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How Light Interacts with Matter
Victor F. Weisskopf
Scientific American Article, 1968

"The everyday objects around us are white, colored or black, opaque or transparent, depending on how the electrons in their atoms or molecules respond to the driving force of electromagnetic radiation."

Here are some questions to answer:

- Why is the sky blue?
- Why is paper white?
- Why is water transparent?
- What causes an object to appear colored?
- Why are metals shiny?

In this familiar quantum picture, the atom only interacts with light by *resonance fluorescence*, requiring $\hbar\omega = E_n - E_0$.

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4-5 THE SEMICLASSICAL MODEL

Jackson Section 7.5
"Frequency dispersion characteristics of dielectrics, conductors, and plasmas"

$$m\,[\ddot{\mathbf{x}} + \gamma\,\dot{\mathbf{x}} + \omega_0^2\,\mathbf{x}\,] = -e\,\mathbf{E}(\mathbf{x},t)$$

$$\frac{\epsilon(\omega)}{\epsilon_0} = 1 + \frac{N e^2}{\epsilon_0 m} \sum_i f_i\,[\,\omega_i^2 - \omega^2 - i\omega\gamma_i\,]^{-1} \quad (\ast)$$

The equation ($\ast$) will be our theoretical model for the dielectric constant in a macroscopic medium as a function of frequency ω.

Jackson Figure 7.8, assuming there are two significant resonances, at ω_1 and ω_2.

Plot $\epsilon(\omega)/\epsilon_0$.

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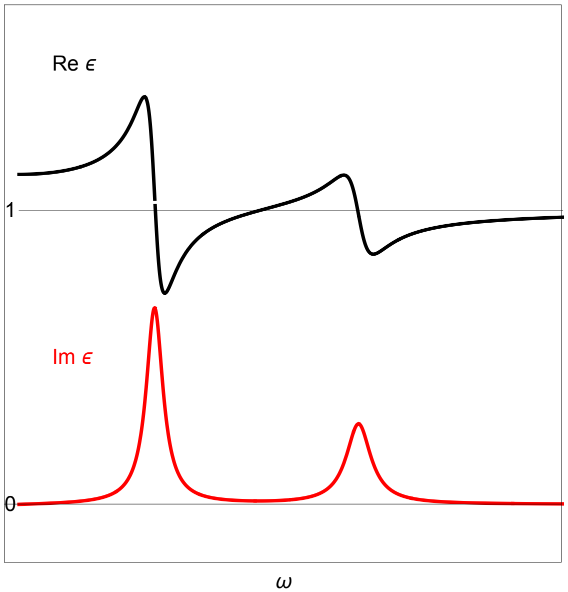

Normal dispersion, anomalous dispersion and resonant absorption.

The wave vector

Recall the “harmonic” spacetime dependence.

$$\vec{E} \text{ or } \vec{H} \propto \exp\{ i\, (\vec{k} \bullet \vec{x} - \omega\, t) \}$$

Maxwell's equations give this "dispersion relation"

$$\omega = v_{\text{phase}}\, k$$

$$\text{where } v_{\text{phase}} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\, \epsilon(\omega)}}$$

In the Lorentz model of dispersion, the interesting thing is that $\epsilon(\omega)$ is complex. Then k is complex.

Write $\vec{k} = k_z\, e_z$ and $k_z = \beta + i\, \frac{\alpha}{2}$.

Then $e^{i\, \vec{k} \bullet \vec{x}} = e^{i\, \beta\, z}\, e^{-\alpha z/2}$

Parameter β = wave vector = $2\pi/\lambda$.

Parameter α = absorption coefficient.

The intensity is proportional to $|\, E\, |^2 \propto e^{-\alpha\, z}$ so we have a damped wave.

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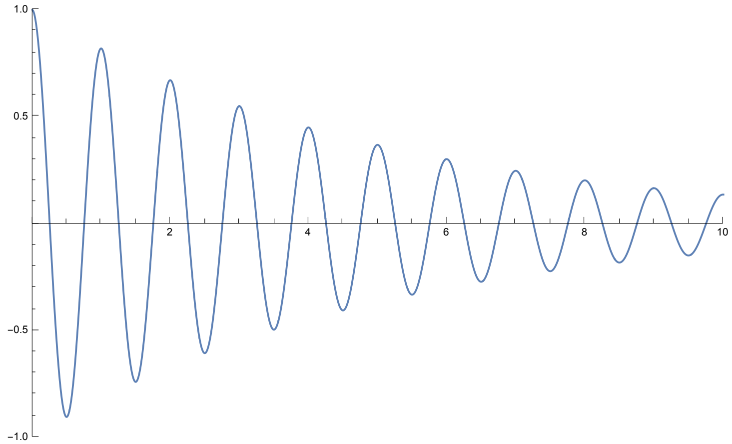

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$$k = \beta + i\,\frac{\alpha}{2} = \frac{\omega}{v_{\text{phase}}} = \omega\,\sqrt{\mu_0\,\epsilon}$$

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$$k^2 = \beta^2 - \frac{\alpha^2}{4} + 2\,i\,\alpha\beta = \omega^2\mu_0\,(\,\text{Re}\,\epsilon + i\,\text{Im}\,\epsilon\,\}$$

$$\beta^2 - \frac{\alpha^2}{4} = \frac{\omega^2}{c^2}\,\text{Re}\,\frac{\epsilon}{\epsilon_0} \quad \text{and} \quad \beta\alpha = \frac{\omega^2}{c^2}\,\text{Im}\,\frac{\epsilon}{\epsilon_0}$$

Example: For weak absorption, i.e., $\alpha \ll \beta$

$$\alpha \approx \frac{\text{Im}\,\epsilon}{\text{Re}\,\epsilon}\,\beta \quad \text{where } \beta = \sqrt{\text{Re}\,(\epsilon\,/\,\epsilon_0)}\;\frac{\omega}{c}$$

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Exercise. For weak absorption the intensity decreases for each wavelength ($\delta z = \lambda$) by the factor $X = Im(\epsilon)/Re(\epsilon)$.

7.5.C.
Low-frequency behavior and electric conductivity

$$\frac{\epsilon(\omega)}{\epsilon_0} = 1 + \frac{N e^2}{\epsilon_0 m} \sum_i f_i [\, \omega_i^2 - \omega^2 - i\omega\gamma_i \,]^{-1} \quad (\ast)$$

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● **Dielectrics:** all the electrons are bound charges.

Consider the contribution of one resonant frequency, ω_b.
For $\omega \ll \omega_b$,

$$\frac{\epsilon(\omega)}{\epsilon_0} \approx 1 + \frac{N e^2}{\epsilon_0 m} \frac{f_b}{\omega_b^2}$$

We have seen this before:

A model for molecular polarizability in electrostatics.

$$\gamma_{mol} = \frac{e^2}{\epsilon_0 m \omega_b^2} \quad = \text{molecular polarizability (4.73)}$$

● **Conductors:** Suppose some fraction f_0 of the electrons in each molecule are "free"; i.e., they can move arbitrary distances; for example the conduction electrons in a metal. In the Lorentz model they have restoring force = $-m\omega_0^2 \vec{x} = 0$; for these electrons $\omega_0 = 0$ and their contribution to permittivity is

$$\frac{\epsilon_{free}(\omega)}{\epsilon_0} = \frac{N e^2}{\epsilon_0 m} \frac{f_0}{(-i\omega\gamma_0 - \omega^2)}$$

$$\epsilon_{free}(\omega) = i \frac{N e^2 f_0}{m\omega(\gamma_0 - i\omega)}$$

In general we can write

$$\epsilon(\omega) = \epsilon_b(\omega) + i \frac{N e^2 f_0}{m\omega(\gamma_0 - i\omega)}$$

The Ampère-Maxwell equation

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

& postulate conductivity $\vec{J} = \sigma \vec{E}$

$$\nabla \times \vec{H} = \sigma \vec{E} - i\,\omega\,\epsilon_b(\omega)\,\vec{E}(\omega)$$

$$= -i\omega\left(\epsilon_b + i\frac{\sigma}{\omega}\right)\vec{E}$$

Compare to the complex $\epsilon(\omega) \Longrightarrow$

$$\sigma = \frac{f_0 N e^2}{m(\gamma_0 - i\omega)} \quad \text{(Drude model)}$$

Implications:

▮ for low frequencies ($\omega \lesssim 10^{11}\ s^{-1}$; microwaves) the conductivity is real and independent of frequency; ~ $6 \times 10^{-7}\ (\Omega \text{m})^{-1}$

▮ for frequencies infrared and higher, σ is complex and $\propto 1/\omega$

▮ We are missing something important: Pauli exclusion principle and band structure — truly quantum mechanical.

▮ For nonzero ω, the distinction between dielectrics and conductors is "artificial".

7.5.D.
High frequency limit and plasma frequency

$$\frac{\epsilon(\omega)}{\epsilon_0} = 1 + \frac{N e^2}{\epsilon_0 m} \sum_i f_i \left[\omega_i^2 - \omega^2 - i\omega\gamma_i \right]^{-1} \quad (\ast)$$

For high frequencies,
i.e., $\omega \gg$ resonance frequencies,
we may approximate

$$\frac{\epsilon(\omega)}{\epsilon_0} \approx 1 - \frac{\omega_p^2}{\omega^2} \quad \text{where} \quad \omega_p^2 = \frac{NZe^2}{\epsilon_0 m}$$

ω_p is called the *plasma frequency* of the medium,

$$\omega_p = \sqrt{NZ}\ (56.3\ \text{m}^{3/2}\ \text{s}^{-1})$$

NZ = electron density

The dispersion relation for high frequencies

$$k = \frac{\omega}{1/\sqrt{\mu_0 \epsilon}} = \frac{\omega}{c}\sqrt{\frac{\epsilon}{\epsilon_0}} = \frac{\omega}{c}\sqrt{1 - \omega_p^2/\omega^2}$$

$$ck = \sqrt{\omega^2 - \omega_p^2}$$

$$\text{Or,} \quad \omega = \sqrt{\omega_p^2 + c^2 k^2}\,,$$

For $\omega < \omega_p$, there is no wave propagation because then k is purely imaginary.

For $\omega > \omega_p$,

$$\text{Phase velocity} = \frac{\omega}{k} = \sqrt{c^2 + (\omega_p/k)^2} > c$$

$$\text{Group velocity} = \frac{d\omega}{dk} = \frac{c^2}{\sqrt{c^2 + (\omega_p/k)^2}} < c$$

$$v_{phase} \times v_{group} = c^2$$

E.M. waves in plasmas

In a dilute plasma, the electrons are free and the damping is negligible. Then

$$\epsilon(\omega) = \epsilon_b(\omega) + i\,\frac{N e^2 f_0}{m\omega(\gamma_0 - i\omega)}$$

$$\approx \epsilon_0\left(1 - \frac{\omega_p^2}{\omega^2}\right) \text{ where } \omega_p^2 = \frac{NZe^2}{\epsilon_0\, m}$$

If $\omega < \omega_p$ then k is ***purely imaginary***;

- the electromagnetic wave cannot propagate in the plasma;
- incident waves with $\omega < \omega_p$ can only reflect from the surface of the plasma;
- Oliver Heaviside and the Ionosphere ("the Heaviside Layer")
- AM and FM radio transmissions

Why are metals shiny?
(And, the ultraviolet transparency of metals.)

$$\epsilon(\omega) \approx \epsilon_{\text{bound}}(\omega) - \epsilon_0 \frac{\omega_p^2}{\omega^2}$$

where $\omega_p^2 = \frac{n\,e^2}{\epsilon_0\, m_{\text{eff}}}$,

n = density of conduction electrons

Electromagnetic waves cannot propagate in the metal, so an incident wave must reflect from the surface. Typically ω_p is in the ultraviolet range of frequencies; so optical and infrared waves are reflected. However, far ultraviolet waves are transmitted.