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Let's go back and review a couple of topics ...

Skin depth; and Attenuation in a wave guide

From Chapter 5, Section 5.18:
"Quasi static magnetic fields in conductors, eddy currents, and magnetic diffusion"

Start with these field equations

$$\nabla\times \vec{H} = \vec{J} \qquad \textit{(neglect displacement current!)}$$

$$\nabla\bullet \vec{B} = 0 \text{ and } \nabla\bullet \vec{E} = 0$$

$$\nabla\times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$\vec{J} = \sigma \vec{E}$$

- $\vec{B} = \nabla\times \vec{A}$ and $\vec{E} = -\nabla\Phi - \frac{\partial \vec{A}}{\partial t}$ and $\nabla\bullet \vec{A} = 0$.
- If there is no free charge then we can set $\Phi = 0$.
- $\nabla\times \vec{B} = \mu \vec{J} = \mu\,\sigma \vec{E}$.

$$\nabla\times\nabla\times \vec{A} = \nabla(\nabla\bullet \vec{A}) - \nabla^2 \vec{A}$$

$$= \mu\sigma \left(-\frac{\partial \vec{A}}{\partial t}\right)$$

$\Longrightarrow$ the diffusion equation

$$\nabla^2 \vec{A} = \mu\sigma \frac{\partial \vec{A}}{\partial t}$$

The potential (and ∴ the fields) diffuse into the conductor by a distance L during the time τ, where

$$\tau = O(\mu\sigma L^2).$$

$\tau = \mathcal{O}(\mu\sigma L^2)$.

Assume there is a tangential magnetic field

$H_x = H_0 \cos\omega t$

at the surface of a conductor.

Then what is the field inside the conductor?

We could guess that the field is in a thin layer of width L with

$L = \mathcal{O}\left(\frac{1}{\sqrt{\mu\sigma}} \frac{2\pi}{\omega} \right)$.

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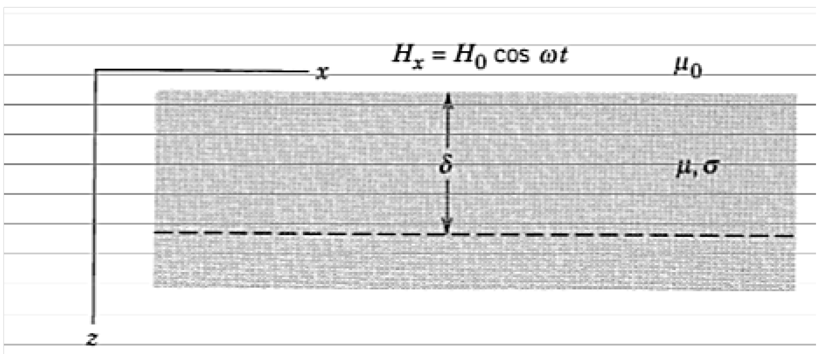


Let's derive it…

We have a boundary value problem,

$H_x(z,t) = h(z)\, e^{-i\omega t}$ *(real part understood)*

$$\left(\frac{d^2}{dz^2} + i\,\mu\sigma\omega \right) h(z) = 0$$

$$h(z) = e^{ikz}$$

$$k^2 = i\,\mu\sigma\omega \implies k = \pm(1+i)\sqrt{\frac{\mu\sigma\omega}{2}}$$

$$h(z) = e^{i\,z/\delta}\ e^{-z/\delta} \text{ where } \delta = \sqrt{\frac{2}{\mu\sigma\omega}}$$

i.e., $H_x(z) = H_0\, e^{-z/\delta} \cos(z/\delta - \omega t)$

a damped wave with decay length δ and wavelength $2\pi/\delta$.

– propagating but damped in the z direction

– polarized in the x direction

– an associated $\vec{E} = E_y\, \hat{e}_y$

Example: Copper at room temperature,

$\sigma = (1.68 \times 10^{-8}\ \Omega\text{m})^{-1}$

$\delta = 0.0652\ m \sqrt{\frac{(2\pi\,\text{Hz})}{\omega}} \propto \frac{1}{\sqrt{\omega}}$

- Resistive heating

Power per unit volume
$= \langle \sigma E^2 \rangle = \frac{1}{2} \mu\omega H_0^2\ e^{-2z/\delta}$

Waveguides
–boundary conditions (Section 8.1)

To a first approximation, we treat the conducting walls as perfect conductors to calculate the fields in the interior.

For real metals ...

Jackson :
"A good conductor behaves effectively like a perfect conductor, with the idealized surface current replaced by an equivalent surface current , which is actually distributed throughout a small thickness at the surface."

Read Jackson Section 8.1: "Fields at the surface of and inside a conductor"

Perfect conductor

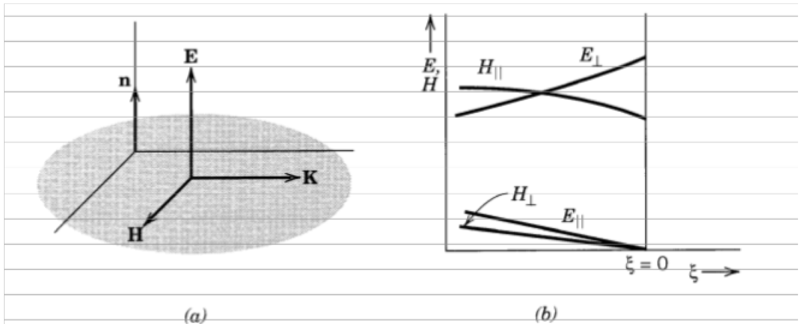
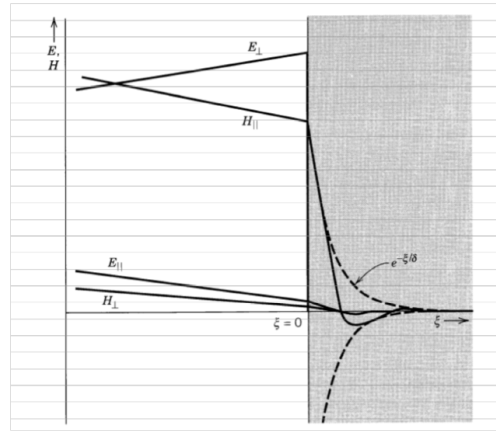


Figure 8.1 Fields near the surface of a perfect conductor.

The shaded region is a small area on the conductor surface; $\hat{n}$ = unit normal;
$\vec{K}$ = surface current density;
$\vec{E} = 0$ and $\vec{H} = 0$ in the perfect conductor,
so $\vec{E}$ is normal and $\vec{H}$ is tangential at the outer surface.

For a real metal, the conductivity σ is large but finite; so the skin depth $\delta = \sqrt{2/(\mu_0\,\omega\sigma)}$ is small .



- The energy loss from electrical resistance is calculated from these equations:

$\vec{H}_c = \vec{H}_{\parallel}\, e^{-\xi/\delta}\, e^{i\xi/\delta}$,

Out[•]= where ξ = distance inside the conductor,

and $\vec{H}_{\parallel}$ = tangential field at the surface

Out[=]= $\vec{E}_c = \sqrt{\frac{\mu_c\,\omega}{2\,\sigma}}\,(1 - i\,)(\hat{n} \times \vec{H}_\parallel\,)\, e^{-\xi/\delta}\, e^{i\xi/\delta}\,;$

this follows from

$$\nabla\times \vec{H}_c = \sigma\,\vec{E}_c\; = -\,\hat{n} \times \frac{\partial \vec{H}_c}{\partial \xi}$$

■ Power loss

The guided wave loses energy; and the power loss = the power flowing into the surface of the conductor.

$$\left\langle\frac{dP_{loss}}{da}\right\rangle = -\,\frac{1}{2}\,\mathrm{Re}\{\,\hat{n} \bullet \vec{E} \times (\vec{H})^*\,\}$$

$$= \frac{\mu_c\,\omega\delta}{4}\;|\,\vec{H}_\parallel\,|^2$$

where ⟨..⟩ means time averaged.

$$\left\langle\frac{dP_{loss}}{da}\right\rangle = \frac{\mu_c\,\omega\delta}{4}\;\left|\vec{H}_\parallel\,\right|^2$$

Or, $\mu\omega\delta = \dfrac{2}{\sigma\delta}$

- For a waveguide, P(z) $= P_0\, e^{-2\,\beta\, z}$;

$$\beta = \frac{-1}{2\,P}\ \frac{\mathrm{dP}}{\mathrm{dz}}$$

$$\mathrm{P} = \int_A \frac{1}{2}\,(\vec{E} \times \vec{H}^{*}) \bullet \hat{e}_z\,\mathrm{da}$$

$$-\frac{\mathrm{dP}}{\mathrm{dz}} = \frac{1}{2\,\sigma\delta} \oint_C |\,\hat{n} \times \vec{H}\,|^2\, dl$$

and the fields for TE or TM waves are known!

- For a resonant cavity, $\frac{\omega_0}{Q} = \frac{-\mathrm{dU/dt}}{U}$;

Out[◦]=

$$U = \frac{d}{4}\begin{Bmatrix}\epsilon\\ \mu\end{Bmatrix}\left[1 + \left(\frac{p\pi}{\gamma_\lambda d}\right)^2\right]\int_A |\psi|^2\, da \qquad (8.92)$$

Out[◦]=

$$P_{\mathrm{loss}} = \frac{1}{2\sigma\delta}\left[\oint_C dl \int_0^d dz\, |\mathbf{n} \times \mathbf{H}|^2_{\mathrm{sides}} + 2\int_A da\, |\mathbf{n} \times \mathbf{H}|^2_{\mathrm{ends}}\right] \qquad (8.93)$$

and the fields for TE and TM waves are known!