GLUON CORRECTIONS TO DRELL-YAN PROCESSES *

H. David POLITZER †

Lyman Laboratory, Harvard University, Cambridge, Massachusetts 02138

Assuming that the Drell-Yan picture is approximately valid, I compute the $O(g^2)$ gluon radiative corrections in QCD. The corrections are self-consistent, i.e. they satisfy several non-trivial constraints necessary for the validity of the whole picture. There are logarithmic scaling violations in the quark distribution functions, precisely equal to those found in electroproduction. Large transverse momenta (of order of the c.m. energy) are produced, balanced by radiation in the opposite direction. I compute the transverse momentum differential cross section for μ pairs as an example.

1. Introduction

The status of the Drell-Yan analysis of production of massive μ-pairs in hadron-hadron collisions [1] remains uncertain from the standpoint of quark-gluon gauge theory, QCD. This in no way detracts from its significance or urgency. Lepton pair production certainly tells something about the interactions of hadron constituents. And most expectations for the production of W bosons in hadron colliding beams are based on this picture [2]. It is also possible to estimate the hadronic production of new quantum numbers in the same way [3]. Yet there is no comprehensive theoretical analysis that goes much beyond the original work. At least there is agreement that the Drell-Yan picture is not obviously wrong [4].

In the absence of a general analysis, it is not immediately clear how to compute corrections. I have formulated an algorithm for these corrections based on Feynman graphs of the basic parton scattering cross sections. The key issue is whether all infrared sensitivity can be absorbed into the parton distribution functions. This algorithm reproduces the standard results in electroproduction [5] and can be applied directly to Drell-Yan processes. I present here the corrections to order g^2, the quark-gluon coupling constant. Aspects of this problem have been discussed recently by

several authors [6–9]. The detailed calculations to $O(g^2)$ agree with all of the general-al conclusions reached earlier.

These $O(g^2)$ corrections satisfy several non-trivial consistency conditions which are necessary for the validity of the Drell-Yan picture. And if that picture is correct and if it has anything to do with asymptotic freedom, then these must be the leading corrections to the naive parton results for high energies. The algorithm suggests a method for proving its validity: proving its consistency order-by-order to all orders in g^2. That would at least put it on a par with the electroproduction analysis.

This paper is organized as follows: I first discuss the phenomenological consequences and present the lepton-pair differential cross sections to $O(g^2)$. Then to introduce the method, I sketch a quark-gluon-parton analysis of electroproduction. The same method is then applied to Drell-Yan processes. Finally, and only briefly, the extension to higher orders and questions of principle are discussed.

2. Phenomenological consequences

The basic idea is that, given the quark and gluon distribution functions (determinable in varying degrees from lepton-hadron scattering), one can predict lepton pair, W boson, or charm production by convoluting the distribution functions with the basic parton cross section. See fig. 1. (I will use μ-pair production as an example; the generalization to the other cases is straightforward, although charm production will merit some additional comments at the very end). To $O(g^2)$ there is a logarithmic dependence on M^2, the mass-squared of the produced pair. This takes the form of M^2 dependent effective parton distributions. The M^2 dependence is precisely that found in electroproduction structure functions. There are also large transverse momentum processes that scale. If $k_\perp$ is the component of the μ-pair momentum k orthogonal to the collision axis, then $\langle k_\perp^2 \rangle \sim g^2 M^2$. These come from graphs such as in figs. 2 and 3. $O(g^4)$ corrections will undoubtedly turn the g^2 into an effective M^2 dependent coupling; whence $\langle k_\perp^2 \rangle \sim M^2 / \log M^2$.

For $k_\perp^2 \neq 0$, the differential cross section can be computed from the squares of the amplitudes of figs. 2 and 3. For example, let $P_{1,2}$ be the initial hadron momenta and let $M^2 = k^2$, the μ-pair mass squared; the cross section for producing a μ-pair
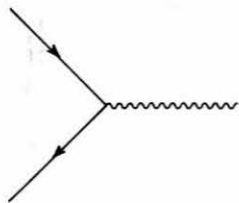

Fig. 1. The amplitude for $d\sigma_{\text{parton}}$ in the free parton approximation: quark + antiquark virtual photon (μ-pair) or any other local current.
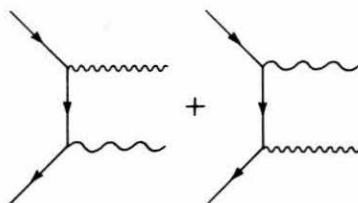
Fig. 2. The $O(g^2)$ real gluon radiative corrections to quark + antiquark production of a virtual photon.

plus anything is

$$d\sigma = \int dx_1 f_1(x_1) \, dx_2 \, f_2(x_2) \, d\sigma_{\text{parton}} . \qquad (1)$$

For quark-antiquark scattering, the diagrams of fig. 2 give

$$d\sigma_{q\bar{q}} = \frac{4\pi\alpha^2}{3M^4} Q^2 \frac{g^2}{4\pi^2} \frac{4}{9} dM^2 \, dz \, \frac{1}{4} \frac{M^2}{s} \left\{ \frac{2s + (s/M^2 - 2)(s + M^2 - z)}{2s(s - M^2 - z)} \right.$$

$$\left. + \frac{2s + (s/M^2 - 2)(s + M^2 + z)}{2s(s - M^2 + z)} + \frac{1}{M^2} \frac{8M^4 - (s - M^2)^2 + z^2}{(s - M^2)^2 - z^2} \right\}, \qquad (2)$$

where s is the parton-parton c.m. energy squared, $s = x_1 x_2 S$ (with S being the hadron energy squared) and $z = +\sqrt{(s - M^2)^2 - 4sk_\perp^2}$. I have spin and color averaged (summed) over initial (final) states. The factor $\frac{4}{9}$ is the resulting color factor. Q^2 is the relevant quark charge-squared in units of e^2. Quark masses are neglected in the above. The differential cross section for quark- or antiquark-gluon scattering can be likewise evaluated from the square of the amplitude depicted in fig. 3:

$$d\sigma_{\substack{\text{(anti) quark} \\ + \text{ gluon}}} = \frac{4\pi\alpha^2}{3M^4} Q^2 \frac{g^2}{4\pi^2} \frac{1}{6} dM^2 \, dz \, \frac{1}{2s^2}$$

$$\times \left(\frac{1}{2}s + \frac{1}{4}(s + M^2)\left(\frac{s}{M^2} - 2\right) + \left\{ s + \frac{(s + M^2)^2 - z^2}{4M^2} \right\} \frac{s(s - M^2)}{(s - M^2)^2 - z^2} \right.$$

$$- \frac{1}{8((s - m^2)^2 - z^2)} \left((s - M^2 + z)^2 \left(4M^2 - \frac{s}{M^2}(s - M^2 - z) \right) \right.$$

$$\left. \left. + (s - M^2 - z)^2 \left(4M^2 - \frac{s}{M^2}(s - M^2 + z) \right) \right) \right). \qquad (3)$$

It is often of interest to know the differential cross section as a function of the μ-pair longitudinal (three-) momentum, k_L, also. That information is contained in eqs. (1) and (2) and is straightforward to extract. It is a consequence of momentum
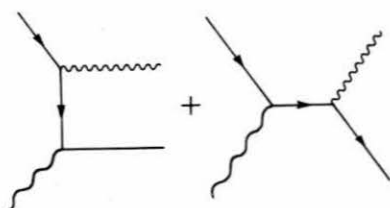
Fig. 3. The lowest order contribution of quark + gluon → virtual photon + anything.

conservation because to $O(g^2)$ the μ-pair recoils against a single parton of mass zero. Hence, for each x_1, x_2, M^2 and $k_\perp^2$, there is a unique k_L.

These differential cross sections blow up as $k_\perp^2 \to 0$. In practice this means that the above expressions are valid for $k_\perp^2 \gtrsim 1$ GeV^2, but for $k_\perp^2 \lesssim (400$ MeV$)^2$ there are other effects which must also be considered. These other effects include the non-zero "primordial" transverse momenta of the quarks, coherence effects within the hadrons, and final state interactions. But all of these go like $\sim (400$ MeV$)^2/M^2$ relative to the effects of figs. 2 and 3, which scale in M^2.

The mean value of $k_\perp^2$ predicted by eqs. (2) and (3) is finite and proportional to g^2M^2. For $M \gtrsim 2$ GeV, this will dominate over the $k_\perp$ coming from the primordial quark transverse momentum. To compute $\langle k_\perp^2 \rangle$, one must integrate over z (or $k_\perp^2$) with the weight factor

$$k_\perp^2 = \frac{(s - M^2)^2 - z^2}{4s} \tag{4}$$

and then convolute with the distribution functions.
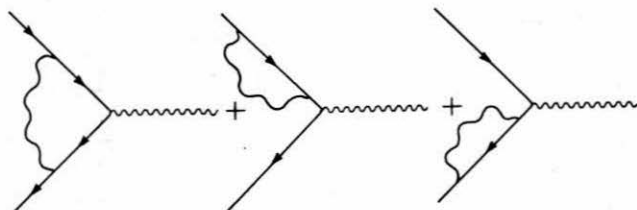
The exact forms are complicated and probably best done numerically. However, the problem simplifies for $x_i \to 1$, where we can pick out the smallest power of $(1 - x)$. The result can be stated most simply as an equivalent initial quark transverse momentum $p_\perp$ which would reproduce the $\langle k_\perp^2 \rangle$ of eqs. (2) and (3) when inserted in the naive $(g = 0)$ parton model (fig. 1). The result is, as $x \to 1$, The initial quark effectively has a

$$\langle p_\perp^2(x_i, M^2) \rangle = \frac{g^2}{12\pi^2} M^2(1 - x_i) = \frac{4}{27} \frac{M^2(1 - x)}{\log(M^2/\Lambda^2)} . \tag{5}$$

Λ^2 is determined by the size of g. From scaling violations in electroproduction [10], we know $\Lambda^2 \simeq 0.25$ GeV^2. Higher powers of $(1 - x)$ have numerically smaller coefficients, so eq. (5) should be approximately valid for a large range in x.

Eq. (5) agrees with a similar effective, naive, initial quark transverse momentum distribution interpretation of electroproduction. In electroproduction, fig. 5 gives a $\sigma_L/\sigma_T \propto g^2(Q^2)$. In the naive parton model, $\sigma_L/\sigma_T = 4\langle p_\perp^2 \rangle/Q^2$. Eq. (5) with $M^2 \to Q^2$ gives an excellent fit to the detailed QCD prediction for σ_L/σ_T [10].

If we are willing to lump all the small $k_\perp^2$ effects together, it is possible to derive

Fig. 4. $O(g^2)$ virtual gluon corrections to $d\sigma_{parton}$.

a very useful prediction. In practice, [i.e. once $d\sigma_{parton}$ is integrated in eq. (1)], it is impossible to distinguish a low $k_\perp^2$ event with $M^2 < x_1 x_2 S$ for some particular $x_1 x_2$ from an elastic quark scattering (as in fig. 1) with a smaller $x_1 x_2$. As I will show, the divergences in eq. (2) for small k can be reabsorbed into the $f_i(x_i)$, the parton distribution functions. Of course, one must also include virtual gluons to $O(g^2)$ as in fig. (4).

The most useful form follows from several approximations. First, I will integrate over all available $k_\perp^2$, i.e. $0 \leqslant k_\perp^2 \leqslant (s - M)^2/4s$; this result could be combined with equation (2) to predict the integral over a smaller $k_\perp^2$ interval, i.e. $0 \leqslant k_\perp^2 \leqslant \lambda^2$, for some λ^2. Second, the result is analytically simple only if we ignore the possibility of initial gluons as in fig. (3). This is probably a good approximation for $x_{1,2} \gtrsim 0.2$ [10] and for the S accessible in the next generation of machines. And third, to $O(g^2)$ there are effects that go like $\log M^2$, like constants, and like inverse powers of M^2. For simplicity, I record here only the largest, i.e. $g^2 \log M^2$, terms. It is these terms which enter into the theoretical questions of principle. The computation of the constant terms is straightforward from figs. 1–4. Under all these approximations, I find

$$\int \frac{d\sigma_{q\bar{q}}}{dk_\perp^2} \, dk_\perp^2 = \frac{4\pi\alpha^2}{3M^4} \, Q^2 \, dM^2 \int_\tau^1 dx_1 f_1(x_1, M_0^2) \int_{\tau/x_1}^1 dx_2 f_2(x_2, M_0^2)$$

$$\frac{1}{3}\left[\delta\left(1 - \frac{\tau}{x_1 x_2}\right) - \frac{g^2}{4\pi^2} \frac{4}{3} \cdot \frac{1}{2} \log\left(\frac{M^2}{M_0^2}\right)\left\{-3\delta\left(1 - \frac{\tau}{x_1 x_2}\right)\right.\right.$$

$$\left.\left. + \left(2 + \frac{\tau}{x_1 x_2} + \left(\frac{\tau}{x_1 x_2}\right)^2\right) + 4\int_0^1 \frac{dz}{1-z}\left(\delta(1 - \frac{\tau}{x_1 x_2}) - \delta\left(z - \frac{\tau}{x_1 x_2}\right)\right)\right\}\right], \quad (6)$$

$$\int \frac{d\sigma_{q+gluon}}{dk_\perp^2} \, dk_\perp^2 = \frac{4\pi\alpha^2}{3M^4} \, Q^2 \, dM^2 \int_\tau^1 dx_1 f_1(x_1, M_0^2) \int_{\tau/x_1}^1 dx_2 f_2(x_2, M_0^2)$$

$$\times \frac{1}{3} \frac{g^2}{4\pi^2} \frac{1}{2} (4x^3 - 4x^2 + 2x) \log\frac{M^2}{M_0^2}. \quad (7)$$

τ is the dimensionless variable $\tau = M^2/S$. The z-integral term is a distribution which

is well defined when inserted into one of the x_i integrals and the order of integration is interchanged. The quark probability distributions $f_i(x_i)$ have been replaced by a simple generalization $f_i(x_i, M_0^2)$. The M_0^2 dependence, implicit in eq. (6) is precisely what is required to allow the identification $xf_i(x, Q^2) \sim \nu W_{2i}(x, Q^2)$, i.e. the ith quark's contribution to νW_2 including the logarithmic Q^2 dependence. The association of the $\log(M^2/M_0^2)$ factor with $d\sigma_{parton}$ is completely arbitrary. Eq. (6) could just as well be written with the free quark $d\sigma_{parton}$ convoluted with the M^2 dependent quark distribution. As it is written eq. (6) can be interpreted as summing the contributions of quark pairs with $s \geqslant M^2$ to the production of a μ pair at M^2. The $O(g^2)$ effects diminish the cross section with increasing M^2 at fixed τ. Hence the production of very massive objects ($M \gtrsim 0.4\sqrt{S}$) will be significantly smaller at high S than expected from the naive parton model. For $M \sim (0.2 - 0.4)\sqrt{S}$, the scaling violations are in fact very mild. For $M \lesssim 0.2\sqrt{S}$ there will be a net enhancement from radiative corrections at large S, but the details here are necessarily entwined with the nature of the gluon distribution.

5. The algorithm for electroproduction

In the absence of an orthodox analysis, I will put the QCD version of electroproduction in a parton form such that an extension to Drell-Yan processes is possible. (An alternate parton approach which includes all the details of the operator product analysis of electroproduction has been described by Parisi [11]. Of course the two are ultimately equivalent).

Quark-gluon perturbation theory stands a chance of being a reliable guide to nature only if successive terms in the expansion are successively smaller. This requires that the coupling constant be small and that the coefficients in the expansion are not uncontrolably large. In most naive applications, apparent infrared and threshold divergences appear to render perturbation theory useless. The challenge is to find processes or features of processes which are amenable to an orderly expansion. Scaling behavior in lepton-hadron scattering is one such example. But the analysis requires the care typically needed in infrared problems to separate observable from unobservable effects.

Feynman suggests that in the impulse approximation the inclusive cross section for scattering a virtual photon of momentum q off a hadron of momentum P takes the form [12]

$$d\sigma(P + q) = \int dz \, f(z) \, d\sigma_{parton}(zP + q) , \qquad (8)$$

where $d\sigma_{parton}$ is computed in free field theory.

There are many different physical effects which give non-scaling power corrections, i.e. powers of m^2/Q^2 for various m^2, to eq. (8). Only certain of these (but, in practice, the largest [13, 10]) are amenable to theoretical analysis. In general they are of

so many different origins that no cogent analysis is possible – short of computing the structure functions themselves from first principles. In this respect, the field theoretic analysis has added precious little to the earliest parton picture.

The field theory of interacting gluons does suggest that there are corrections to eq. (8) which scale (up to logarithms) and hence survive long after the power corrections die away. The parton model derivation of the form of eq. (8) for the leading contribution to lepton-hadron scattering is correct. What must be modified is the insistence on using free field theory for computing $d\sigma_{\text{parton}}$. It is the perturbative corrections to $d\sigma_{\text{parton}}$ that give almost-scaling terms in the cross section.

The graphs that contribute to $O(g^2)$ are shown in fig. 5. The computation of the square of the sum of these amplitudes is straightforward, particularly in the approximation $m_{\text{quark}} \approx 0$. However, the loop graphs are infrared divergent as are the phase space integrals in the inclusive cross section. It is convenient to regulate these infrared divergences by first evaluating the graphs off shell. If p^2 is the non-zero momentum-squared of the off-shell massless quarks and gluons, then graph-by-graph (squared and integrated over phase space) there are terms that go like $\log^2(Q^2/p^2)$, $\log(Q^2/p^2)$, constants, and positive powers of p^2/Q^2. The $\log^2(Q^2/p^2)$ terms cancel when all terms are summed. This is a manifestation in the present context of the cancelation of the infrared divergences between real and virtual emissions. The remaining $\log(Q^2/p^2)$ divergences are associated only with the initial quarks (or gluons). This reflects the general disease of massless, charged particles. Specifically, there is a type of "infrared" divergence which occurs here which cannot occur for massive particles. A massless particle can emit a hard massless particle and still remain on its mass shell as long as the particles are colinear. This process is kinematically allowed and will
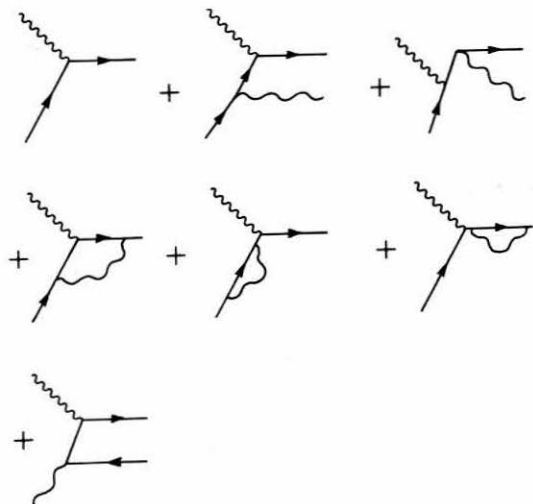

Fig. 5. Electroproduction off partons to $O(g^2)$: photon + quark (or gluon) → anything.

produce a divergence if the phase space is large enough. Allowing the quarks a small mass will replace the $\log(Q^2/p^2)$ associated with initial quarks by $\log(Q^2/m^2)$. But for light quarks, $m^2 \ll Q^2$, such terms still spoil any naive use of perturbation theory, and there still remain massless, colored gluons, which will give infrared divergences in $O(g^2)$ even if all the quarks are massive.

The key observation in handling the $\log(Q^2/p^2)$ as $p^2 \to 0$ is that $d\sigma_{\text{parton}}$ essentially factorizes into a Q^2 dependent well-behaved piece times a Q^2 independent infrared divergent piece. The latter, since it is Q^2 independent, should be absorbed into the parton distribution functions.

The factorization is a generalization of the following form:

$$1 + ag^2 \log(Q^2/p^2) + ... = [1 - ag^2 \log(p^2/Q_0^2) + ...][1 + ag^2 \log(Q^2/Q_0^2) + ...] . \tag{9}$$

To $O(g^2)$, factorizability requires only the cancellation of the $\log^2$ terms. In higher orders, factorization of the Q^2 dependence from the p^2 dependence is a far more complex issue, but it can be proven order by order in perturbation theory. (That is the essential content of the operator product expansion).

Implementation of factorization of the type indicated in eq. (9) is somewhat complicated by the x dependence of $d\sigma_{\text{parton}}$. The problem naturally simplifies in terms of x moments. As an example, consider electroproduction off a quark of momentum $zP = p$:

$$d\sigma_{\text{quark}} \propto \delta(x/z - 1) + a(x/z) g^2 \log(Q^2/p^2) + ... \tag{10}$$

where $x = Q^2/2P \cdot q$ and I have dropped the terms that are constant or vanish as $p^2 \to 0$. $d\sigma_{\text{quark}}$ is not of the form of eq. (9), but its moments are.

To see the significance of moments, consider the moments of eq. (4).

$$\int_0^1 dx \, x^n d\sigma(x, Q^2) = \int_0^1 dx \, x^n \int_x^1 dz \, f(z) \, d\sigma_{\text{parton}}(x/z, Q^2) \tag{11}$$

$$= \int_0^1 dz \, z^n \, zf(z) \int_0^1 dy \, y^n d\sigma_{\text{parton}}(y, Q^2) .$$

For a struck quark,

$$\int_0^1 dy \, y^n d\sigma_{\text{quark}}(y, Q^2) \propto 1 + a_n g^2 \log(Q^2/p^2) + ... , \tag{12}$$

where

$$a_n = \int_0^1 dy \, y^n \, a(y) \tag{13}$$

and $a(y)$ comes from eq. (10). So n-by-n, the infrared sensitivity can be factored and reabsorbed into the f's:

$$\int_0^1 x^n d\sigma(x, Q^2) \propto \int_0^1 dz \, z^n zf(z, Q_0^2)[1 + a_n g^2 \log(Q^2/Q_0^2) + ...] \, , \tag{14}$$

where

$$\int_0^1 dz \, z^n zf(z, Q_0^2) = \int_0^1 dz \, z^n f(z)\left[1 + z_n g^2 \log \frac{Q_0^2}{p^2} + ...\right]. \tag{15}$$

The fact that the terms on the right-hand side of eq. (15) make no sense as $p^2 \to 0$ is irrelevant because it is only $f(z, Q_0^2)$, and not $f(z)$, that is experimentally observable. The moment prediction can be inverted to give a rather simple result:

$$d\sigma(x, Q^2) \propto \int_x^1 dz \, f(z, Q_0^2)\left[\delta\left(\frac{x}{z} - 1\right) + a\left(\frac{x}{z}\right)g^2 \log \frac{Q^2}{Q_0^2} + ...\right]. \tag{16}$$

Taking moments is not only a theoretical tool used in justifying eq. (16), but is a very practical calculational tool. This is because $a(x/z)$ is not a well-behaved function for $x/z \to 1$ but is in fact a distribution. Hence its meaning is made precise only by its integrals.

There are three comments that must be made regarding the above discussion. First, no reference was made to the renormalization group or to asymptotic freedom. Second, the orthodox analysis is fraught with pitfalls and comes with many qualifications, none of which appeared above. And third, why do we choose x and Q^2 as variables instead of x and ν? I will discuss these issues in order.

The renormalization group, operator mixing, and asymptotic freedom enter the discussion when we ask what happens for $Q^2 \gg Q_0^2$ or $Q^2 \ll Q_0^2$ or, equivalently, if we raise the issue that nothing should really depend on Q_0^2, an arbitrary parameter introduced for convenience. If $Q^2 \gg Q_0^2$, then the power series in g^2 in eq. (16) is not well behaved in that successive terms are not small. Conversely, as $Q^2 \to Q_0^2$ the power series should be good as long as g^2 is small. The $Q^2 \to Q_0^2$ limit is a differential equation in $Q^2 = Q_0^2 + \Delta Q^2$. But to integrate this equation in Q^2 up to $Q^2 \gg Q_0^2$, we must know how $f(x, Q_0^2)$, depends on Q_0^2. The Q_0^2 dependence of the f's is a matrix problem because the quark and gluon distributions are coupled; the presence of one induces the presence of the other. The full structure of the quark-gluon mixing manifests itself first only in $O(g^4)$ in the naive Feynman graphs for $d\sigma_{parton}$. See fig. 6. However, $O(g^4)$ is the first appearance of the $O(g^2)$ dependence of $f_{glue}(x, Q_0^2)$ on $\log Q_0^2$, and this $O(g^2)$ dependence is needed in the Q^2 differential equation if it is to be accurate to $O(g^2)$. Similarly, only in $O(g^4)$ of $d\sigma_{parton}$ do we encounter the $O(g^2)$ $\log Q_0^2$ dependence of $g(Q_0^2)$. The issue of asymptotic freedom is whether $g(Q^2)$ increases or decreases with Q^2, which in turn determines the con-
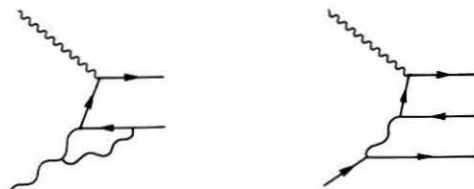
Fig. 6. Examples of the lowest order occurrences of the infrared renormalization of the gluon distribution.

sistency of using perturbation theory at large Q^2, given that the interactions must be strong at long distances. In practice, though, the form given in eq. (16) will only differ from the integrated differential form by terms of yet higher order in $g^2(Q_0^2) \log(Q^2/Q_0^2)$. So if $g^2(Q_0^2)$ is already small, Q^2 will have to be exponentially large to invalidate the unimproved form.

The second issue concerns all of the caveats that attend the QCD analysis of electroproduction. Why must we smear over final states, avoid $x \to 0$, etc? Where are the non-uniformities in n versus g^2? The answer is that higher orders of perturbation theory contain yet worse infrared problems than suggested by the lowest order calculation. In higher and higher orders, the attractive forces between members of color-neutral subsets of the produced quarks and gluons will give stronger and stronger threshold divergences in the sub-energy. Similar effects involving the spectator quarks will give arbitrarily large coefficients to the terms that drop like $1/Q^2$. A variety of approximations and guesses must be introduced to bring any order to this chaos [13,10]. One clear lesson that comes out of the most careful analyses to date is that any feature that is sharp as a function of energy must be smeared over. That applies to both theory and experiment. Only smeared quantities have any reason to be comparable. All such caveats must be carried over into the analysis of Drell-Yan processes. The question I wish to face, and will only begin to answer, is whether the parton model can be justified in the context of μ-pair or W boson production to the same extent that it is justified in lepton-hadron scattering.

The calculation sketched above concentrated on the $\log(p^2/Q^2)$ terms and dropped constant pieces. Hence it was ambiguous as to whether it should be $\log(p^2/Q^2)$ or $\log p^2/\nu$, as these differ only by constants. The correct variable is Q^2, as I will explain below. Remember, however, that there still will be constant pieces and that there is a strong x dependence multiplying the log Q^2.

There is a physical answer to why x and Q^2 rather than x and ν, which translates into a technical, field-theoretic answer. As noted, we calculate x moments. The x moments for fixed ν go down to $Q^2 = 0$, i.e. they touch on photoproduction, which is not really describable in parton language. In contrast, x moments for fixed Q^2 range over $Q^2/2m < \nu < \infty$. This region is more amenable to a parton analysis. Technically speaking, the question rests on the analytic structure in the variables Q^2 and ν. The moments are computed by analytic continuation. We use x and Q^2 to avoid the photoproduction singularities.

4. Drell-Yan processes

I propose that corrections to the Drell-Yan mechanism can be evaluated in the same way. Here, too, there are infrared divergences that thwart any naive application of perturbation theory. However, we can ask whether the infrared dependence can be factored from the dependence on the observable kinematic variables, leaving a well-behaved expansion. For the specifics of the Drell-Yan picture to be self-consistent, we must further inquire whether the infrared term itself factors into two pieces, one to be absorbed into the wave function of each incident hadron. And finally, for the parton distribution functions to be the same functions encountered in electroproduction, the infrared factors must be the same for the two processes. (This ensures the same Q^2 dependence.)

All of these necessary conditions are satisfied to $O(g^2)$. I will now discuss that calculation.

Begin with the form suggested by Drell and Yan for production of μ-pairs, for example, of mass M[1]:

$$d\sigma = \int dx_1 f_1(x_1) \, dx_2 f_2(x_2) \, d\sigma_{\text{parton}} . \tag{17}$$

If P_1 and P_2 are the momenta of the incident hadrons, then $d\sigma_{\text{parton}}$ is the production cross section of the μ pair given partons of momenta $p_1 = x_1 P_1$ and $p_2 = x_2 P_2$. The free field expression for $d\sigma_{\text{parton}}$ (from fig. 1) gives the original Drell-Yan result.

Consider now $O(q^2)$ and figs. 2–4 [There are in fact other quark-gluon processes to $O(g^2)$ which do not give the form of eq. (13) involving the spectators, but these are down by powers of M^2 or $S = (P_1 + P_2)^2$ and are infrared finite in $O(g^2)$.] Because the partons are massless, infrared singularities arise whenever one of the virtual quanta becomes parallel to one of the massless external lines and consequently itself goes on shell. These singularities can be regulated by taking $p_1^2 \neq 0, p_2^2 \neq 0$. It is also necessary to keep $p_1^2 \neq p_2^2$ or at least to distinguish p_1^2 from p_2^2 to know which divergence goes into which hadron.

Since the singularities arise from being parallel to p_1 or p_2, they take the form of $\log p_1^2$ or $\log p_2^2$ and not $\log(p_1^2 + p_2^2)$. This feature is necessary for the factorization. The two logarithms can coalesce when a radiated (real or virtual) gluon is soft. Then the internal quark momentum can be parallel to both initial momenta. This possibility gives terms proportional to $\log p_1^2 \log p_2^2$. These terms do not satisfy the necessary factorization property, but they exactly cancel between the virtual gluon exchange and the real gluon emission. (In particular, it is the $\log^2$ in the cross term in the square of real emissions that cancels the virtual emissions.)

The evaluation of the virtual gluon correction is done easiest as it was done first [14], using a particular set of Feynman parameters. These can be specified by writing the virtual gluon momentum q as $q = z_1 p_1 + z_2 p_2 + q_\perp$. The $q_\perp$ integral is trivial, and the z_1, z_2 integral is straightforward.

This graph contributes only to $k_\perp = 0$, where k is the μ-pair momentum and $M^2 = (p_1 + p_2)^2 = s$. In contrast, the real bremsstrahlung graphs give $0 \leqslant k \leqslant (s - M^2)^2/4s$ and $0 \leqslant M^2 \leqslant s$. And the log divergences only occur when $k_\perp$ is integrated over an interval containing $k_\perp = 0$ (i.e. parallel emission), and the $\log^2$ occurs only when, in addition, M^2 is integrated over an interval containing $M^2 = s$ (i.e. soft emission). The coefficients of these logs are evidently distributions, and must be evaluated by taking integrals. As in electroproduction, we will find that the set of moments are not only a convenient basis of integrals for evaluating these distributions but that they also facilitate the factorization of infrared sensitive pieces.

Consider therefore τ^n moments of eq. (17) where $\tau = M^2/S$:

$$\int_0^1 d\tau\, \tau^n d\sigma(\tau) = \int_0^1 d\tau\, \tau^n \int_\tau^1 dx_1 f_1(x_1) \int_{\tau/x_1}^1 dx_2 f_2(x_2)\, d\sigma_{\text{parton}}(\tau/x_1 x_2)\,. \tag{18}$$

For simplicity, consider $d\sigma$ integrated over all available $k_\perp^2$. (The $k_\perp^2$ distribution could be extracted using $k_\perp^2$ moments, but in practice the $k_\perp^2 \neq 0$ distribution, e.g. eq. (2), and the integral over all $k_\perp$ are sufficient for phenomenological purposes). On changing orders of integration and variables:

$$\int_0^1 d\tau\, \tau^n d\sigma(\tau) = \int_0^1 dx_1 x_1^n x_1(x) \int_0^1 dx_2 x_2^n x_2 f(x_2) \int_0^1 dz\, z^n d\sigma_{\text{parton}}(z)\,. \tag{19}$$

The trickiest piece of $d\sigma_{\text{parton}}$ to evaluate is the bremsstrahlung cross term. I found it simplest to imitate the Sudakov variables and write the emitted gluon momentum q as $q = y_1 p_1 + y_2 p_2 - k_\perp$. The phase-space integral becomes an integral over y_1 and y_2, and $z^n = (M^2/s)^n = (1 - y_1 - y_2)^n$. The infrared sensitive pieces are easy to identify. The constant pieces are more difficult. Since the constant pieces neither effect the factorization nor significantly enter the phenomenology, I have dropped them here. For the cross term, I find

$$\int_0^1 dz\, z^n d\sigma_{\text{cross term}}(z) \propto \frac{g^2}{4\pi^2} \frac{4}{3} \left[\log \frac{-p_1^2}{M^2} \log \frac{-p_2^2}{M^2} + \log \frac{-p_2^2}{M^2} \sum_{j=1}^{n+2} \frac{1}{j} \right]\,. \tag{20}$$

Since the $\log^2$ term is n independent it is evidently a δ function at $M^2 = s$, and it exactly cancels the $\log^2$ in the vertex correction. The sum of all the contributions to quark-antiquark scattering is

$$\int_0^1 dz\, z^n d\sigma_{q\bar{q}}(z) = \frac{4\pi\alpha^2 Q^2}{3M^4} \frac{1}{3} \left[1 + \frac{g^2}{4\pi^2} \frac{4}{3} \cdot \frac{1}{4} \left(\log \frac{-p_1^2}{M^2} + \log \frac{-p_2^2}{M^2} \right) \right.$$

$$\left. \times \left\{ -3 + 4 \sum_{j=1}^{n+2} \frac{1}{j} + \frac{2}{n+3} - \frac{2}{n+2} \right\} \right] \tag{21}$$

$$= \frac{4\pi\alpha^2 Q^2}{3M^4} \left[1 + \frac{g^2}{4\pi^2}\frac{4}{3}\cdot\frac{1}{4}\log\frac{-p_2^2}{M_0^2}\left\{-3 + ...\right\}\right]$$

$$\times \left[1 + \frac{g^2}{4\pi^2}\frac{4}{12}\log\frac{-p_2^2}{M_0^2}\left\{-3 + ...\right\}\right]\left[1 - \frac{g^2}{4\pi^2}\frac{4}{12}\log\frac{M^2}{M_0^2}\left\{-3 + ...\right\}\right].$$

The infrared renormalization is precisely that encountered in electroproduction in the absence of gluons as initial partons. Continuing with equation (21), if the p_i^2 sensitive terms are absorbed into the f_i distribution functions, the remaining well-behaved piece can be inverted to give eq. (3). The Mellin transform of the $\sum 1/j$ term is facilitated by the following representations:

$$\sum_{j=1}^{n}\frac{1}{j} = \int_0^1 \frac{dx}{1-x}\frac{1-x^n}{1-x} = \int_0^\infty e^{-z}\,dz\,\frac{1-e^{-nz}}{1-e^{-z}}. \tag{22}$$

The graphs of fig. 3 present a slight complication. They contain only a single $\log p_2^2$ when integrated, where p_2 is the initial gluon momentum. This occurs when the virtual quark travels along the gluon direction. There is no $\log p_1^2$ to $O(g^2)$. This is simply the fact that the cross section to produce a μ-pair from a quark-gluon collision begins $O(g^2)$. Only in $O(g^4)$ would we see the consequences of the radiatively modified quark distribution. The cross section of fig. 3 measures how effective gluons are at producing quarks, and is not quite a modification ot the gluon distribution function.

$$\int dz\, z^n d\sigma_{q+gluon}(z) = -\frac{4\pi\alpha^2 Q^2}{3M^4}\frac{1}{3}\frac{g^2}{4\pi^2}\log\frac{-p_2^2}{M^2}\left[\frac{2}{n+4} + \frac{4}{(n+2)(n+3)(n+4)}\right]. \tag{23}$$

All these awkward words can be summarized succinctly by noting that this is really a matrix problem. The distribution functions form a vector whose components are the various quark and gluon distributions. The infrared renormalizations are matrices: the effective quark or gluon content depends on all the quark and gluon distributions. And finally $d\sigma_{parton}$ is a matrix giving the μ-pair cross section for the various combinations of partons from the incident hadrons. If Z_{ij} is the infrared renormalization matrix, then the general form is

$$f_{1i}Z_{ij}f_{2k}Z_{kl}d\sigma_{parton}^{jl}. \tag{24}$$

Since I have computed the whole cross section only to $O(g^2)$ and since only the quark-antiquark entry of $d\sigma_{parton}$ is $O(g^0)$, only the quark-quark and quark-gluon entries of Z enter into the calculation. These entries are precisely the factors encountered in the electroproduction anomalous dimension matrix. The other two entries of Z, also $O(g^2)$, only enter the cross section to $O(g^4)$ and $O(g^6)$ respectively. Presumably, they will also agree with the electroproduction anomalous dimensions.

In keeping only logarithmic terms and dropping constants, there is an ambiguity

between $\log M^2$ and $\log S$. I write the result in terms of M^2 by analogy with electro-production. The τ moments at fixed M^2 sweep over $M^2 < S < \infty$. In contrast, for fixed S, we would sweep over $0 < M^2 < S$. Perturbation theory does not distinguish these, but the real physics does. In particular, τ moments for fixed S may pick up significant contributions from hadronic resonances like ρ, ω, ϕ and ψ.

5. Higher orders and questions of principle

I would like to discuss why I think the factorization observed to $O(g^2)$ persists to all orders and to what extent a proof of that factorization would constitute a proof of the Drell-Yan picture.

The first question is whether inclusive μ-pair production in parton-parton scattering (where the partons are either quark, gluon, or antiquark) factorizes into an infrared-sensitive term appropriate to each incident parton which is independent of the observable kinematic variables times a factor that does depend on the kinematic variables and does admit a well behaved power series expansion in g. The $O(g^2)$ results are certainly suggestive, indicating that the leading behavior in parton-parton scattering is well-behaved when expressed in terms of the effective parton distributions derived in electroproduction. It should be possible to prove the factorization to all orders in g by induction, much as the operator product expansion can be proven.

I have an independent physical argument as to why the factorization should take place. Infrared divergences occur in perturbation theory when we ask unphysical or unobservable questions about systems containing massless fields. We are used to this in electrodynamics where we must sum over indistinguishable final states to get sensible predictions. We don't have to worry much about initial states because the massless particles (photons) are neutral and the charged particles (electrons) have a finite mass. The finite mass guarantees that an electron can always be separated from the real photons that were produced of necessity when the electron itself was produced They travel at different speeds. After a finite time interval, the collision of the electron with another system does not necessarily involve the photons that were originally produced with that electron. Massless charged particles present a new feature. They can never get away from the massless radiation that accompanies their creation. To eliminate spurious infrared divergences, one must sum over indistinguishable states. An initial state of light-like momentum can contain a variety of combinations of light-like colinear particles. That such a summation over initial massless particles will eliminate the infrared divergences is a consequence of the classic infrared analyses [15, 16].

What does this have to do with the case at hand? In some sense, the infrared divergences encountered in inclusive μ-pair production in parton-parton scattering are artificial. The point to note is that the initial states that must be added to eliminate the divergences are physically indistinguishable from the undressed partons. The di-

vergences are associated with incoming particles that we have neglected to sum over that are colinear with one parton (hadron) or the other. That is to say that each piece of the infrared divergence can be associated unambiguously with one hadron (parton) or the other. And the infrared divergences are not tangled up in functions involving the momenta of both hadrons (partons) simultaneously.

The spirit of the above discussion transcends perturbation theory [15]. But the theoretical tools available within the framework of QCD are meager and inadequate, and they will so remain until we better understand the relation between quarks and hadrons. Perturbation theory (to all orders) is one of these inadequate tools that could be used to verify the picture described in the preceeding paragraph. But even if that picture is true for parton-parton scattering order by order, we may justly ask whether it is in fact true and, if so, is it sufficient to justify the Drell-Yan picture. It was recognized at the outset [1] that effects which invalidate the jump from parton-parton to hadron-hadron scattering are all down by powers of M, at least order by order. So there is really only one issue: should perturbation theory be our guide or is it physical nonsense?

That the latter is a possibility is obvious if we try to describe the total proton-proton cross section in parton language. The contribution to the cross section from collisions of all partons with non-vanishing fractions of the hadrons' momenta drops like $1/S$ to all orders in perturbation theory. Presumably that is true in general, so to keep the proton-proton cross section approximately constant, we must conclude that there are arbitrarily many soft quanta (wee partons) in each hadron, whose interactions remain strong, independent of S. Since they are soft, asymptotic freedom is no guide to their behavior.

Do the wee partons spoil the Drell-Yan picture? Quite possibly they do not. More recent arguments, e.g. ref. [4], are more precise than the original parton approach, but I find the parton line quite convincing. The amplitude for wee partons to do something may be quite non-trivial, but the probability to do anything (including nothing) is always one. And it is that "one" that the Drell-Yan picture depends on.

The QCD perturbative analysis can add something to this discussion, particularly when renormalization-group improved. If factorization persists to all orders, it will
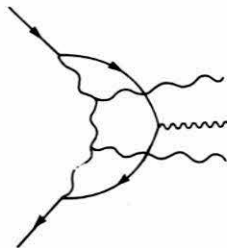

Fig. 7. An example of the occurrence of wee partons (here, soft gluons) in quark + antiquark → virtual photon + anything.

include processes of the type illustrated in fig. 7. The integration over all phase space includes the region where all the gluons are arbitrarily soft, and the net effect will only be a tiny correction if factorization perdists to that order. More generally, the perturbation expansion does generate a quark sea and wee partons of all types, and presumably the factorization persists to all orders.

One may counter by noting that to any finite order, the number of effective partons is finite, and hence the momentum distributions (i.e. $xq(x)$ where $q(x)$ is a parton probability distribution) go to zero as $x \to 0$. So there is nothing to keep hadron-hadron cross-sections from falling like $1/s$. However, the effective distributions that arise from integrating the renormalization group differential equations (even using only $O(g^2)$ input parameters) are non-zero as $x \to 0$. This is because the integration procedure sums the leading pieces of all higher orders. There is no reason to believe that the behavior so arrived at [17] corresponds to the real world or even to QCD, but it serves as an example in which the interactions of an infinite number of wee partons does not spoil the Drell-Yan picture.

6. Color neutrality of the initial states

The scheme outlined in the previous sections for describing Drell-Yan processes in terms of parton-parton scattering makes no mention of the quantum numbers of the initial hadronic states. One may ask, "Isn't the color neutrality of the initial hadrons and absence of long range forces between them crucial to the Drell-Yan picture?" The answer is, of course, "Yes". I wish to describe to what extent that is incorporated into the suggested analysis.

As long as we don't know how hadrons are built out of quarks, theoretical arguments must allow the structure of hadrons to be quite general. It is assumed that the bound states of quarks form asymptotic single particle states, i.e. with no strong, long range interaction. Perturbation theory gives a scale invariant force (up to logarithms) which is weaker than the presumed strong force between unshielded colors, but it is in fact stronger than the longest range force that is observed between hadrons. Hence, the study of parton-parton collisions in perturbation theory overestimates the long range force that exists between quarks in different hadrons. And in four dimensions, perturbation theory for high energy collisions is insensitive to the charge of color of colliding bound states. (This is familiar from electrodynamics).

7. Hadronic charm production

It has been suggested [3] that the Drell-Yan mechanism can give an estimate for the inclusive production of particles with new, heavy quantum numbers, such as charm. The production would proceed analogously to μ-pair production, with the substitution of a gluon for the virtual photon and a heavy quark pair for the μ-pair.

We are typically interested in the cross section for producing any state containing particles with new quantum numbers. Since this cross section is relatively small, it should be safe to count any state containing a heavy quark pair as one that will evolve into a hadronic state with new particles. To get the total cross section we must integrate the pair mass from S down to its production threshold, M_{thr}^2. For S not too much greater than M_{thr}^2, the Drell-Yan picture should be useful. But for $S \gg M_{thr}^2$, there will be a contribution from wee partons. As $M_{thr}^2/S \to 0$ the number of contributing wee partons presumably grows fast enough to compensate for the $1/S$ in the parton cross section to give a constant production cross section. However, the integral over non-zero x_i, i.e. down to some non-zero M^2/S may be useful as a lower bound on the production cross section.

That charm production will be a fixed fraction of the total cross section as $S \to \infty$, and not go like $1/S$, is also a consequence of a string picture. The energy in a collision goes first into strings connecting outgoing colored partons. The amplitude per unit length for the string to break by producing a quark pair is some fixed number that is a rapidly decreasing function of the quark mass. So the quarks produced from string breaking will be a specific fraction of the total, depending on their mass, while the total grows (logarithmically) with the energy available to make string. This constrasts sharply with the short distance process of hard quark annihilation, i.e. $q + \bar{q} \to q' + \bar{q}'$, which soon becomes quark mass independent as the available energy exceeds the produced quark mass.

The contribution of hard annihilations is computable and gives a lower bound on the total. Unfortunately there is not much known, theoretically or experimentally, about the degree of suppression of heavy quark production from string breaking.

A second computable, positive contribution to new particle production comes from processes such as illustrated in fig. 3 where the quark is itself a heavy quark in the "sea" carried by an incident hadron and the virtual photon is replaced by a real gluon. This is $O(g^4)$ times the probability of finding such a quark. The same "sea" distribution enters in electroproduction of new quantum numbers and can be estimated reliably [18,19], at least in an integrated form (integrated over all x_i, $0 \leqslant x_i \leqslant 1$). For charmed quarks this mechanism will yield only some 3-5% as much as the quark annihilation mechanism at presently accessible energies. But with higher and higher S, its relative importance will grow as the relative importance of the "sea" itself grows.

8. Conclusion

An explicit computation has verified that the salient features of QCD in Drell-Yan processes are large transverse momenta $[\langle k_\perp^2 \rangle \sim O(g^2 M^2)]$ and logarithmic scaling violations in the effective parton distributions. I have shown here how to compute the μ-pair (or W or Z) differential cross section, particularly as a function of $k_\perp^2$.

The computational algorithm suggests a method of proof of the Drell-Yan picture which deserves further attention.

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