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Jackson Chapter 8

“Waveguides, Resonant Cavities, and Optical Fibers”

General Introduction

What is a waveguide?

- Waves in an infinite space spread out in all directions (think: radio antenna) and the energy flux decreases as R^{-2} (inverse square law)
- A waveguide is a structure that guides waves, such as electromagnetic waves, conserving the energy by restricting the wave propagation to one dimension.
- A hollow conducting metal pipe used to carry high-frequency radio waves, particularly microwaves.
- Boundary conditions keep the fields inside the waveguide; and they determine how the wave propagates down the waveguide.

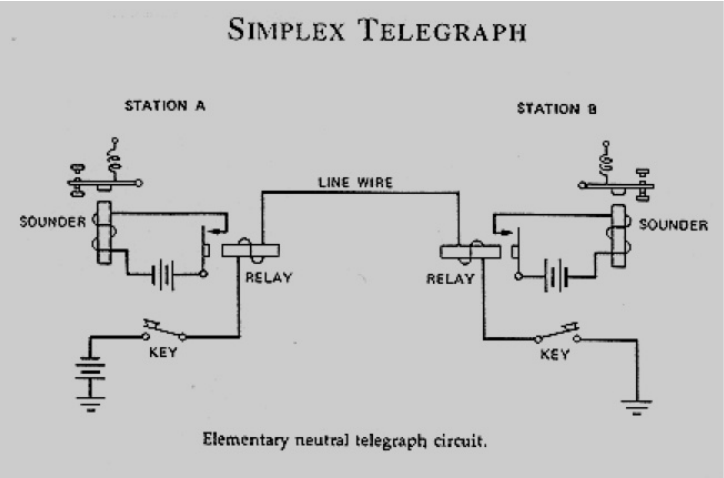
History of Waveguides (Wikipedia)

- The first structure for guiding waves was proposed by J.J.Thomson in 1893, and was first experimentally tested by Oliver Lodge in 1894.
- The first mathematical analysis of electromagnetic waves in a metal cylinder was performed by Lord Rayleigh in 1897.
  (Rayleigh was already an expert on sound waves.)
- The study of dielectric waveguides (such as optical fibers) began as early as the 1920s, by several people, most famous of which were Rayleigh, Sommerfeld and Debye.
  (Today optical fibers are widely used in the communications industry; optical frequencies versus radio frequencies)
- Radar in World War II led to waveguide research and development.
  (• the invention of the magnetron as a source of microwaves; • the use of waveguides to transmit information from the antenna to the receiver circuit.)

Brief history of electrical communication

The telegraph (Samuel Morse, 1838)

`sptel (*people.seas.harvard.edu*)`

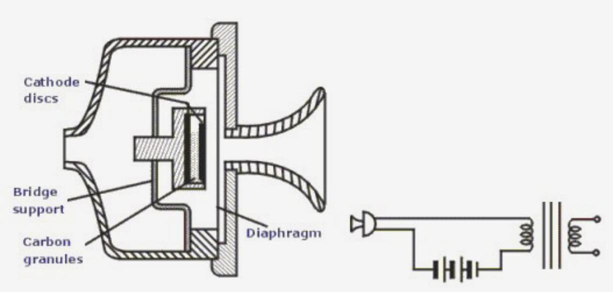


Elementary neutral telegraph circuit.

He called it an “electromagnetic telegraph”, because the receiver used an electromagnet.

The telephone
(Alexander Graham Bell, 1876)
(Thomas Edison, Carbon microphone, 1878)

`Cmic (*wired.com *)`

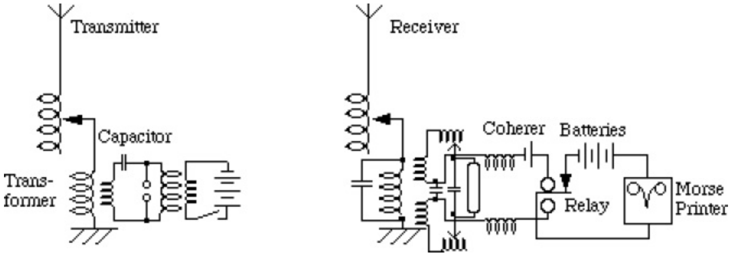


Signal is carried on a *two-conductor transmission line*; twisted pair of wires;

human voices : 80 Hz to 14 kHz
telephone : 300 Hz to 3.4 kHz
HD audio : 50 Hz to 7 kHz

Wireless
(Gugliemo Marconi, 1890’s; transatlantic, 1901)

`radio (*bnrg.cs.berkeley.edu *)`

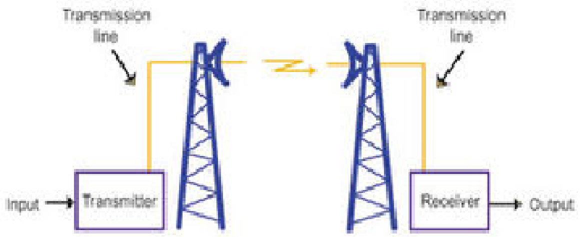


Frequencies

AM radio : 540 kHz to 1600 kHz
FM radio : 88.1 MHz to 108.1 MHz

Microwave links

link



Point to point transmission through the atmosphere with microwaves;
line-of-sight limitation;
the advantage is that the energy flux is not inverse square law;
transmission lines carrying microwaves

Transmission lines

popsci

British cable ship "Monarch" will lay line on uneven ocean bed, which in some places is 15,000 feet deep

Starting about 1920 companies laid submarine cables for telephone communication across the ocean. Diagram above is from 1954.
Today there are many submarine cables, for communication and internet, carrying microwaves.

Radio waves and Microwaves

Radio waves

$$f \in (30 \text{ kHz}, 300 \text{ GHz})$$

Microwaves

$$f \in (300 \text{ MHz}, 300 \text{ GHz})$$

the spectrum

Mobile phones, phone mast antennas (base stations), DECT cordless phones, Wi-Fi, WLAN, WiMAX and Bluetooth have *carrier wave frequencies* within the microwave band of the electromagnetic spectrum, and are pulsed/modulated to transmit information.

The TEM mode of a coaxial cable

– a kind of transmission line that is used to carry a signal in the microwave frequency range.

picture of a coaxial cable

Electromagnetic waves propagate in the space between the cylinders; charge and current waves move on the cylinder surfaces.

We'll analyze the fields.

By cylindrical symmetry, we can anticipate that $\vec{E}$ points radially and $\vec{B}$ points azimuthally. Try

$$\vec{E}(\vec{x},t) = \hat{r}\, E_r(r) \exp[i\,(kz-\omega t)\,]$$

$$\vec{B}(\vec{x},t) = \hat{\phi}\, B_\phi(r) \exp[i\,(kz-\omega t)\,]$$

and now substitute these into Maxwell's equations.

- (ρ,ϕ,z) are cylindrical coordinates
- $\partial/\partial t$ can be replaced by $-i\omega$
- material parameters ϵ, μ_0
- $\nabla \bullet \vec{E} = 0$

$$\frac{1}{r}\frac{d}{dr}(r\,E_r) = 0 \implies E_r = \frac{C}{r}$$

- $\nabla \bullet \vec{B} = 0$ is automatic
- $\nabla \times \vec{E} = i\,\omega\,\vec{B}$

$$\nabla \times \vec{E} = \hat{\phi}\ \ ik\,E_r(r)\,e^{i(kz-\omega t)}$$

$$\therefore\ B_\phi(r) = \frac{k}{\omega}\,E_r(r) = \frac{kC}{\omega r}$$

- $\nabla \times \vec{B} = -i\,\omega\,\mu_0\,\epsilon\,\vec{E}$

$$\nabla \times \vec{B} = -\,\hat{r}\ \ ik\,B_\phi(r)\,e^{i(kz-\omega t)}$$

$$\therefore\ B_\phi(r) = \frac{\mu_0\,\epsilon\,\omega}{k}\,E_r(r) = \frac{\mu_0\,\epsilon\,\omega\,C}{kr}$$

- The dispersion relation (relates k and ω)

$$\frac{k}{\omega} = \frac{\mu_0\,\epsilon\,\omega}{k}$$

$$\implies \omega = v\,k \quad \text{where} \quad v = \frac{1}{\sqrt{\mu_0\,\epsilon}}$$

which looks familiar.
Also, $B_0 = E_0/v$.

Potential V(z,t)

By definition,

$$V(z,t) = \int_a^b \vec{E}\cdot\hat{r}\,dr = C\ln(b/a)\,e^{i[(kz-\omega t)]}$$

therefore $V_0 = C\ln(b/a)$

$$\vec{E}(\vec{x},t) = \hat{r}\,\frac{V_0}{r\ln(b/a)}\,e^{i[(kz-\omega t)]}$$

$$\vec{B}(\vec{x},t) = \hat{\phi}\,\frac{V_0}{v\,r\ln(b/a)}\,e^{i[(kz-\omega t)]}$$

Boundary Conditions

Take the cylinders to be perfect conductors.

Then $\vec{E} = 0$ and $\vec{B} = 0$ inside the conductor.

- B_{normal} and $E_{\text{tangential}}$

$B_r = 0$ at r = a and r = b; OK

$E_\phi = E_z = 0$ at r = a and r = b; OK

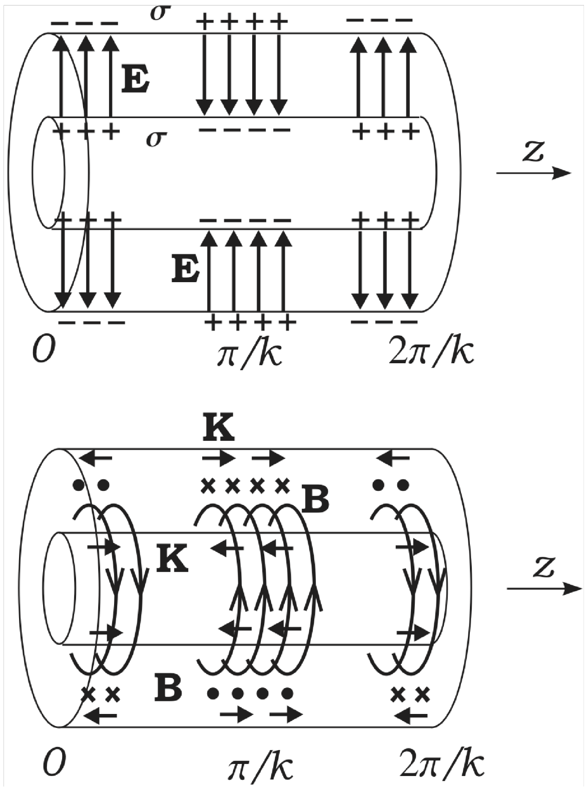
- D_{normal} and $H_{\text{tangential}}$

$$\sigma = \epsilon E_r = \begin{cases} \dfrac{\epsilon V_0}{a\ln(b/a)}\,e^{i\Phi} & \text{for } r = a \\ \dfrac{-\epsilon V_0}{b\ln(b/a\}}\,e^{i\Phi} & \text{for } r = b \end{cases}$$

$$\vec{K} = \frac{\hat{r}\times\vec{B}}{\mu_0} = \begin{cases} \dfrac{\hat{z}\,v\,\epsilon V_0}{a\ln(b/a)}\,e^{i\Phi} & \text{for } r = a \\ \dfrac{-\hat{z}\,v\,\epsilon V_0}{b\ln(b/a\}}\,e^{i\Phi} & \text{for } r = b \end{cases}$$

These are *charge and current waves* on the conductor surfaces.

TEMcoax



Line Impedance

$$Z \equiv \frac{V(z,t)}{I(z,t)} = \sqrt{\frac{\mu_0}{\epsilon}}\ \frac{\ln(b/a)}{2\,\pi}$$

or,

$$Z = \frac{\ln(b/a)}{2\,\pi\,\sqrt{\epsilon_R}} \square (377\ \Omega)$$

Homework problems and questions to be assigned later.

- In physics, what is ether?
  What is ethernet?
- What are the frequencies used by your cell phone?
- What are the frequencies used for WiFi communication?
- Jackson Problem 8.2
- What is the impedance of free space?