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How Light Interacts with Matter
Victor F. Weisskopf
Scientific American Article, 1968

"The everyday objects around us are white, colored or black, opaque or transparent, depending on how the electrons in their atoms or molecules respond to the driving force of electromagnetic radiation."

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- Most of what we see is by *reflection.*
- But what are the *atomic and molecular* mechanisms that occur when light hits matter?
- It is ultimately a question of *quantum mechanics*, but we can understand some things from a semi-classical model.
- Absorption and reemission



Here are some questions to answer:
- Why is the sky blue?
- Why is paper white?
- Why is water transparent?
- What causes an object to appear colored?
- Why are metals shiny?

The Simplest Unit of Matter

What happens when *an isolated atom* is exposed to light?

Light is a stream of "energy packets" called photons; $E_\gamma = \hbar\,\omega$

The energy of an atom is quantized;
$E_n$ with n = 0 1 2 3 4 ....

Normally the atom is in the ground state with energy $E_0$. Exposed to light it can absorb *a single photon* and be left in an excited state with energy $E_n$;
this requires $\hbar\omega \approx E_n - E_0$ ;
more precisely, $\hbar\omega - (E_n - E_0) \sim O(\delta E)$
where $\delta$E is the spectral line width.



Example : H atom

E(2p) – E(1s) = –3.4 – (–13.6) eV = 10.2 eV

$\lambda_\gamma = \frac{c}{\omega/(2\pi)} = \frac{2\pi\hbar c}{10.2\text{ ev}} = 121.4$ nm (ultraviolet)

$\delta E \cdot \delta t \geq \hbar/2$ and $\delta t = \tau = 1.6 \times 10^{-9}$ sec

$\therefore \delta E = \frac{\hbar c}{2c\tau} = 2 \bullet 10^{-7}$ eV (natural line width)

But then (almost immediately!) the atom decays back to the ground state and emits a photon (in any direction) with energy $\hbar\omega \approx$ 10.2 eV.

In this familiar quantum picture, the atom only interacts with light by *resonance fluorescence*, requiring $\hbar\omega = E_n - E_0$ .

***But this picture is inadequate to describe the interaction of light with a macroscopic sample of matter.***

## 4-5 THE SEMICLASSICAL MODEL

- the Lorentz dispersion model (1878)
- the Drude dispersion model (1900) for metals

Jackson Section 7.5
"Frequency dispersion characteristics of dielectrics, conductors, and plasmas"

We have been considering plane waves in simple linear media ( $\epsilon, \mu$ ). This can describe reflection and refraction, but there is neither absorption nor dispersion.



Now we study a simple model for $\epsilon(\omega)$.
We' ll use complex field functions, such as

$$\vec{E}(\vec{x},t) = \vec{E}(x,\vec{\omega})\, e^{-i\omega t}$$

Remember : the real part of the function is the physical field.
Also we'll have

$$\vec{D}(\vec{x},\omega) = \epsilon\,(\vec{\omega})\, E(\vec{x},\omega)$$

and $\epsilon(\omega)$ will be complex.

## The Model

▪ $\mu = \mu_0$ ; not interested in magnetic effects

▪ An atom (or molecule) has positive charge that does not move.

▪ and it has an electron with mass m and charge –e that does move ( ⇒ *polarization*).

▪ The electron has:

▪▪ an equilibrium position;

▪▪ and displacement from equilibrium = $\vec{x}(t)$;

▪▪ and a linear restoring force = – m $\omega_0^2\, \vec{x}$;

▪▪ and a damping force = – m$\gamma\, \vec{v}$ , which decribes the transfer of energy to the medium.



$$\boxed{m\,[\,\ddot{\mathbf{x}} + \gamma\,\dot{\mathbf{x}} + \omega_0^2\,\mathbf{x}\,] = -e\,\mathbf{E}(\mathbf{x},t)}$$

The atom is so small that $\vec{E}$ can be approximated as a constant .

*(Which is larger—an atom or a wavelength of visible light? Is the difference of sizes a large difference or a small difference?)*

$$m\,[\,\ddot{\mathbf{x}} + \gamma\,\dot{\mathbf{x}} + \omega_0^2\,\mathbf{x}\,] = -e\,\mathbf{E}\,e^{-i\omega t}$$

You know how to solve the equation of motion from mechanics ⟶

EQUATION: linear inhomogeneous equation

SOLUTION: steady-state oscillations with frequency $\omega$ + a damped transient solution

The transients are not important.

$$m\,[\,\ddot{\mathbf{x}} + \gamma\,\dot{\mathbf{x}} + \omega_0^2\,\mathbf{x}\,] = -e\,\mathbf{E}\,e^{-i\omega t}$$

The steady-state solution is

$$\vec{x}(t) = \vec{x}_0\,e^{-i\,\omega\,t} \quad \text{(Re is implied)}$$

$$m\,[\,-\omega^2 - i\omega\gamma + \omega_0^2\,]\,\vec{x}_0 = -e\,\vec{E}$$

$$\vec{x}(t) = -\,\frac{e}{m}\,[\,\omega_0^2 - \omega^2 - i\omega\gamma\,]^{-1}\,\vec{E}\,e^{-i\omega t}$$

The dipole moment

$$\vec{p} = -e\,\vec{x} = \alpha\,\vec{E}$$

$$\alpha = \frac{e^2}{m}\,[\,\omega_0^2 - \omega^2 - i\omega\gamma\,]^{-1}$$

That is for one atom with one electron.

Now let N = number of molecules per unit volume, and Z = the number of electrons per molecule $\Longrightarrow$



$$\vec{P} = \epsilon_0\,\chi_e\,\vec{E} \quad \text{where} \quad \chi_e = N\,\alpha$$

$$\vec{D} = \epsilon_0\,\vec{E} + \vec{P} = \epsilon\,\vec{E}$$

$$\frac{\epsilon(\omega)}{\epsilon_0} = 1 + \chi_e = 1 + \frac{N\,e^2}{\epsilon_0\,m}\sum_i f_i\,[\,\omega_i^2 - \omega^2 - i\omega\gamma_i\,]^{-1} \quad (\ast)$$

Sum rule: $\sum_i f_i = Z$

$$\frac{\epsilon(\omega)}{\epsilon_0} = 1 + \frac{N e^2}{\epsilon_0 m} \sum_i f_i \left[ \omega_i^2 - \omega^2 - i\omega\gamma_i \right]^{-1} \quad (\ast)$$

The equation ($\ast$) will be our theoretical model for the dielectric constant in a macroscopic medium as a function of frequency $\omega$.

It has these parameters:

strengths $f_i$ ,

frequencies $\omega_i$ ,

damping constants $\gamma_i$ ,

i = 1 2 3 ... the number of electron states.

Ultimately these parameters come from quantum theory, or from experimental measurements of atomic physics.



## ANOMALOUS DISPERSION AND RESONANT ABSORPTION

Atoms have spectral frequencies

$$\omega_i \equiv (E_i - E_0) / \hbar$$

from electron transitions, which are typically in the ultraviolet part of the EM spectrum.

Molecules also have vibrational energy levels, from relative motion of the nuclei, which are typically in the infrared part of the EM spectrum.

(This explains why some materials are transparent; there is no resonant absorption of optical photons.)

For Jackson Figure 7.8, assume there are two significant resonances, at $\omega_1$ and $\omega_2$.

Plot $\epsilon(\omega)/\epsilon_0$ .

Figure 7.8
The real and imaginary parts of $\epsilon(\omega)/\epsilon_0$ .

$$\frac{\epsilon(\omega)}{\epsilon_0} = 1 + \frac{N e^2}{\epsilon_0 m} \sum_i f_i [\omega_i^2 - \omega^2 - i\omega\gamma_i]^{-1} \quad (\ast)$$

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