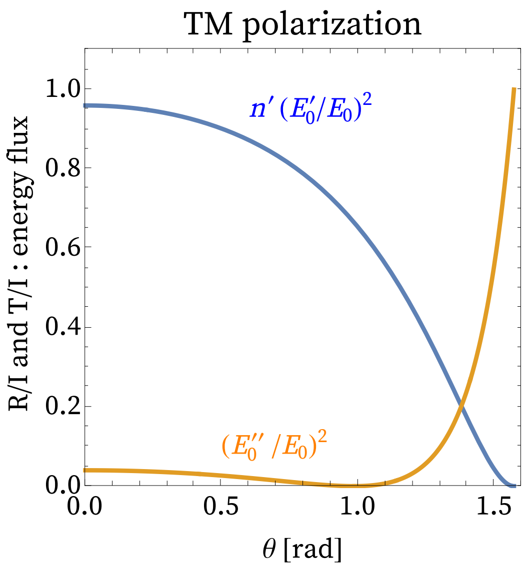
Interesting : ∃ phase change for $\theta > \theta_B$ but no phase change for $\theta < \theta_B$; Brewster's angle.

Intensities

In[•]:=
```
l1R = {Blue, Text["n′ (E′0/E0)^2", {0.6, 0.95}, {-1, 0}]};
l2R = {Orange, Text["(E″0 /E0)^2", {0.5, 0.1}, {-1, 0}]};
```

In[•]:=
```
sh12 = Show[
   Plot[{np * (Etransm[θ]) ^ 2, (Erefl[θ]) ^ 2}, {θ, 0, Pi / 2},
    PlotRange → {{0, 1.6}, {0, 1.1}},
    Frame → True, ImageSize → 450,
    FrameLabel → {"θ [rad]",
      " R/I and T/I : energy flux"},
    PlotLabel → "TM polarization",
    PlotStyle → Thickness[0.01], AspectRatio → 1,
    BaseStyle → {FontFamily → "Source Serif Pro", 24}],
   Graphics[{l1R, l2R}]];
```

In[•]:= sh12



normal incidence... grazing incidence... Brewster's angle.

Energies

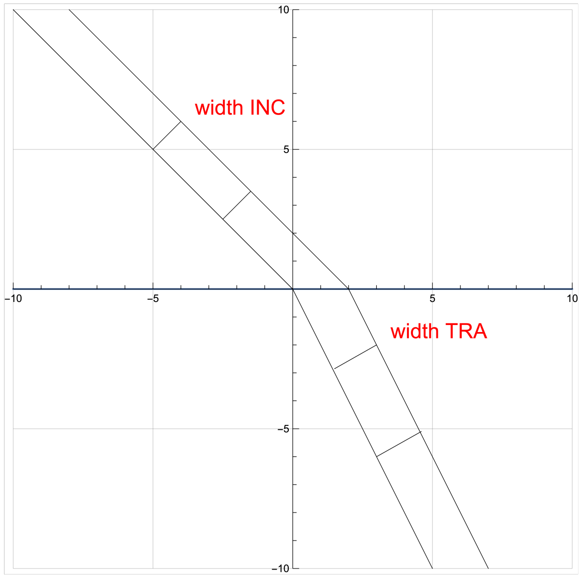
The equation for conservation of energy is

$$\frac{\cos(\theta\prime)}{\cos(\theta)}\,T + R = 1$$

where T = S'/S and R = S''/S

The factor cos(θ')/cos(θ) reflects the fact that a transmitted beam has a larger area (by that factor) than the incident beam; because of the refraction.

In[•]:= shf

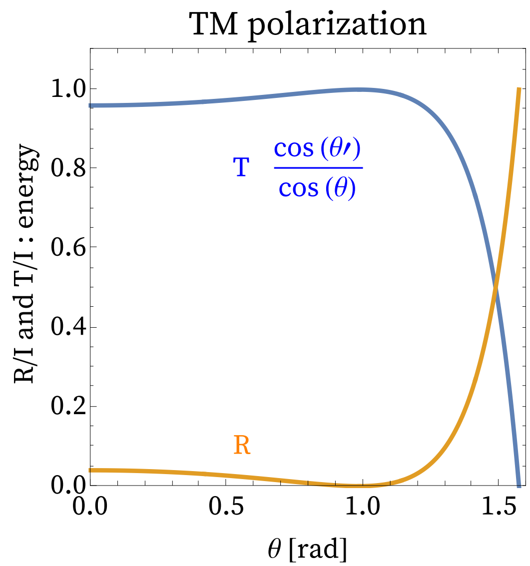


In[•]:=
```
T[θ_] = np * (Etransm[θ]) ^ 2;
R[θ_] = (Erefl[θ]) ^ 2;
fac[θ_] =
  Sqrt[1 - n ^ 2 / np ^ 2 * Sin[θ] ^ 2] / Cos[θ];
```

In[•]:=
```
lT2 = {Blue, Text[" T  cos (θ′)/cos (θ) ", {0.5, 0.8}, {-1, 0}]};
lR2 = {Orange, Text[" R ", {0.5, 0.1}, {-1, 0}]};
```

In[•]:=
```
sh13 = Show[
   Plot[{T[θ] * fac[θ], R[θ]},
    {θ, 0, Pi / 2},
    PlotRange → {{0, 1.6}, {0, 1.1}},
    Frame → True, ImageSize → 450,
    FrameLabel → {"θ [rad]", " R/I and T/I : energy"},
    PlotLabel → "TM polarization",
    PlotStyle → Thickness[0.01], AspectRatio → 1,
    BaseStyle → {FontFamily → "Source Serif Pro", 24}],
   Graphics[{lT2, lR2}]];
```

In[]:= sh13



“Why do fishermen wear polarized sunglasses?”
The answer to this question is “Brewster’s angle”.

Explanation

Calculate Brewster's angle for water:

$E_0'' =$ (n' cosθ - n cos θ ') / (n' cosθ + n cos θ ') E_0

$E_0'' = 0$ implies n’cosθ = n cos θ’

Also, n sinθ = n’ sinθ’ (Snell’s law)

Solve for $\theta \equiv \theta_B$

$\tan\theta_B = \frac{n'}{n}$

For water θ_B = arctan(1.33) = 53.06 degrees.
If the sun is 36.94 degrees above the horizontal then sunlight reflected from the lake surface is 100 percent TE polarized.
That is why fishermen wear polarized sunglasses.