enter

TOTAL INTERNAL REFLECTION

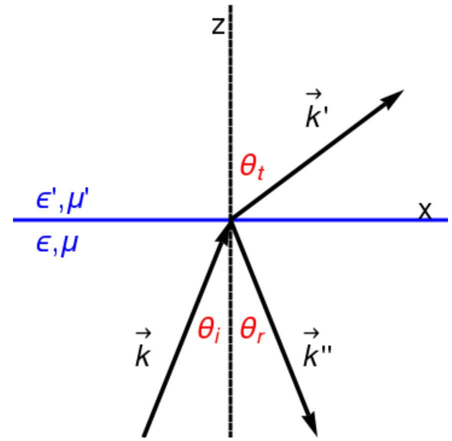
Jackson Section 7.3

Now consider reflection and refraction from an interface, in which the incident and reflected waves propagate in the optically more dense medium (ϵ, μ , n) , and the transmitted wave propagates in the optically less dense medium (ϵ', μ', n'); i.e., n > n'.

Think of an E.M. wave inside a piece of glass, that hits the boundary separating the glass from air.

Reflection and refraction from glass into air; $n = n_{\rm glass} = 1.5$ and $n' = n_{\rm air} = 1.0$.

,



- The *interface* is the xy–plane; i.e., z = 0.
- Lower Region (z < 0) is glass; Higher Region (z > 0) is air.
- The *plane of incidence* is the xz–plane.

Snell's Law

$$n \sin\theta = n' \sin\theta'$$

Or... $$1.5 \sin\theta_{\rm inc.} = 1.0 \sin\theta_{\rm transm.}$$

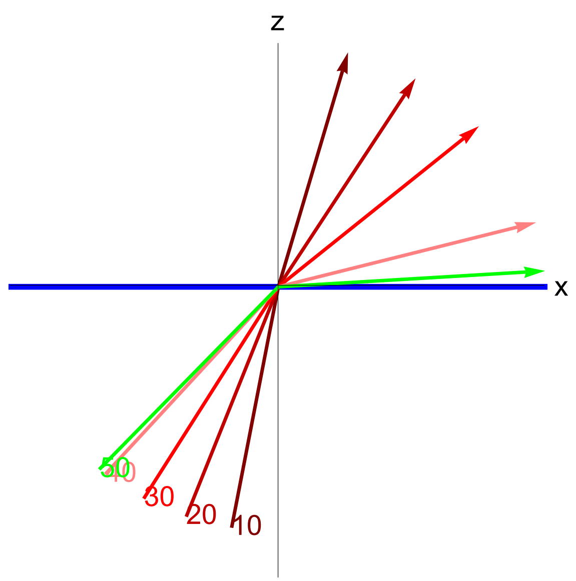
Glass to air ⇒ $\sin\theta_{transm.} = 1.5 \sin\theta_{inc.}$

,

The angles θ { $=\theta_{inc.}$} and θ' { $=\theta_{transm.}$} should be real angles, in the range $\theta, \theta' \in \{0, \pi/2\}$. **[Rememeber these angles are measured from the normal direction.]**

For n (=1.5) > n' (=1.0) *the transmitted wave bends away from the normal*.

In[•]:= sh



,

- Normal incidence is $\theta = 0$.

  Then by Snell's Law, $\theta' = 0$.

,

- At $\theta = 30$ degrees, i.e., the incident wave vector makes an angle of 60 degrees with respect to the surface, $\theta' =$ 48.6 degrees.

,

- As $\theta \longrightarrow 41.8$ degrees, i.e., the incident wave vector makes an angle of 48.2 degrees with respect to the surface, $\theta' \longrightarrow 90$ degrees; i.e., the reflected wave goes in the direction tangent to the surface.

The critical angle

n sin $\theta = n'$ sin θ'

n sin $\theta_{cr} = n'$; i.e., $\theta' = 90$ degrees.

$$\sin \theta_{critical} = \frac{n'}{n}$$

,

For glass to air,

θ_{cr} = arcsin(1/1.5) = 41.8 degrees.

What happens for $\theta = 42$ degrees?

sin(θ') = 1.5 *Sin(42 deg) = 1.0037

$\Rightarrow \theta'$ = arcsin(1.0037) = 90 – 4.92 i deg

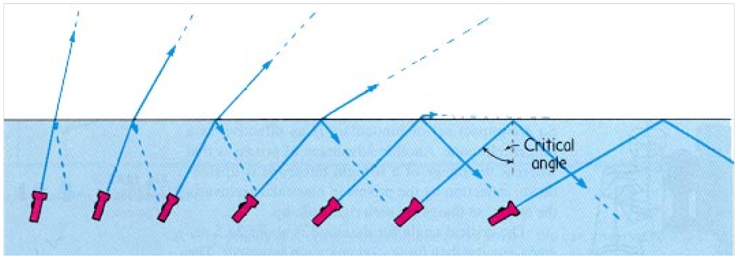
Snell's Law has no real solution for $\theta > \theta_{cr}$.

,

For $\theta > \theta_{cr}$, there is no propagating wave in the second medium (i.e., in the air). The light is trapped in the glass. There must be 100% reflection back into the first medium (i.e., in the glass).

,

Total ***internal*** reflection occurs for $\theta > \theta_{cr}$.



We analysed Snell's Law.

But what about the field equations?

What are the solutions of the field equations for $\theta > \theta_{cr}$?

Given

INCIDENT WAVE; z < 0

$$\vec{E}(\vec{x},t) = \vec{E}_0\, e^{i\left(\vec{k}\cdot\vec{x} - \omega t\right)}$$

$$\vec{B}(\vec{x},t) = \sqrt{\mu\epsilon}\; \hat{k} \times \vec{E}(\vec{x},t)$$

..... the real part is the physical field

.

What are the rest of the electric and magnetic fields?

Evanescent Waves in Total Internal Reflection

We can solve the field equations in the usual way, by writing

$$\vec{E} = E_0\, \hat{\epsilon}\, e^{i\left(\vec{k}.\vec{x} - \omega t\right)} \qquad \text{for } z < 0$$

$$\vec{E}_{tr} = E_0'\, \epsilon'\, e^{i\left(\vec{k}'.\vec{x} - \omega t\right)} \qquad \text{for } z > 0$$

$$\vec{E}_{refl} = E_0''\, \epsilon''\, e^{i\left(\vec{k}''.\vec{x} - \omega t\right)} \qquad \text{for } z < 0$$

with $\omega \vec{B} = \vec{k} \times \vec{E}$, etc.;

and solving the boundary conditions.

.

In fact we have alraady done this calculation.

For the transmitted wave we will have the spatial function

$$e^{i\vec{k}'\cdot\vec{x}} = \exp(\,i\,k_x\,x\,)\,\exp\,(\,i\,k_z'\,z\,)$$

where $k_x = \mathrm{k}\sin\theta$ and $k_z' = k'\cos\theta'$

.

Here $\cos\theta' = \sqrt{1-\sin^2\theta'}$

$$= \sqrt{1-(n/n')^2\sin^2\theta} \qquad \text{(Snell)}$$

$$= \mathrm{i}\,\sqrt{(n/n')^2\sin^2\theta - 1}\ \ \text{if}\ \theta > \theta_{\mathrm{cr}}$$

$e^{i\vec{k}'\cdot\vec{x}} = \exp(\,i\,k_x\,x\,)\,\exp\,(\,-\,z/\delta\,)$ ***for z > 0***

= oscillating in the x direction and damped in the z direction

Exercise: Check Jackson's equation, $\delta^{-1} = \mathrm{k}\sqrt{\sin^2\theta - \sin^2\theta_{\mathrm{cr}}}$

.

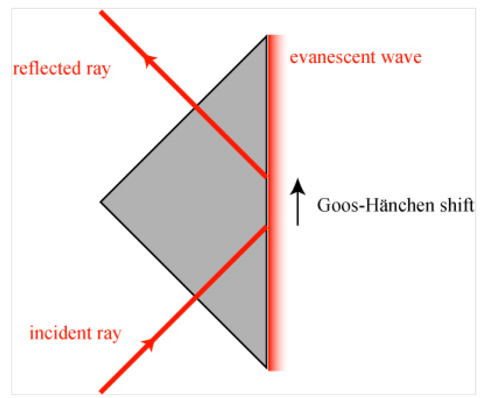
The fields in the region z > 0 are called the ***evanescent wave***. *(Draw a picture.)*

We could take this further and solve the boundary conditions for the field amplitudes and the energy fluxes.

The Goos Hänchen effect (page 308)

Wikipedia:

The Goos–Hänchen effect (named after Hermann Fritz Gustav Goos (1883–1968)[1] and Hilda Hänchen (1919–2013) is an optical phenomenon in which linearly polarized light undergoes a small lateral shift when totally internally reflected. The shift is perpendicular to the direction of propagation in the plane containing the incident and reflected beams.This effect is the linear polarization analog of the Imbert–Fedorov effect.



Applications of total internal reflection

Jackson mentions several applications.

TIR is useful where "it is desired to transmit light without loss of intensity".

∎ Experimental physics: ***Plastic light pipes*** carry light from scintillator detectors to photomultiplier tubes.

∎ Telecommunications: ***Optical fibers*** transmit modulated (binary) signals over long distances.

′

Or, Google it:
—endoscopy
—prisms in binoculars
—mirages
—diamonds (why does a diamond sparkle?)
—rainbows (role of TIR in a rainbow?)
—the "fish-eye lens" (R W Wood, 1906)
—many other things involve TIR.