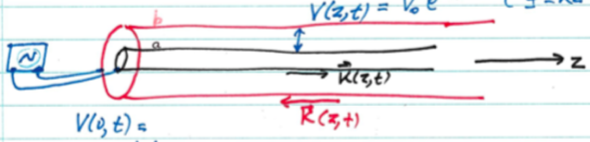
enter

z

Line Impedance of a coaxial Cable (a, b, ϵ)

$V(z,t) = V_0 e^{i\Phi}$ $\quad (\Phi = kz - \omega t)$



$V(0,t) = V_0 e^{-i\omega t}$

$$\vec{E}(\vec{x},t) = \frac{\hat{r} V_0}{r \ln(b/a)} e^{i\Phi}$$

$$\vec{B}(\vec{x},t) = \frac{\hat{\phi} V_0 e^{i\Phi}}{v\, r \ln(b/a)} \quad \text{where } v = \frac{1}{\sqrt{\mu_0 \epsilon}}$$

$$\vec{K}(z,t) = \frac{\hat{r} \times \vec{B}}{\mu_0} \qquad (\Delta H_t = K)$$

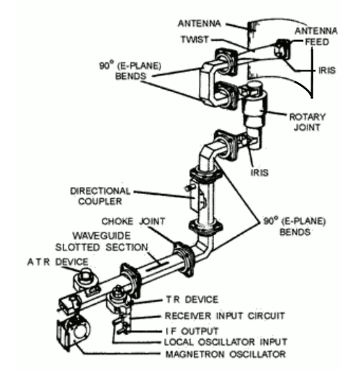
$$\vec{K}(z,t) = \frac{\hat{z} V_0 e^{i\Phi}}{\mu_0 v a \ln(b/a)}$$

$$I(z,t) = \oint K_z\, a\, d\phi = \frac{V_0 \cdot 2\pi e^{i\Phi}}{\mu_0 v \ln(b/a)}$$

$$Z = \frac{V}{I} = \frac{V_0 e^{i\Phi} \mu_0 v \ln(b/a)}{V_0 \cdot 2\pi e^{i\Phi}} = \frac{\ln(b/a)}{2\pi} \sqrt{\frac{\mu_0}{\epsilon}}$$

In[=]:=
```
GraphicsGrid[{{wg, RD}}]
```

Out[=]=



I'm not sure why we are studying waveguides, but it is included in the PHY 842 course description.

A coaxial cable is a transmission line.

Sometimes a waveguide is better than a coaxial cable (why?) (homework question)

Waveguides
–boundary conditions (Section 8.1)

The calculation of the TEM mode of a coaxial cable shows the importance of boundary conditions.

We assumed that the conductors are perfect conductors, so **E** = 0 and **B** = 0 inside the conductor.

What about real metals?

Jackson :

“A good conductor behaves effectively like a perfect conductor, with the idealized surface current replaced by an equivalent surface current , which is actually distributed throughout a small thickness at the surface.”

Read Jackson Section 8.1: “Fields at the surface of and inside a conductor”

f81

Perfect conductor

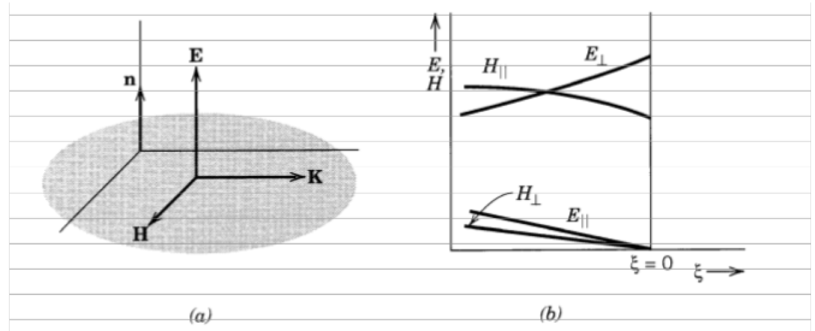


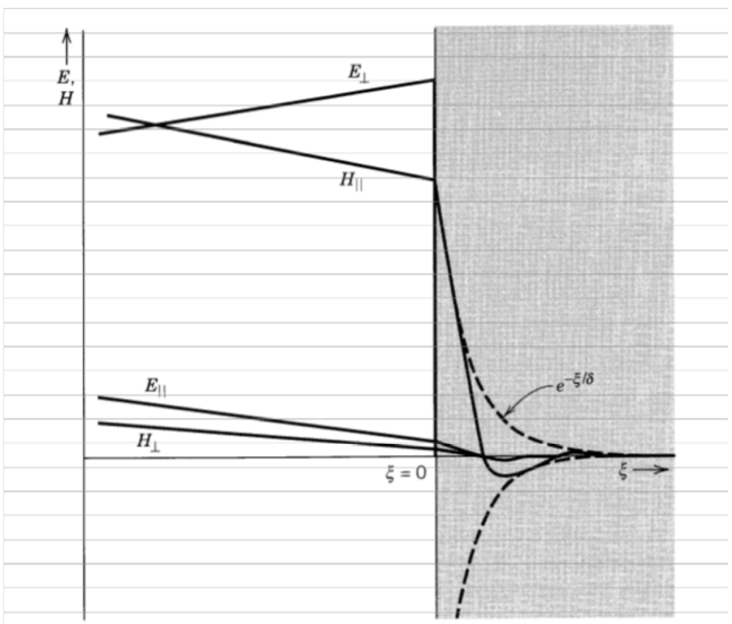
Figure 8.1 Fields near the surface of a perfect conductor.

The shaded region is a small area on the conductor surface; $\hat{n}$ = unit normal; $\vec{K}$ = surface current density; $\vec{E} = 0$ and $\vec{H} = 0$ in the perfect conductor, so $\vec{E}$ is normal and $\vec{H}$ is tangential at the outer surface.

For a real metal, the conductivity σ is large but finite; so the skin depth $\delta = \sqrt{2/(\mu_0\,\omega\sigma)}$ is small .

Figure 8.1
(perfect conductor; $\sigma = \infty$, $\delta = 0$)
Figure 8.2
(good conductor; δ is “small”)

f82



We' ll analyze waveguides in a two-step calculation called "successive approximations":

▪ First we solve the problem for the ideal conductor; i.e., $\sigma \to \infty$. Here the boundary conditions are

$$\hat{n} \cdot \vec{B} = 0 \quad \text{and} \quad \hat{n} \times \vec{E} = 0$$

$$\hat{n} \cdot \vec{D} = \Sigma \quad \text{and} \quad \hat{n} \times \vec{H} = \vec{K}$$

where Σ = ideal surface charge density and $\vec{K}$ = ideal surface current density.

That idealization means that $\vec{E}$ and $\vec{B}$ are 0 inside the conductors — the first approximation.

▪ Then, to calculate energy loss from electrical resistance, we'll calculate the fields in a thin layer just inside the conductor — the second approximation.

These are the results from Jackson ...

$$\vec{H}_c = \vec{H}_{\parallel}\, e^{-\xi/\delta}\, e^{-i\xi/\delta},$$

where ξ = distance inside C.

and $H_{\parallel}$ = tangential field at the surface

$$\vec{E}_c = \sqrt{\frac{\mu_c\,\omega}{2\,\sigma}}\,(1-i)(\hat{n}\times\vec{H}_{\parallel})\, e^{-\xi/\delta}\, e^{i\xi/\delta},$$

At the outer surface we can approximate $E_{normal} = \Sigma/\epsilon$ and $H_{tang.} = K_{eff}$; and we can neglect H_{normal} and $E_{tang.}$; see Figure 8.2 and Equation 8.11:

$$\vec{E}_{\parallel} = \sqrt{\frac{\mu_c\,\omega}{2\,\sigma}}\,(1-i)(\hat{n}\times\vec{H}_{\parallel})$$

■ Power loss

The guided wave loses energy.

Power loss = the power flowing through the surface of the conductor.

$$\left\langle\frac{dP_{loss}}{da}\right\rangle = -\frac{1}{2}\,\mathrm{Re}\{\,\hat{n}\cdot\vec{E}\times(\vec{H})^*\,\}$$

$$= \frac{\mu_c\,\omega\delta}{4}\,|\vec{H}_{\parallel}|^2$$

where $\langle..\rangle$ means time averaged

The "loss of energy" is due to resistance of the conductor (finite σ) ; ohmic loss; current density in the conductor, $\vec{J} = \sigma\,\vec{E}_c$.

■ OR, in terms of the "surface current",

$$\vec{K}_{eff} = \int_0^{\infty}\vec{J}\,d\xi = \hat{n}\times\vec{H}_{\parallel}$$

$$\left\langle\frac{dP_{loss}}{da}\right\rangle = \frac{1}{2\,\sigma\delta}\,|\vec{K}_{eff}|^2$$

Cylindrical waveguides (Section 8.2)

Figure 8.3

f83

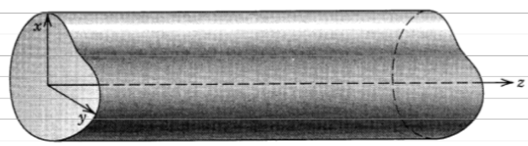


Figure 8.3 Hollow, cylindrical waveguide of arbitrary cross-sectional shape.

Field equations for harmonic time dependence, $F(\vec{x},t) = F(\vec{x})\, e^{-i\omega t)}$,

$$\text{curl}\, \vec{E} = i\omega \vec{B} \quad \text{and} \quad \text{div}\, \vec{B} = 0$$
$$\text{curl}\, \vec{B} = -i\, \mu\epsilon\, \omega\, \vec{E} \quad \text{and} \quad \text{div}\, \vec{E} = 0$$

We know that these imply

$$(\nabla^2 + \mu\epsilon\, \omega^2)\, \{ \vec{E} \text{ and } \vec{B} \} = 0$$

Using the symmetry —
translation invariance in z —
we'll consider waves propagating down the wave guide,

$$\vec{E}(\vec{x},t) = \vec{E}(x,y)\, e^{i\,(kz-\omega t)}$$
$$\vec{B}(\vec{x},t) = \vec{B}(x,y)\, e^{i\,(kz-\omega t)}$$

Remember, many kinds of superpositions are possible.

Now,

$$(\nabla_T^2 - k^2 + \mu\epsilon\omega^2)\, (\vec{E} \text{ and } \vec{B}) = 0$$

where $\nabla_T^2 \;=\; \partial_x^2 + \partial_y^2$.

So, we have a 2D partial differential equation to solve inside the waveguide, along with appropriate boundary conditions.

The boundary value problem for a hollow waveguide

There are two types of solution.

TE = transverse electric: $E_z(x,y) = 0$

TM = transverse magnetic: $B_z(x,y) = 0$

For a waveguide in the form of a hollow cylinder, there are no TEM waves!

txt358

The final consequence is that the TEM mode cannot exist inside a single, hollow, cylindrical conductor of infinite conductivity. The surface is an equipotential; the electric field therefore vanishes inside. It is necessary to have two or more cylindrical surfaces to support the TEM mode. The familiar coaxial cable and the parallel-wire transmission line are structures for which this is the dominant mode. (See Problems 8.1 and 8.2.)

TM modes; $B_z = 0$

$$\vec{B} = \vec{B}_T(x,y)\, e^{ikz} \quad e^{-i\omega t}$$

$$\vec{E} = [\vec{E}_T(x,y) + \hat{e}_z E_z(x,y)] e^{ikzt} \quad e^{-i\omega t}$$

Please verify ...

TM modes	$B_z = 0$
$\nabla \bullet \mu\vec{H} = 0$	$\nabla_T \bullet \vec{H}_T = 0$
$\nabla \bullet \epsilon\vec{E} = 0$	$\nabla_T \bullet \vec{E}_T + ik E_z = 0$
$i\omega\,\mu\vec{H} = \nabla \times \vec{E}$	$0 = \hat{e}_z \bullet (\nabla_T \times \vec{E}_T)$
	$i\omega\mu\,\vec{H}_T = -\hat{e}_z \times \vec{\nabla}_T E_z - ik\hat{e}_z \times \vec{E}_T$
$-i\omega\,\epsilon\,\vec{E} = \nabla \times \vec{H}$	$-i\omega\epsilon E_z = \hat{e}_z \bullet (\nabla_T \times \vec{H}_T)$
	$-i\omega\epsilon\,\vec{E}_T = -ik\,\hat{e}_z \times \vec{H}_T$

These are PDE's. What are the boundary conditions? $E_{\text{tangential}}$ is continuous, and it is 0 in the conductor; so $E_z|_S = 0$.

Since $\nabla_T \times \vec{E}_T = 0$ we can write

$$\vec{E}_T = \frac{\text{ik}}{\gamma^2}\, \nabla_T\, \psi\ .$$

By the wave equation,

$$(\nabla_T^2 - k^2 + \mu\epsilon\,\omega^2)\, \psi = 0$$

$$= (\nabla_T^2 + \gamma^2)\, \psi$$

where $\gamma^2 \equiv \mu\epsilon\omega^2 - k^2$

We also have

$$\text{ik}\, E_z = -\, \nabla_T \bullet \vec{E}_T \implies E_z = \frac{-1}{\gamma^2} \nabla_T^2 \psi = \psi;$$

this implies the boundary condition

$$\psi\, |_S = 0\ .$$

Wave equation + boundary condition
$\implies$ eigenvalues and eigenfunctions.

TE modes ; E_z = 0

Please verify ...

TE modes	$E_z = 0$
$\nabla \bullet \mu\vec{H} = 0$	$\nabla_T \bullet \vec{H}_T + \text{ik}\, H_z = 0$
$\nabla \bullet \epsilon\vec{E} = 0$	$\nabla_T \bullet \vec{E}_T = 0$
$\text{i}\omega\, \mu\vec{H} = \nabla \times \vec{E}$	$\text{i}\omega\, \mu H_z = \hat{e}_z \bullet (\nabla_T \times \vec{E}_T)$
`	$\text{i}\omega\, \mu\vec{H}_T = -\,\text{ik}\, \hat{e}_z \times \vec{E}_T$
$-\text{i}\omega\, \epsilon\, \vec{E} = \nabla \times \vec{H}$	$0 = \hat{e}_z \bullet (\nabla_T \times \vec{H}_T)$
`	$-\text{i}\omega\epsilon\, \vec{E}_T = \hat{e}_z \times \nabla_T H_z - \text{ik}\hat{e}_z \times \vec{H}_T$

Write $\vec{H}_T = \frac{\text{ik}}{\gamma^2} \nabla_T\, \psi$. Then show $H_z = \psi$.

The boundary condition: B_{normal} is continuous at S and =0 in the conductor;
$\therefore$ $\mathbf{n} \bullet \boldsymbol{H_T} = 0$ on the surface S;
this implies $\mathbf{n} \bullet \nabla_T\, \psi = 0$; or, $(\partial\psi / \partial n)\, |_S = 0$.

Modes of propagation in a rectangular waveguide

Out[•]=