How does light interact with matter?

V. F. Weisskopf, Sci Am 1968

Examples:

Why is water transparent?

Why is the sky blue?

Why are clouds white?

We have been using the Lorentz-Drude model to describe the interactions of matter and light.

The Lorentz-Drude model for electromagnetic dispersion

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Review $\vec{E}(\vec{x},t) = \vec{E}_0 \, e^{i(\vec{k}\cdot\vec{x}-\omega t)}$

$$\vec{D}(\vec{x},\omega) = \epsilon(\omega)\,\vec{E}_0\, e^{i\vec{k}\cdot\vec{x}}$$

$$\nabla\times\vec{H} = \frac{\partial \vec{D}}{\partial t} = -i\omega\vec{D}$$

The Lorentz-Drude model for dispersion

$$m\left[\ddot{\vec{x}} + \gamma\dot{\vec{x}} + \omega_0^2\,\vec{x}\right] = -e\vec{E}$$

$$\Rightarrow \quad \frac{\epsilon(\omega)}{\epsilon_0} = 1 + \frac{Ne^2}{\epsilon_0 m}\sum_{\text{states}} \frac{f_i}{\omega_i^2-\omega^2-i\omega\gamma_i}$$

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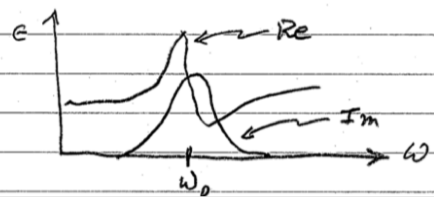
For a conductor, add contribution with $\omega_0 = 0$

$$-\frac{Ne^2}{\epsilon_0 m}\frac{f_0}{\omega^2 + i\omega\gamma_0} = \frac{iNe^2 f_0}{\epsilon_0 m\omega(\gamma_0 - i\omega)}$$

Apply to dielectrics, metals, plasmas

Propagation $\quad k = \frac{\omega}{v_{phase}} = \omega\sqrt{\mu_0 \epsilon(\omega)} = \beta + \frac{i\alpha}{2}$

$$\alpha = \frac{\omega^2}{\beta^2 c^2} Im\left(\frac{\epsilon}{\epsilon_0}\right) \quad \text{and} \quad \beta \approx \frac{\omega}{c}\sqrt{Re\left(\frac{\epsilon}{\epsilon_0}\right)}$$



We've seen that this can be applied to dielectrics, metals and plasmas.

Why is the sky blue?
And give the answer at grad-school level.

Possible answers

• **Rayleigh scattering**

Correct, but it doesn't explain anything. It just gives the name of the persn who first understood why the sky is blue.

Lord Rayleigh; AKA John Strutt
(Nobel prize for the discovery of Argon)

• **The atmosphere is a dilute gas of molecules and atoms.**

Correct, but it's doesn't explain anything. What do N_2 and O_2 and Ar have to do with blue light?

• **According to the theory of radiation, the scattered power is proportional to ω^4.**

Correct, but it's doesn't explain enough. What is the radiation process?

• **Sunlight interacts with molecules and atoms in the atmosphere. The oscillaing electric field causes oscillations of an electron's position, and it radiates. According to Larmor's Formula, the power is**

$$P = \frac{e^2 a^2}{6 \pi \epsilon_0 c^3}$$

where a = acceleration = $\ddot{x} = -\omega^2 x$.
The scattering increases with frequency as ω^4 .

A good answer, but it doesn't mention blue light!

• **The sky is blue because it is reflecting blue light from the ocean.**

False

History

Rayleigh wrote several papers on the color of the sky: 1871; again in 1881; again in 1899.

At first he assumed that light scatters from "small particles" (ie, what?) in the atmosphere; but eventually he realized that it has nothing to do with particulates nor with diffraction.

It's just from the *molecules* that compose the atmosphere.

Classical electron theory of light scattering

■ As light passes an electron in a molecule, the electron motion may be modeled by a classical equation of motion,

$$m\,[\,\ddot{x} + \gamma\,\dot{x} + \omega_0^2\,x\,] = -e\,E_x\,e^{-i\,\omega\,t}$$

Here ω — the frequency of the driving force — is a particular frequency in the sunlight.

■ The steady-state solution

$$x(t) = \frac{-e\,E_x}{m(\omega_0^2 - \omega^2 - i\,\gamma\,\omega)}\;e^{-i\,\omega\,t}$$

■ The electron accelerates, so it must radiate; this is a classical picture of the "scattering process"

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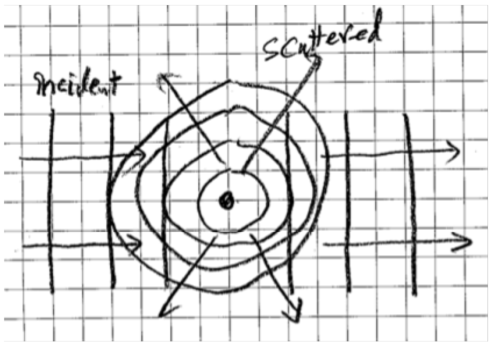

■ ***Larmor's formula (1897).*** The instantaneous power radiated by an accelerating charge (*non-relativistic limit*) is

$$P = \frac{1}{4\pi\epsilon_0}\,\frac{2\,q^2\,a^2}{3\,c^3}$$

■ ***Taking the real part,***

$$a_x = C_1 \cos\omega t + C_2 \sin\omega t$$

$$C_1 = \frac{e\,E_0\,\omega^2\,m\left(\omega_0^2 - \omega^2\right)}{m^2\left[\left(\omega_0^2 - \omega^2\right)^2 + \gamma^2\,\omega^2\right]}$$

$$C_2 = \frac{e\,E_0\,\omega^3\,\gamma}{m^2\left[\left(\omega_0^2 - \omega^2\right)^2 + \gamma^2\,\omega^2\right]}$$

■ The average power (averaged over a period of ocsillation) of e.m. waves radiated by the electron is

$$P_{\text{avg}} = \frac{e^2\left(C_1^2 + C_2^2\right)}{12\,\pi\,\epsilon_0\,c^3}$$

$$= \frac{e^4\,E_0^2}{12\,\pi\,\epsilon_0\,c^3}\;\frac{\omega^4}{m^2\left[\left(\omega_0^2 - \omega^2\right)^2 + \gamma^2\,\omega^2\right]}$$

■ The scattering cross section is defined by

$\sigma = \frac{P_{\text{avg}}}{S_{\text{inc}}}$ where $S_{\text{inc}} = c\,\epsilon_0\,E_0^2/2$.

■ ***RESULT***

The cross section for light scattering by a bound electron, in this classical model, is

$$\sigma = \frac{8\,\pi\,r_e^2}{3}\;\frac{\omega^4}{\left(\omega_0^2 - \omega^2\right)^2 + (\gamma\omega)^2}$$

where r_e is called the "classical radius of the electron" is

$$r_e = \frac{1}{4\,\pi\,\epsilon_0}\,\frac{e^2}{mc^2} = 2.8 \times 10^{-15}\ \text{m}$$

$$\sigma = \frac{8\,\pi\, r_e^2}{3} \quad \frac{\omega^4}{\left(\omega_0^2 - \omega^2\right)^2 + (\gamma\omega)^2} \sim \begin{cases} \omega^4 & \text{small } \omega \\ 1 & \text{large } \omega \end{cases}$$

Plot the function:

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f[x_] = x^4 * Power[(1 - x^2)^2 + κ^2 * x^2, -1];
lp = LogPlot[f[x] /. {κ → 0.1}, {x, 0.02, 3}, PlotRange → All,
  Frame → True, GridLines → Automatic,
  BaseStyle → 18, PlotStyle → {Blue, AbsoluteThickness[3]},
  FrameLabel → {"ω/ω₀", "cross section"}, ImageSize → Large]
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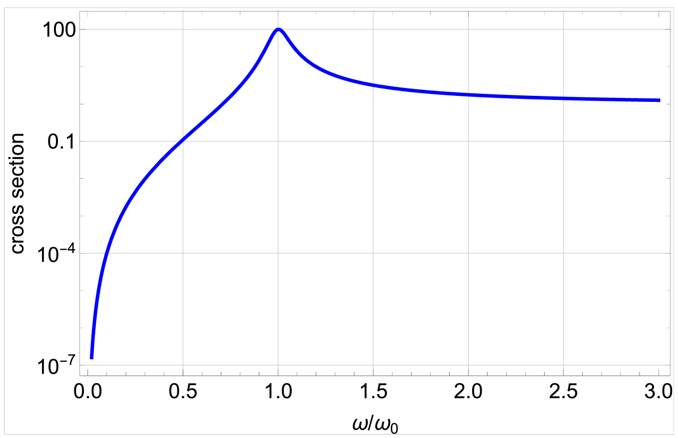

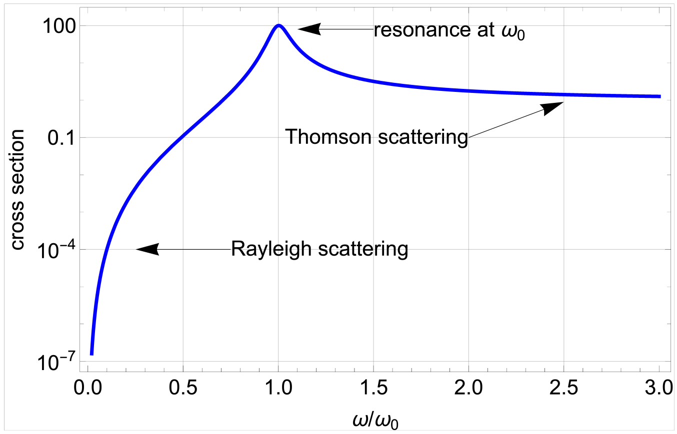

Comments

▪▪■▪▪ You will not find this calculation in Jackson.
However, see Section 10.2: "Perturbation Theory of Scattering, Rayleigh's Explanation of the Blue Sky, Etc."
Jackson shows $P \propto \omega^4$ by Rayleigh's method, which was published before Larmor's Formula.

▪▪■▪▪ In reality, Rayleigh scattering must be an example in *quantum electrodynamics*:
a single photon is absorbed, and a new photon is emitted, by a single electron in a molecule.
(Draw the Feynman diagram!)
The result of the QED calculation,
for low energies, ...

is identical to the classical formula**!**

Weisskopf

We are now ready to understand one of the most beautiful colors in nature: the blue of the sky. The action of sunlight on the molecules of oxygen and nitrogen in air is the same as the action on the two kinds of oscillator. Both oscillators will vibrate under the influence of visible sunlight. The amplitude of the infrared oscillators, however, will be much smaller than the amplitude of the ultraviolet oscillators because of their higher vibrating mass. Accordingly we

We must now take into account the fact that a vibrating charge is an emitter of light. According to a principle of electrodynamics an electron oscillating with an amplitude *A* emits light in all directions with an intensity given by a formula in which the intensity of the radiation is proportional to the fourth

sunlight. The scattered light is predominantly blue because the reradiation varies with the fourth power of the frequency; therefore higher frequencies are reemitted much more strongly than the lower ones.

The complementary phenomenon is the color of the setting sun. Here we see solar rays that have traveled through the air a great distance. The higher-frequency light is attenuated more than light of lower frequency; therefore the reds and yellows come through more strongly than the blues and violets. The

In actuality Rayleigh scattering is a very weak phenomenon. Each molecule scatters extremely little light. A beam of green light, for example, goes about 150 kilometers through the atmosphere before it is reduced to half its intensity.

Now we know why the sky is blue. Why, then, are clouds so white? Clouds are small droplets of water suspended in air. Why do they react differently to sunlight? The water molecule