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LARGE PERTURBATIVE CORRECTIONS TO THE DRELL-YAN PROCESS IN QCD *

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The total cross section $d\sigma/dQ^2$ for the production of a muon pair of invariant mass Q^2 *via* the Drell-Yan mechanism and the Feynman x_F differential cross section $d^2\sigma/dQ^2dx_F$ are calculated in QCD retaining all terms up to order $\alpha_s(Q^2)$. The calculations are performed using dimensional regularisation of the intermediary infrared and collinear singularities, but we present our results in a form independent of such details. The corrections to both these cross sections coming from radiative corrections to the lowest-order $q\bar{q}$ annihilation diagram are found to be large at present values of Q^2 and S when the cross section is expressed in terms of parton densities derived from leptoproduction, for all Drell-Yan processes of practical interest. Numerical calculations are presented which show, for any reasonable parametrisation of the parton densities, that the neglect of higher-order terms in $\alpha_s(Q^2)$ is not justifiable. The quark-gluon diagrams on the other hand give small corrections in this order and are only important for PP scattering.

1. Introduction

In a preceding paper [1] we have considered deep inelastic leptoproduction and the Drell-Yan process in quantum chromodynamics [2,3]. We defined quark and

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gluon densities beyond the leading order in QCD and in terms of these scale-dependent parton densities [4,5] we calculated correction terms to the leading-order results. These correction terms are suppressed by one power of ln Q^2 relative to the dominant terms in the parton cross sections and hence are negligible in the asymptotic region. At the subasymptotic energies of present day experiments they play an important role.

The higher-order corrections in the Drell-Yan process [6,7]

$$\text{hadron} + \text{hadron} \to \mu^+\mu^- + \text{anything}\ ,$$

are particularly striking because they turn out to be large at present energies. The modifications of the Drell-Yan formula (expressed in terms of scale-dependent parton densities derived from measurements in deep inelastic leptoproduction) are so large that the retention of only the first-order correction terms in $\alpha_s(Q^2)$ is unjustified. We stress that this is *not* an artifact of our particular definition of parton densities beyond the leading order but rather a result which we will show to be true in any expression for the Drell-Yan cross section written in terms of parton distribution functions measured in deep inelastic lepton-hadron scattering. The numerically important terms in our result are independent of details of how the distribution functions are defined from lepton-hadron scattering.

In this paper we study and extend our previous results on the Drell-Yan process. We consider all the contributions of order α_s, that is both those involving an initial quark and antiquark as well as those with an initial quark (antiquark) and gluon. Using the technique of dimensional regularisation [8,9] of the infrared and collinear singularities encountered in the massless quark-gluon theory we derive in detail the $O(\alpha_s)$ corrections to the total cross section * and to the cross-section differential in Feynman x_F or equivalently in the massive photon rapidity y_R. These latter cross sections are of special interest because the experimental data are normally presented in this form. We thus calculate cross sections which are directly comparable with experimental results.

In our treatment of the quark and gluon densities **, which we feel to be most convenient for many reasons, large corrections appear in the $q\bar{q}$ annihilation piece of the Drell-Yan cross section both in proton-nucleon and antiproton-nucleon collisions. By way of contrast, for reasonable choices of the gluon densities the quark-gluon corrections are always small even for PP scattering or other processes where the leading term is proportional to small sea densities. These results run contrary to the naive expectation that the lowest-order Drell-Yan formula is a good approximation at least for πP and P$\bar{\text{P}}$ scattering (i.e., valence-valence processes). Such intuition

* Corrections to the total cross section have also been considered in refs. [10,21].

** This caveat is only necessary because we could choose to have small corrections in the Drell-Yan process and large corrections to the parton-model expression for F_2. Nothing would be gained by such a shuffling of the large correction terms and it would correspond less to the normal operational procedure. Full details of our definition of parton densities are given in sect. 2.

is thus positively misleading in the order in $\alpha_s(Q^2)$ in which we calculate.

The algorithm of perturbative QCD [12] and in particular its application to the Drell-Yan process [13] are now well-understood. All the mass-singularity logarithms encountered in quark-gluon perturbation theory may be factored from the perturbative partonic cross section and consistently absorbed into the parton distribution functions [14,15], which thereby acquire a calculable dependence on the scale size of the interaction, determined in the case of muon production by the invariant mass Q^2 of the muon pair and the intermediate massive photon. We argue that renormalisation group improved *lowest*-order perturbation theory from which the mass singularities have been exorcised according to the above procedure is not adequate to describe the Drell-Yan process at present values of Q^2. A reliable treatment of the Drell-Yan process will require the inclusion of higher-order terms and hence depends crucially on the factorisability of both leading and subleading mass singularity logarithms *.

The structure of this paper is as follows. In sect. 2 we remind the reader of our definition of the parton densities beyond the leading order in terms of the deep inelastic structure function $\mathcal{F}_2$. In sect. 3 we discuss the advantages of the dimensional scheme for the regularisation of the infrared and collinear divergences and calculate the deep inelastic structure function $\mathcal{F}_2$ and the Drell-Yan cross section in order α_s. In sect. 4 we derive the corrections to the total Drell-Yan cross section $d\sigma/dQ^2$ and discuss the reason why the corrections are large. In sect. 5 we derive the results for cross-section differential in Feynman x_F. A discussion of the numerical size of these corrections in relation to the available experimental data is given in sect. 6.

2. Parton densities beyond the leading order

In this section we establish our notation and remind the reader of our definition of the parton densities beyond the leading order. Let us consider first deep inelastic leptoproduction. Q^2 is the absolute value of q^2, where q is the momentum carried by the current and x and t are defined by:

$$x = \frac{Q^2}{2P \cdot q}, \qquad t = \ln Q^2/\mu^2 , \tag{1}$$

P is the four-momentum of the target and μ is an arbitrary scale of mass. We also define the quantities $\mathcal{F}_i(x, Q^2)$ related to the normal structure functions by **

$$(\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3) = (2F_1, F_2/x, F_3) . \tag{2}$$

We have defined the script $\mathcal{F}_i$ because they have a more immediate relationship with

* See refs. [14,16].

** See for example, ref. [17].

the densities of partons. For example, in the naive parton model the electroproduction structure functions are given by

$$\mathcal{F}_1(x) = \mathcal{F}_2(x) = \sum_l e_l^2 q_{0l}(x) , \tag{3}$$

where $q_{0l}(x)$ is the "bare" distribution of the lth type of quark *or* antiquark inside the hadron and sum on l runs over all flavours of quarks and antiquarks.

Our definition of the parton densities beyond the leading order in QCD is the requirement that, in terms of the "renormalised" scale-dependent parton densities $q_l(x, t)$ the form of eq. (3) is preserved for the structure function $\mathcal{F}_2$ with no corrections proportional to $\alpha_s(Q^2)$:

$$\mathcal{F}_2(x, t) \equiv \frac{F_2(x, t)}{x} = \sum_l e_l^2 q_l(x, t) . \tag{4}$$

Calculating the leptoproduction structure functions $\mathcal{F}_i(x, t)$ in perturbation theory we find, ($G_0(y)$ is the "bare" gluon distribution function):

$$\mathcal{F}_i(x, t) = \int_x^1 \frac{dy}{y} \left\{ \sum_l a_l^i \left[\delta\left(1 - \frac{x}{y}\right) + \frac{\alpha_s}{2\pi} t P_{qq}\left(\frac{x}{y}\right) + \alpha_s f_{q,i}\left(\frac{x}{y}\right) \right] q_{0l}(y) \right.$$

$$\left. + \left(\sum_l a_l^i\right) \left[\frac{\alpha_s}{2\pi} t P_{qG}\left(\frac{x}{y}\right) + \alpha_s f_{G,i}\left(\frac{x}{y}\right) \right] G_0(y) \right\} . \tag{5}$$

In the above formula the index l runs over quarks and antiquarks of any flavour and the numbers a_l^i are the appropriate coupling factors. In particular in electroproduction the a_l^i are given by the squares of the quark (or antiquark) charges. Eq. (4) implies that to first order in α_s the relationship between bare and renormalised quark densities is given by:

$$q_k(x, t) = q_{0k}(x) + \int_x^1 \frac{dy}{y} \left\{ \left[\frac{\alpha_s}{2\pi} t P_{qq}\left(\frac{x}{y}\right) + \alpha_s f_{q,2}\left(\frac{x}{y}\right) \right] q_{0k}(y) \right.$$

$$\left. + \left[\frac{\alpha_s}{2\pi} t P_{qG}\left(\frac{x}{y}\right) + \alpha_s f_{G,2}\left(\frac{x}{y}\right) \right] G_0(y) \right\} . \tag{6}$$

Defining the moments of the quark and gluon distributions,

$$q_k^{(n)}(t) = \int_0^1 dx\, x^{n-1} q_k(x, t) ,$$

$$G^{(n)}(t) = \int_0^1 dx\, x^{n-1} G(x, t) , \tag{7}$$

we may rewrite eq. (6) as,

$$q_k^{(n)} = \left[1 + \frac{\alpha_s}{2\pi} t\, \tfrac{4}{3}\gamma_{qq}^{(n)} + \alpha_s f_{q,2}^{(n)}\right] q_{0k}^{(n)} + \left[\frac{\alpha_s}{2\pi}\, \tfrac{1}{2} t \gamma_{qG}^{(n)} + \alpha_s f_{G,2}^{(n)}\right] G_0^{(n)}\,, \tag{8}$$

where

$$\tfrac{4}{3}\gamma_{qq}^{(n)} = \frac{4}{3}\left(\frac{3}{2} + \frac{1}{n(n+1)} - 2\sum_{j=1}^{n} \frac{1}{j}\right), \qquad \tfrac{1}{2}\gamma_{qG}^{(n)} = \frac{1}{2}\,\frac{n^2+n+2}{n(n+1)(n+2)}\,. \tag{9}$$

The generalisation of eq. (8) to include the effects of all leading logarithms may be written as follows:

$$q_k^{(n)} = (1 + \alpha_s f_{q,2}^{(n)})\, \tilde{q}_k^{(n)} + \alpha_s f_{G,2}^{(n)} \tilde{G}^{(n)}\,, \tag{10}$$

where $\tilde{q}$ and $\tilde{G}$ are related to q_0 and G_0 by the matrix equation

$$\begin{bmatrix} \tilde{q} \\ \tilde{\bar{q}} \\ \tilde{G} \end{bmatrix} = \exp \int_{\alpha_s(0)}^{\alpha_s(t)} \mathrm{d}\alpha\, \frac{\gamma^{(n)}(\alpha)}{\beta(\alpha)} \begin{bmatrix} q_0 \\ \bar{q}_0 \\ G_0 \end{bmatrix}, \tag{11}$$

and $\gamma^n(\alpha)$ is the standard anomalous dimension matrix given in lowest order by

$$\gamma^{(n)}(\alpha) = \frac{\alpha_s}{2\pi} \begin{bmatrix} \tfrac{4}{3}\gamma_{qq}^{(n)} & 0 & \tfrac{1}{2}\gamma_{qG}^{(n)} \\ 0 & \tfrac{4}{3}\gamma_{qq}^{(n)} & \tfrac{1}{2}\gamma_{qG}^{(n)} \\ \tfrac{4}{3}\gamma_{Gq}^{(n)} & \tfrac{1}{2}\gamma_{Gq}^{(n)} & 3\gamma_{GG}^{(n)} \end{bmatrix}. \tag{12}$$

In terms of these densities the electroproduction structure function $\mathcal{F}_1$, for example, may be written

$$\mathcal{F}_1(x,t) = \int_x^2 \frac{\mathrm{d}y}{y} \left\{ \sum_l e_l^2 \left[\delta\left(\frac{x}{y} - 1\right) + \alpha_s(t)\left[f_{q,1}\left(\frac{x}{y}\right) - f_{q,2}\left(\frac{x}{y}\right)\right]\right] q_l(y,t) \right.$$
$$\left. + \left(\sum_l e_l^2\right) \alpha_s(t) \left[f_{G,1}\left(\frac{x}{y}\right) - f_{G,2}\left(\frac{x}{y}\right)\right] G(y,t) \right\}. \tag{13}$$

In this equation α_s has been replaced by the running coupling constant $\alpha_s(t)$.

There are many advantages to this definition of the quark densities. One of the most important is that it appears to be the most natural choice since a large fraction of the information on parton distribution functions inside hadrons comes from measurements of F_2 in electroproduction and (anti-) neutrino experiments. There are also technical advantages; the corrections to the lowest-order results are independent of gauge and the method of regularisation of the infrared divergences. For example, in eq. (13) the correction terms to the leading-order results are given by [18],

$$\alpha_s [f_{q,2}(z) - f_{q,1}(z)] = \frac{\alpha_s}{2\pi}\frac{4}{3} 2z \,, \tag{14}$$

$$\alpha_s [f_{G,2}(z) - f_{G,1}(z)] = \frac{\alpha_s}{2\pi}\frac{1}{2} 4z(1-z) \,. \tag{15}$$

These corrections are independent of the method of regularisation of the divergences, because in this order in perturbation theory they require no regularisation (i.e., they are finite). Since they correspond to on-shell matrix elements they are manifestly gauge invariant. Thus, whilst any single function f is, in general, convention dependent, the differences of the functions f are well-defined.

Another requirement of parton densities is that they should obey the conservation of charge:

$$\int_0^1 dx\,[q_l(x,t) - \bar{q}_l(x,t)] = v_l \,, \tag{16}$$

where v_l is the valence value of the lth quark in the hadron. Using our definition of parton densities this condition is automatically satisfied beyond the leading order. This can be seen by considering the Adler sum rule [19] which, neglecting powers of M^2/Q^2, may be written

$$\int_0^1 \frac{dx}{x}\,[F_2^{\nu p}(x,t) - F_2^{\nu n}(x,t)] = A_0 \,. \tag{17}$$

A_0 is a constant dependent only on the flavour content of the theory. The Adler sum rule is true for any combination of vector and axial vector currents. Therefore, in order that the Adler sum rule be free from scale-breaking corrections we must have

$$\tfrac{4}{3}\gamma_{qq}^{(1)} = \int_0^1 dz\, P_{qq}(z) = 0 \,, \tag{18}$$

$$f_{q,2}^{(1)} = \int_0^1 dz\, f_{q,2}(z) = 0 \,. \tag{19}$$

These conditions will be explicitly verified in our calculations in sect. 3. Using these two conditions it is clear from eq. (6) that if the "bare" quark densities satisfy eq. (16), then this condition will be maintained by our renormalised parton densities.

There still remains a considerable degree of ambiguity in the definition of the gluons. However, since we have no probes which couple directly to the gluon field, the exact definition of the gluon field beyond the leading order is irrelevant for calculations performed only up to order α_s. All reaction cross sections initiated by gluons start in order α_s. A sensible requirement to demand of the gluon field is the conservation of momentum:

$$\int_0^1 dx\, x[\sum_l q_l(x, t) + G(x, t)] = 1 \;. \tag{20}$$

Since the quark distribution function has been completely defined this condition fixes the second moment of the gluon distribution (f is the number of flavors):

$$G^{(2)}(t) = [1 - 2f\,\alpha_s(t) f^{(2)}_{G,2}]\,\tilde{G}^{(2)}(t) - (\alpha_s(t) f^{(2)}_{q,2}) \sum_l \tilde{q}^{(2)}_l(t)\,. \tag{21}$$

This condition ensures that eq. (20) is free from corrections of order $\alpha_s(t)$ (if it is satisfied by the bare distributions). A possible complete definition of the gluon field would be to extend eq. (21) to all moments of the gluon distribution. In order $\alpha_s^2(Q^2)$ the exact definition of the gluon density will be important, but for the order α_s calculation which we present here it is immaterial.

The absorption of the f's into the distribution functions only changes the derivatives with respect to t in next order in $\alpha_s(t)$ since

$$\frac{d}{dt} = \beta(\alpha_s(t))\frac{d}{d\alpha_s(t)} \simeq -b\,\alpha_s^2(t)\frac{d}{d\alpha_s(t)}\,. \tag{22}$$

The quark and gluon densities therefore continue to satisfy the standard evolution equations in order $\alpha_s(t)$ [5]:

$$\frac{dq_l(x, t)}{dt} = \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y}\left[P_{qq}\left(\frac{x}{y}\right) q_l(y, t) + P_{qG}\left(\frac{x}{y}\right) G(y, t)\right]\,, \tag{23}$$

$$\frac{dG(x, t)}{dt} = \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y}\left[P_{Gq}\left(\frac{x}{y}\right)\sum_l q_l(y, t) + P_{GG}\left(\frac{x}{y}\right) G(y, t)\right]\,. \tag{24}$$

We now move on to consider the implications of these definitions for the Drell-Yan process. In the naive parton model the total cross section for the production of a lepton pair of mass Q^2 in the collision of two hadrons is given by

$$\frac{d\sigma^{DY}}{dQ^2} = \frac{4\pi\alpha^2}{9SQ^2}\int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\,[\sum_f e_f^2 q^{[1]}_{0f}(x_1)\,\bar{q}^{[2]}_{0f}(x_2) + (1 \leftrightarrow 2)]\;\delta\left(1 - \frac{\tau}{x_1 x_2}\right)\,. \tag{25}$$

In this formula $\sqrt{S}$ is the invariant mass of the incoming hadron system, $\tau = Q^2/S$, the flavour index f sums over quarks alone and the superscripts [1] and [2] label the incoming hadrons. If we assume that such a formula is true in zeroth order in

α_s we may investigate the modifications caused by the order α_s processes:

$$G + q(\bar{q}) \to \gamma^* + q(\bar{q}) \,, \tag{26}$$

$$q + \bar{q} \to \gamma^* + G \,, \tag{27}$$

together with the virtual gluon corrections to the lowest-order process $q + \bar{q} \to \gamma$. Omitting all sums and other obvious factors, the perturbative corrections to the Drell-Yan process may be cast in the form:

$$\begin{aligned}\frac{d\sigma^{DY}}{dQ^2} = \int_0^1 \frac{dx_1}{x_1} \int_0^1 \frac{dx_2}{x_2} \Bigg\{ & [q_0^{[1]}(x_1)\, \bar{q}_0^{[2]}(x_2) + (1 \leftrightarrow 2)] \\ & \times \left[\delta(1-z) + \theta(1-z) \left(\frac{\alpha_s}{2\pi} 2P_{qq}^{DY}(z)\, t + \alpha_s f_{q,DY}(z) \right) \right] \\ & + [(q_0^{[1]}(x_1) + \bar{q}_0^{[1]}(x_2))\, G_0^{[2]}(x_2) + (1 \leftrightarrow 2)] \\ & \times \theta(1-z) \left[\frac{\alpha_s}{2\pi} P_{qG}^{DY}(z)\, t + \alpha_s f_{G,DY}(z) \right] \Bigg\} \,, \end{aligned} \tag{28}$$

where z has the meaning,

$$z = \frac{\tau}{x_1 x_2} = \frac{Q^2}{s} \,, \tag{29}$$

and s is the incoming four-momentum squared of the partonic subprocess. Suppressing similar coupling factors we may also write the perturbative correction to the $\mathcal{F}_2$ structure function due to the interaction of the virtual photon with a quark as:

$$\begin{aligned}\mathcal{F}_2(x,t) \simeq \int_0^1 \frac{dy}{y} \Bigg\{ & \left[\delta(1-z) + \theta(1-z) \left(\frac{\alpha_s}{2\pi} P_{qq}(z)\, t + \alpha_s f_{q,2}(z) \right) \right] q_0(y) \\ & + \theta(1-z) \left[\frac{\alpha_s}{2\pi} P_{qG}(z)\, t + \alpha_s f_{G,2}(z) \right] G_0(y) \Bigg\} \,, \end{aligned} \tag{30}$$

where $z = x/y$. Eq. (30) also defines our quark densities beyond the leading order:

$$q(x,t) = \mathcal{F}_2(x,t) \,. \tag{31}$$

It is known that the functions in eqs. (28) and (30) are in fact equal [13]:

$$P_{qq}^{DY}(z) = P_{qq}(z) \,, \tag{32}$$

$$P_{qG}^{DY}(z) = P_{qG}(z) \,. \tag{33}$$

Expressing eq. (28) in terms of our scale-dependent parton densities we obtain (to order α_s)

$$\frac{d\sigma^{DY}}{dQ^2} = \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\Bigg\{[q^{[1]}(x_1,t)\,\bar{q}^{[2]}(x_2,t) + (1 \leftrightarrow 2)]$$

$$\times\,[\delta(1-z) + \alpha_s(t)\,\theta(1-z)(f_{q,DY}(z) - 2f_{q,2}(z))]$$

$$+\,[(q^{[1]}(x_1,t) + \bar{q}^{[1]}(x_1,t))\,G^{[2]}(x_2,t) + (1 \leftrightarrow 2)]$$

$$\times\,[\alpha_s(t)\,\theta(1-z)(f_{G,DY}(z) - f_{G,2}(z))]\Bigg\}\,, \tag{34}$$

where α_s has been renormalisation-group improved to $\alpha_s(t)$. Restoring all factors we find the Drell-Yan formula including corrections up to order $\alpha_s(Q^2)$ is given by

$$\frac{d\sigma^{DY}}{dQ^2} = \frac{4\pi\alpha^2}{9SQ^2}\int_0^1 \frac{dx_1}{x_1}\int_0^1 \frac{dx_2}{x_2}\Bigg\{[\sum_f e_f^2 q_f^{[1]}(x_1,t)\,\bar{q}_f^{[2]}(x_2,t) + (1 \leftrightarrow 2)]$$

$$\times\,[(1-z) + \alpha_s(t)\,\theta(1-z)(f_{q,DY}(z) - 2f_{q,2}(z))] \tag{35}$$

$$+\,[\mathcal{F}_2^{[1]}(x_1,t)\,G^{[2]}(x_2,t) + (1 \leftrightarrow 2)]\,\alpha_s(t)\,\theta(1-z)(f_{G,DY}(z) - f_{G,2}(z))\Bigg\}\,.$$

We therefore see that, at least as far as the cross section $d\sigma/dQ^2$ is concerned, the problem reduces to the identification of the two terms $(f_{q,DY} - 2f_{q,2})$ and $(f_{G,DY} - f_{G,2})$. These correction differences are obviously independent of the infrared singularities and the regularisation prescription. For the correction to the differential rapidity cross section we will need a slight generalisation of these correction terms in which the Drell-Yan cross section is not integrated over angle. Full details of this latter problem will be given in sect. 5.

3. Corrections to leptoproduction and the Drell-Yan process

To fulfil our stated aim of defining the parton densities beyond the leading order in terms of the structure function $\mathcal{F}_2$ we must calculate current-parton deep inelastic scattering cross sections up to order α_s to identify the terms $f_{q,2}(z)$ and $f_{G,2}(z)$. The former quantity is calculated from the graphs shown in fig. 1 with an incoming quark and the latter from the graphs of fig. 2 with an incoming gluon.

In calculating these partonic cross sections we encounter divergences. We may regulate these divergences in any way we choose since, in our method of definition of the parton densities, any regularisation dependence will cancel in the physical corrections which are given by differences of the functions f. Thus we could introduce parton masses, or continue the external parton legs slightly off-shell as we did

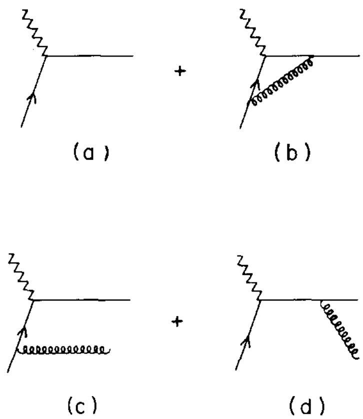

Fig. 1. Diagrams giving the corrections of order α_s to the point-like quark-current cross sections. The incoming current is denoted by a wavy line, the gluon by a spiralling line and the quarks by a continuous line. In calculating (a) the quark wave-function renormalization must be taken into account in order α_s. (a) + (b): the virtual-gluon corrections $\gamma^* + q \to q$; (c) + (d): the real-gluon corrections $\gamma^* + q \to q + G$.

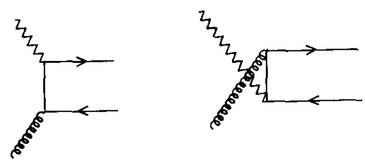

Fig. 2. Diagrams of order g_s contributing to gluon-current scattering.

in our previous paper [1]. In this paper we choose to regulate the divergences by performing the calculation of real and virtual gluon corrections in a number of space-time dimensions different from four. This method has several technical advantages. Using dimensional regularisation the gauge invariance of the theory is assured at all stages. The phase-space integrals are considerably simplified because of the presence of only massless partons.

The absorbtive part of the forward photon-parton scattering amplitude may be expanded in the usual structure function expansion:

$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) W_1 + \left(p^\mu - \frac{p\cdot q}{q^2} q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2} q^\nu\right) W_2 \,, \tag{36}$$

where p is the incoming parton momentum and q is the momentum carried by the photon current. The structure functions W_1 and W_2 are in general functions of the variable z and Q^2, where Q^2 is the absolute value of current momentum q^2 and z is

$$z = \frac{Q^2}{2p\cdot q} \,. \tag{37}$$

In analogy to the relations for virtual photon-hadron scattering we have that,

$$\mathcal{F}_2(z, Q^2) = \frac{p \cdot q\, W_2}{z}\,, \tag{38}$$

$$\mathcal{F}_1(z, Q^2) = 2W_1\,. \tag{39}$$

It is convenient to project out two different linear combinations of the parton structure functions by saturating the virtual-photon indices with the tensors $-g^{\mu\nu}$ and $p^\mu p^\nu$. From eq. (36) we obtain in n dimensions ($\epsilon = 2 - \frac{1}{2}n$).

$$-g^{\mu\nu} W_{\mu\nu} = (1-\epsilon)\, \mathcal{F}_2(z, Q^2) - (\tfrac{3}{2} - \epsilon)(\mathcal{F}_2(z, Q^2) - \mathcal{F}_1(z, Q^2))\,, \tag{40}$$

$$p^\mu p^\nu W_{\mu\nu} = \frac{Q^2}{8z^2}\,(\mathcal{F}_2(z, Q^2) - \mathcal{F}_1(z, Q^2))\,. \tag{41}$$

The second of these combinations is proportional to the longitudinal cross section. The results for the longitudinal cross section require no regularisation and are well-known [18]. They have been given in our notation in sect. 2. Extraction of the structure function $\mathcal{F}_2$ requires only that we calculate the quantity in eq. (40) from the relevant graphs.

We consider first of all the graphs with incoming quarks. The lowest-order graph fig. 1a gives the result

$$\mathcal{F}_2 = \delta(1-z)\,. \tag{42}$$

This result defines the normalisation of our partonic cross section. The calculation of the cross section in the next order requires the evaluation of the real-gluon emission graphs figs. 1c,d and the interference of the lowest-order graph, fig. 1a, with the virtual graph, fig. 1b. In order that the coupling constant remain dimensionless in an arbitrary number of dimensions we make the replacement $g \to g(\mu)^\epsilon$, where μ is an arbitrary parameter with the dimensions of mass.

The real gluon emission graphs describe the reaction

$$\gamma^*(q) + \mathrm{q}(p) \to \mathrm{q}(p') + \mathrm{G}(k)\,, \tag{43}$$

where the symbols in brackets are the momenta carried by the fields. In n dimensions the invariant matrix element squared for this reaction is given by

$$|M_{\gamma^*\mathrm{q}\to\mathrm{qG}}|^2 = 4\alpha_s\, \tfrac{4}{3}(1-\epsilon)(\mu^2)^\epsilon \left\{(1-\epsilon)\left(\frac{s}{-t} + \frac{-t}{s}\right) - \frac{2uq^2}{st} + 2\epsilon\right\}\,, \tag{44}$$

where $\alpha_s = g^2/4\pi$, $s = (p+q)^2$, $t = (p-k)^2$ and $u = (p-p')^2$. In this expression the virtual-photon indices have been summed over by contraction with $-g^{\mu\nu}$ and all manipulations performed in n space-time dimensions [9].

The two-particle phase space (PS) in n dimensions for the production of two on-shell massless objects may be written

$$\mathrm{PS} = \int \frac{\mathrm{d}^n p'}{(2\pi)^{n-1}} \int \frac{\mathrm{d}^n k}{(2\pi)^{n-1}} (2\pi)^n \delta^{(n)}(p + q - p' - k)\, \delta^+(p'^2)\, \delta^+(k^2)\,. \tag{45}$$

In the parton-virtual photon c.m.s. with incoming momenta directed along the $(n-1)$th direction we may write

$$k = (|k|, ..., |k| \cos\theta)\,, \tag{46}$$

where the dots indicate $n-2$ unspecified momenta. In this frame we may perform $n-2$ angular integrations so that eq. (45) reads

$$\mathrm{PS} = \frac{1}{4\pi} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \int_0^\infty \mathrm{d}|k| |k|^{1-2\epsilon} \int_{-1}^{1} \mathrm{d}(\cos\theta)(1-\cos^2\theta)^{-\epsilon} \delta(s - 2\sqrt{s}|k|)\,. \tag{47}$$

Performing the $|k|$ integration using the delta function and changing the variable of angular integration $y = \frac{1}{2}(1 + \cos\theta)$ we have finally for the centre of mass two-particle phase space

$$\mathrm{PS} = \frac{1}{8\pi} \left(\frac{4\pi}{s}\right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \int_0^1 \mathrm{d}y (y(1-y))^{-\epsilon}\,. \tag{48}$$

In this frame we may write

$$s = \frac{Q^2(1-z)}{z}\,, \qquad t = \frac{-Q^2}{z}(1-y)\,, \qquad u = \frac{-Q^2}{z} y\,, \tag{49}$$

so that we obtain the contribution to $\mathcal{F}_2|_{\mathrm{real}}$ as

$$\mathcal{F}_2|_{\mathrm{real}} = \frac{\alpha_s}{2\pi} \frac{4}{3} \left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \left\{ 3z + z^\epsilon (1-z)^{-\epsilon} \int_0^1 \mathrm{d}y (y(1-y))^{-\epsilon} \right.$$
$$\left. \times \left[\left(\frac{1-z}{1-y} + \frac{1-y}{1-z} \right)(1-\epsilon) + \frac{2zy}{(1-z)(1-y)} \right] \right\}\,. \tag{50}$$

In the above equation we have inserted the contribution from the longitudinal cross section according to eqs. (14) and (40). The integrations over the angular variable y which would diverge in the limit $y \to 1$ (the region of forward gluon emission) are finite for small negative values of ϵ. Performing the integral over y we have

$$\mathcal{F}_2(z, Q^2)|_{\mathrm{real}} = \frac{\alpha_s}{2\pi} \frac{4}{3} \left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left\{ 3z + z^\epsilon (1-z)^{-\epsilon} \right.$$
$$\left. \times \left[-\frac{1}{\epsilon} \frac{1+z^2}{1-z} + 3 - z - \frac{3}{2} \frac{1}{1-z} - \tfrac{7}{2}\epsilon \frac{1}{1-z} \right] \right\}\,. \tag{51}$$

In this expression we have dropped terms of order ϵ if they do not ultimately lead to a finite contribution. At this stage it is useful to make the singularities in ϵ manifest by defining distributions having finite integrals as $\epsilon \to 0$ using the identity

$$z^{\epsilon}(1-z)^{-1-\epsilon} \equiv -\frac{1}{\epsilon}\,\delta(1-z) + \frac{1}{(1-z)_+} - \epsilon\left(\frac{\ln(1-z)}{1-z}\right)_+ + \epsilon\,\frac{\ln z}{1-z} + O(\epsilon^2)\,. \tag{52}$$

The distributions denoted by the small cross (+) are defined as usual. For example, the distribution $1/(1-z)_+$ is defined such that

$$\int_0^1 \mathrm{d}z\,\frac{h(z)}{(1-z)_+} \equiv \int_0^1 \mathrm{d}z\,\frac{h(z)-h(1)}{1-z}\,. \tag{53}$$

Further details on the distributions and on their moments are given in an appendix at the end of this paper. Expanding the other terms in eq. (51) in the normal way:

$$z^{\epsilon}(1-z)^{-\epsilon} = 1 + \epsilon\ln\frac{z}{1-z} + O(\epsilon^2)\,, \tag{54}$$

we finally obtain the contribution of real-gluon emission graphs, figs. 1c,d, to $\mathcal{F}_2$:

$$\mathcal{F}_2(z,Q^2)\big|_{\mathrm{real}} = \frac{\alpha_s}{2\pi}\frac{4}{3}\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left\{\frac{2}{\epsilon^2}\,\delta(1-z) - \frac{1}{\epsilon}\frac{1+z^2}{(1-z)_+} + \frac{3}{2\epsilon}\,\delta(1-z) + \left[(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - \frac{3}{2}\frac{1}{(1-z)_+} - \frac{1+z^2}{1-z}\ln z + 3 + 2z + \tfrac{7}{2}\delta(1-z)\right]\right\}\,. \tag{55}$$

To complete the calculation of $\mathcal{F}_2$ we must now calculate the interference of the virtual-gluon correction fig. 1b with the lowest-order graph fig. 1a and the associated external quark leg wave-function renormalisation. Since our method of regularisation is explicitly gauge invariant we may calculate in any gauge. It is convenient to calculate in the Landau gauge in which the vertex correction and the quark self-energy are individually ultraviolet finite. In fact in this gauge, using dimensional regularisation, the quark self-energy vanishes in order α_s for massless quarks [9]. Quark wave-function renormalisation is therefore not needed, and in this gauge the problem reduces to the calculation of the order α_s corrections to the photon vertex. Our result is

$$\Gamma^{\mu}(q^2) = \gamma^{\mu}\left\{1 + \frac{\alpha_s}{4\pi}\frac{4}{3}\left(\frac{4\pi\mu^2}{-q^2}\right)^{\epsilon}\frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}\left[\frac{-2}{\epsilon^2} - \frac{3}{\epsilon} - 8\right]\right\}\,. \tag{56}$$

After use of the expansion,

$$\Gamma(1+\epsilon)\,\Gamma(1-\epsilon) = 1 + \epsilon^2\,\tfrac{1}{6}\pi^2 + O(\epsilon^4)\,, \tag{57}$$

we may write the contribution to $\mathcal{F}_2$ from figs. 1a,b as

$$\mathcal{F}_2(z, Q^2)|_{\text{virtual}} = \delta(1-z)\left\{1 + \frac{\alpha_s}{2\pi}\frac{4}{3}\left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left[\frac{-2}{\epsilon^2} - \frac{3}{\epsilon} - 8 - \tfrac{1}{3}\pi^2\right]\right\}. \quad (58)$$

Adding the real and virtual contributions to $\mathcal{F}_2$ we obtain

$$\mathcal{F}_2(z, Q^2) = \delta(1-z) - \frac{1}{\epsilon}\frac{\alpha_s}{2\pi} P_{qq}(z)\left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}$$
$$+ \frac{\alpha_s}{2\pi}\frac{4}{3}\left[(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - \frac{3}{2}\frac{1}{(1-z)_+} - \frac{1+z^2}{1-z}\ln z\right.$$
$$\left. + 3 + 2z - (\tfrac{9}{2} + \tfrac{1}{3}\pi^2)\,\delta(1-z)\right]. \quad (59)$$

The double pole in ϵ has cancelled between real and virtual graphs as it must. This is the familiar cancellation of soft singularities. The coefficient of the single pole in $1/\epsilon$ and hence of the logarithm of Q^2/μ^2 is just the normal anomalous dimension function [5]

$$P_{qq}(z) = \frac{4}{3}\left(\frac{1+z^2}{(1-z)_+} + \tfrac{3}{2}\delta(1-z)\right). \quad (60)$$

Eq. (59) allows us to identify the final object of this calculation which is the function $f_{q,2}$ defined in sect. 2:

$$\alpha_s f_{q,2}(z) = \frac{\alpha_s}{2\pi}\frac{4}{3}\left\{(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - \frac{3}{2}\frac{1}{(1-z)_+} - \frac{1+z^2}{1-z}\ln z + 3 + 2z\right.$$
$$\left. - (\tfrac{9}{2} + \tfrac{1}{3}\pi^2)\,\delta(1-z) + \left(\frac{1+z^2}{(1-z)_+} + \tfrac{3}{2}\delta(1-z)\right)\left(-\frac{1}{\epsilon} + \gamma_E - \ln 4\pi\right)\right\}. \quad (61)$$

The occurrence of the Euler-Mascheroni constant γ_E and the ln 4π in these expressions is an artifact of dimensional regularisation and they will not be present in the physical predictions of the theory. The moments of $f_{q,2}(z)$ defined as

$$f_{q,2}^{(n)} = \int_0^1 dz\, z^{n-1} f_{q,2}(z), \quad (62)$$

are easily derived using the table of Mellin transforms given in the appendix to this paper:

$$\alpha_s f_{q,2}^{(n)} = \frac{\alpha_s}{2\pi}\frac{4}{3}\left[2\sum_{k=1}^{n}\frac{1}{k}\sum_{j=1}^{k}\frac{1}{j} - 2\sum_{j=1}^{n}\frac{1}{j^2} - \frac{1}{n(n+1)}\sum_{j=1}^{n}\frac{1}{j} + \frac{1}{n^2}\right.$$
$$\left. + \tfrac{3}{2}\sum_{j=1}^{n}\frac{1}{j} + \frac{3}{2n} + \frac{2}{n+1} - \tfrac{9}{2} + \gamma_{qq}^{(n)}\left(-\frac{1}{\epsilon} + \gamma_E - \ln 4\pi\right)\right]. \quad (63)$$

$\gamma_{qq}^{(n)}$ is the anomalous dimension [3]

$$\gamma_{qq}^{(n)} = \left[\frac{3}{2} + \frac{1}{n(n+1)} - 2\sum_{j=1}^{n} \frac{1}{j}\right].$$

Two remarks should be made about eq. (63). The first is that the first moment $f_{q,2}^{(1)}$ vanishes:

$$f_{q,2}^{(1)} = 0 . \tag{64}$$

This is a consequence of the Adler sum rule. As explained in sect. 2, the importance of this fact for our definition of parton densities is that the number of valence quarks inside the hadron is maintained beyond the leading order.

The second remark is that discarding the pole in $1/\epsilon$ in eq. (63) we directly obtain the result of Bardeen et al. [20], for the coefficient function of the quark operator in the light-cone expansion. Their calculation took into account the renormalisation of the operator matrix element beyond the leading order necessary in the minimal subtraction scheme.

We now proceed to extract the quantity $f_{G,2}(z)$. The relevant Feynman graphs are shown in fig. 2. With the experience gained in the calculation of $f_{q,2}(z)$ the labour is small. The matrix element, with the virtual-photon indices contracted with $-g^{\mu\nu}$, may be obtained by crossing from eq. (44), after suitable modification of the colour sums and averages

$$|M_{\gamma^* G\to q\bar{q}}|^2 = 4\alpha_s\, \tfrac{1}{2}(1-\epsilon)(\mu^2)^\epsilon \left\{(1-\epsilon)\left(\frac{u}{t}+\frac{t}{u}\right) + \frac{2q^2 s}{ut} - 2\epsilon\right\} . \tag{65}$$

The final particle phase space is the same as before and so, using eq. (49) for the partonic variables s, t and u, we have for the gluon contribution to $\mathcal{F}_2$:

$$\tfrac{1}{2}\mathcal{F}_2(z,Q^2) = \frac{\alpha_s}{2\pi}\frac{1}{2}\left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \left\{6z(1-z)\right.$$

$$+ \tfrac{1}{2}z^\epsilon(1-z)^{-\epsilon}\int_0^1 dy (y(1-y))^{-\epsilon}$$

$$\left.\times\left[(1-\epsilon)\left(\frac{1}{y}+\frac{1}{1-y}-2\right) - 2z(1-z)\left(\frac{1}{y}+\frac{1}{1-y}\right)\right]\right\} . \tag{66}$$

In the above equation we have inserted the appropriate combination of the longitudinal cross section, eq. (15), as required by eq. (40) and dropped terms of order ϵ if they do not ultimately lead to finite contributions. Performing the angular integration in y we have

$$\tfrac{1}{2}\mathcal{F}_2(z,Q^2) = -\frac{1}{\epsilon}\frac{\alpha_s}{2\pi} P_{qG}(z)\left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}$$

$$+\frac{\alpha_s}{2\pi}\frac{1}{2}\left[(z^2+(1-z)^2)\ln\frac{1-z}{z}+6z(1-z)\right]. \tag{67}$$

The function $P_{qG}(z)$ is given as usual by

$$P_{qG}(z)=\tfrac{1}{2}[z^2+(1-z)^2]. \tag{68}$$

From eq. (67) we can identify the quantity $f_{G,2}(z)$:

$$\alpha_s f_{G,2}(z)=\frac{\alpha_s}{2\pi}\frac{1}{2}\left\{(z^2+(1-z)^2)\ln\frac{1-z}{z}+6z(1-z)\right.$$

$$\left.+(z^2+(1-z)^2)\left(-\frac{1}{\epsilon}+\gamma_E-\ln 4\pi\right)\right\}. \tag{69}$$

The expression for $f_{G,2}$ in eq. (67) is appropriate for electroproduction with one quark flavour of unit charge. From our definition of $f_{G,2}$ in eq. (5) is clear that $f_{G,2}$ is defined from the quantity $\frac{1}{2}\mathcal{F}_2$ so that it gives the gluon correction for either a quark or an antiquark. The gluon correction to $\mathcal{F}_2$ always contains the correction for a quark and an antiquark because the gluon dissociates into a quark-antiquark pair. To each quark plus antiquark with unit coupling there corresponds a gluon correction equal to $2\alpha_s f_{G,2}(z)$.

Using the table of moments in the appendix we may take moments of $f_{G,2}(z)$

$$f_{G,2}^{(n)}=\frac{\alpha_s}{2\pi}\frac{1}{2}\left\{\frac{4}{n+1}-\frac{4}{n+2}+\frac{1}{n^2}-\frac{n^2+n+2}{n(n+1)(n+2)}\sum_{j=1}^{n}\frac{1}{j}\right.$$

$$\left.+\gamma_{qG}^{(n)}\left(-\frac{1}{\epsilon}+\gamma_E-\ln 4\pi\right)\right\}, \tag{70}$$

where $\gamma_{qG}^{(n)}$ is the usual anomalous dimension function:

$$\gamma_{qG}^{(n)}=\frac{n^2+n+2}{n(n+1)(n+2)}. \tag{71}$$

This result is again compatible with the results of ref. [20] ⋆.

Thus armed with the functions $f_{q,2}(z)$ and $f_{G,2}(z)$ we may now proceed to calculate the corresponding quantities for the Drell-Yan process. There are two types of contributions in this order; the modifications of the lowest-order quark-antiquark annihilation process shown in fig. 3 and the contributions involving an incoming gluon shown in fig. 4. Since we are not interested in correlations between the plane of the $\mu^+\mu^-$ pair and the incoming scattering plane we sum over the polarisations of

⋆ In the sense defined above for $f_{q,2}$. Our conventions for the gluons correspond in the language of the light-cone expansion to a slightly different definition than normal for the gluon operators.

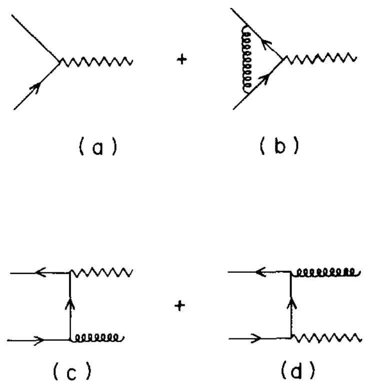

Fig. 3. Diagrams giving corrections of order α_s to the basic quark-antiquark annihilation graph of the Drell-Yan process. In calculating (a) the quark wave-function renormalization must be taken into account in order α_s. (a) + (b): the virtual-gluon corrections $q + \bar{q} \to \gamma^*$; (c) + (d): the real-gluon corrections $q + \bar{q} \to G + \gamma^*$.

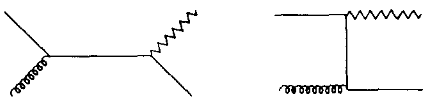

Fig. 4. Diagrams of order g_s contributing to the process $q(\bar{q}) + G \to q(\bar{q}) + \gamma^*$.

the virtual photon by contraction of the massive photon by indices with the tensor $-g^{\mu\nu}$.

We consider first of all the lowest-order quark-antiquark annihilation diagram fig. 3a:

$$q(p) + \bar{q}(p') \to \gamma^*(q) \,.$$

The matrix element for this process (in n dimensions) is given in our normalisation by ($s = (p + p')^2$)

$$|M_{q\bar{q}\to\gamma^*}|^2 = \frac{1-\epsilon}{2N}\frac{s}{2\pi} \,. \qquad (72)$$

In this equation the factor $1/N$ comes from the average over the N initial colours ($N = 3$). The phase-space factor for the production of a photon of mass Q^2 is

$$\mathrm{PS} = \frac{2\pi}{s}\,\delta(1 - z) \,, \qquad (73)$$

where the variable z, here, and throughout our treatment of the Drell-Yan process has the meaning:

$$z = Q^2/s \,. \qquad (74)$$

Combining eqs. (72) and (73) we have the results for the lowest-order quark-antiquark annihilation diagram:

$$\frac{\mathrm{d}\sigma_{\mathrm{q\bar{q}}}(z, Q^2)}{\mathrm{d}Q^2} = \delta(1-z)\,. \tag{75}$$

This equation defines the normalisation of our partonic Drell-Yan cross section used throughout the rest of this paper. It corresponds to multiplying all invariant matrix elements by a factor $2N/(1-\epsilon)$.

The contribution of the real-gluon emission diagrams is easily evaluated. The matrix element for the process

$$\mathrm{q}(p) + \bar{\mathrm{q}}(p') \rightarrow \mathrm{G}(k) + \gamma^*(q)\,, \tag{76}$$

is given by

$$|M_{\mathrm{q\bar{q}}\rightarrow\gamma^*\mathrm{G}}|^2 = 4\alpha_{\mathrm{s}}\,\frac{4}{3}\,\frac{1-\epsilon}{2N}\,(\mu^2)^\epsilon\left\{(1-\epsilon)\left(\frac{u}{t}+\frac{t}{u}\right)+\frac{2Q^2s}{ut}-2\epsilon\right\}. \tag{77}$$

In n dimensions we may write the phase space for the production of a massive photon as

$$\mathrm{PS} = \frac{1}{4\pi}\,\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\int_0^\infty \mathrm{d}|k|\,|k|^{1-2\epsilon}\int_{-1}^{1}\mathrm{d}(\cos\theta)(1-\cos^2\theta)^{-\epsilon}$$

$$\times\,\delta(s-Q^2-2\sqrt{s}\,|k|)\,. \tag{78}$$

Performing the integration over the gluon momentum $|k|$ and making the charge of variables $y=\frac{1}{2}(1+\cos\theta)$ we obtain

$$\mathrm{PS} = \frac{1}{8\pi}\left(\frac{4\pi}{Q^2}\right)^\epsilon\frac{1}{\Gamma(1-\epsilon)}\,z^\epsilon(1-z)^{1-2\epsilon}\int_0^1\mathrm{d}y\,(y(1-y))^{-\epsilon}\,. \tag{79}$$

In terms of the variables Q^2, y and z, the invariants s, t and u are given in the c.m.s. by:

$$s=\frac{Q^2}{z}\,,\qquad t=-\frac{Q^2}{z}(1-z)(1-y)\,,\qquad u=-\frac{Q^2}{z}(1-z)\,y\,. \tag{80}$$

Substituting these values into the matrix element eq. (77), we obtain for the real $\mathrm{q\bar{q}}$ contribution to the Drell-Yan cross section

$$\left.\frac{\mathrm{d}\sigma_{\mathrm{q\bar{q}}}(z,Q^2)}{\mathrm{d}Q^2}\right|_{\mathrm{real}} = \frac{\alpha_{\mathrm{s}}}{2\pi}\,\frac{4}{3}\left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon\frac{1}{\Gamma(1-\epsilon)}\,(1-z)^{1-2\epsilon}z^\epsilon\int_0^1\mathrm{d}y\,y^{-\epsilon}(1-y)^{-c}$$

$$\times\left[(1-\epsilon)\left(\frac{1-y}{y}+\frac{y}{1-y}\right)+\frac{2z}{(1-z)^2\,y(1-y)}-2\epsilon\right]. \tag{81}$$

The contribution of the virtual graphs, figs. 3a,b, can be obtained from the value of the vertex correction given in eq. (56). Note that in the Drell-Yan process $q^2 = Q^2 > 0$ (whereas in deep inelastic scattering $q^2 = -Q^2 < 0$). We therefore obtain an extra factor of $(-1)^\epsilon$ relative to the deep inelastic scattering case. Expanding this factor up to terms of order ϵ^2 we obtain for the real part:

$$\mathrm{Re}\ \Gamma^\mu(Q^2) = \gamma^\mu \left\{1 + \frac{\alpha_s}{4\pi}\frac{4}{3}\left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \frac{\Gamma(1+\epsilon)\,\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}\right.$$
$$\left.\times\left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2\right]\right\}. \tag{82}$$

This difference of π^2 between the Drell-Yan process and deep inelastic scattering will turn out to be of considerable numerical importance and we will discuss it further in sect. 4. Suffice it to say at this point that the coefficient of π^2 is controlled by the magnitude of the double-pole terms in ϵ (the soft divergence) ★. Eq. (82) allows us to write for the virtual $q\bar{q}$ annihilation contribution to the Drell-Yan process

$$\left.\frac{\mathrm{d}\sigma_{q\bar{q}}(z, Q^2)}{\mathrm{d}Q^2}\right|_{\text{virtual}} = \delta(1-z)\left\{1 + \frac{\alpha_s}{2\pi}\frac{4}{3}\left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\right.$$
$$\left.\times\left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \tfrac{2}{3}\pi^2\right]\right\}. \tag{83}$$

Integrating eq. (81) over y and adding it to the virtual contribution eq. (83) we would obtain the contribution of the $q\bar{q}$ annihilation graphs to the total cross section. This will be done in sect. 4. Since we are also interested in the angular distribution of the massive photon we will leave eqs. (81) and (83) as the final result of the $q\bar{q}$ calculation in this section.

Lastly we must calculate the contribution of the quark gluon scattering graphs shown in fig. 4. As before the matrix element for the process

$$q + G \to q + \gamma^* \tag{84}$$

is given by the matrix element for the time-reversed process which we have already given in eq. (44). After suitable modifications of the sums and averages over colours we obtain

$$|M_{qG\to\gamma^* q}|^2 = 4\alpha_s \frac{1}{2}\frac{1-\epsilon}{2N}\left\{(1-\epsilon)\left(\frac{s}{-t} + \frac{-t}{s}\right) - \frac{2uQ^2}{st} + 2\epsilon\right\}. \tag{85}$$

The indices of the massive virtual photon have been contracted with the tensor $-g^{\mu\nu}$. Introducing the dimensional phase-space equation (79) and making the substitu-

★ The coefficient of the soft singularity logarithms is the same in any *on-mass-shell* regularization scheme.

tions eq. (80) we obtain for the quark-gluon Drell-Yan cross section

$$\frac{\mathrm{d}\sigma_{\mathrm{qG}}(z,Q^2)}{\mathrm{d}Q^2}=\frac{\alpha_s}{2\pi}\frac{1}{2}\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon}\frac{1}{\Gamma(1-\epsilon)}\left\{(1-z)^{1-2\epsilon}z^{\epsilon}\int_0^1 \mathrm{d}y(y(1-y))^{-\epsilon}\right.$$

$$\left.\times\left[(1-\epsilon)\left(\frac{1}{(1-z)(1-y)}+(1-z)(1-y)\right)-\frac{2zy}{1-y}\right]\right\}. \tag{86}$$

As usual we have dropped terms of order ϵ where they are innocuous.

4. The Drell-Yan cross section $\mathrm{d}\sigma/\mathrm{d}Q^2$

In this section we complete the calculation of the first-order corrections to the total Drell-Yan cross section; the corresponding results at fixed x_{F} are obtained in sect. 5. We first use the results of sect. 3 to derive the final expression for $\mathrm{d}\sigma_{\mathrm{q}\bar{\mathrm{q}}}/\mathrm{d}Q^2$. Integrating eq. (81) over the angular variable y we obtain:

$$\left.\frac{\mathrm{d}\sigma_{\mathrm{q}\bar{\mathrm{q}}}(z,Q^2)}{\mathrm{d}Q^2}\right|_{\mathrm{real}}=\frac{\alpha_s}{2\pi}\frac{4}{3}\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}$$

$$\times\left\{-\frac{2}{\epsilon}\left[(1-z)^{1-2\epsilon}z^{\epsilon}+2z^{1+\epsilon}(1-z)^{-1-2\epsilon}\right]\right\}. \tag{87}$$

Using distribution identities as in eq. (52) we can make the terms singular as $\epsilon\to 0$ manifest, yielding,

$$\left.\frac{\mathrm{d}\sigma_{\mathrm{q}\bar{\mathrm{q}}}(z,Q^2)}{\mathrm{d}Q^2}\right|_{\mathrm{real}}=\frac{\alpha_s}{2\pi}\frac{4}{3}\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left[\frac{2}{\epsilon^2}\delta(1-z)-\frac{2}{\epsilon}\frac{1+z^2}{(1-z)_+}\right.$$

$$\left.+4(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+-2\frac{1+z^2}{1-z}\ln z\right]. \tag{88}$$

Adding the virtual graph contribution we obtain:

$$\frac{\mathrm{d}\sigma_{\mathrm{q}\bar{\mathrm{q}}}(z,Q^2)}{\mathrm{d}Q^2}=\delta(1-z)-\frac{2}{\epsilon}\frac{\alpha_s}{2\pi}P_{\mathrm{qq}}(z)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}$$

$$+\frac{\alpha_s}{2\pi}\frac{4}{3}\left[4(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+-2\frac{1+z^2}{1-z}\ln z+(\tfrac{2}{3}\pi^2-8)\,\delta(1-z)\right], \tag{89}$$

where $P_{\mathrm{qq}}(z)$ has its usual meaning (eq. (60)). This is the manifestation in this order of the universality of mass singularities. Comparison of eq. (89) with eq. (28) allows us to extract $f_{\mathrm{q,DY}}(z)$:

$$\alpha_s f_{\mathrm{q,DY}}(z)=\frac{\alpha_s}{2\pi}\frac{4}{3}\left\{4(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+-2\frac{(1+z^2)}{1-z}\ln z+(\tfrac{2}{3}\pi^2-8)\,\delta(1-z)\right.$$

$$+ 2\left(\frac{1+z^2}{(1-z)_+} + \tfrac{3}{2}\delta(1-z)\right)\left(-\frac{1}{\epsilon} + \gamma_E - \ln 4\pi\right)\Bigg\} . \tag{90}$$

By subtracting $2f_{q,2}(z)$ (eq. (61)) we obtain the quantity relevant for the corrections to the Drell-Yan formula *:

$$\alpha_s(f_{q,DY} - 2f_{q,2})$$
$$= \frac{\alpha_s}{2\pi}\frac{4}{3}\left[\frac{3}{(1-z)_+} - 6 - 4z + 2(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ + (1 + \tfrac{4}{3}\pi^2)\,\delta(1-z)\right] . \tag{91}$$

The moments of this function are given by:

$$\alpha_s(f^{(n)}_{q,DY} - 2f^{(n)}_{q,2}) = \frac{\alpha_s}{2\pi}\frac{4}{3}\left[-3\sum_{j=1}^{n}\frac{1}{j} - \frac{3}{n} - \frac{4}{n+1} + 4\sum_{k=1}^{n}\frac{1}{k}\sum_{j=1}^{k}\frac{1}{j}\right.$$
$$\left. - \frac{2}{n(n+1)}\sum_{j=1}^{n}\frac{1}{j} + \frac{2}{(n+1)^2} + 1 + \tfrac{4}{3}\pi^2\right] . \tag{92}$$

Performing the integral over y in eq. (86) we obtain the perturbative corrections to $d\sigma_{qG}/dQ^2$:

$$\frac{d\sigma_{qG}(z,Q^2)}{dQ^2} = -\frac{1}{\epsilon}\frac{\alpha_s}{2\pi}P_{qG}(z)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}$$
$$+ \frac{\alpha_s}{2\pi}\frac{1}{2}\left[(z^2 + (1-z)^2)\ln\frac{(1-z)^2}{z} - \tfrac{3}{2}z^2 + z + \tfrac{3}{2}\right] . \tag{93}$$

By comparison with eq. (28) we may extract $f_{q,DY}(z)$:

$$\alpha_s f_{G,DY}(z) = \frac{\alpha_s}{2\pi}\frac{1}{2}\left[(z^2 + (1-z)^2)\ln\frac{(1-z)^2}{z} - \tfrac{3}{2}z^2 + z + \tfrac{3}{2}\right.$$
$$\left. + (z^2 + (1-z)^2)\left(-\frac{1}{\epsilon} + \gamma_E - \ln 4\pi\right)\right] . \tag{94}$$

Taking the difference between this quantity and eq. (69) we obtain

$$\alpha_s(f_{G,DY}(z) - f_{G,2}(z)) = \frac{\alpha_s}{2\pi}\tfrac{1}{2}\,[(z^2 + (1-z)^2)\ln(1-z) + \tfrac{9}{2}z^2 - 5z + \tfrac{3}{2}] . \tag{95}$$

The moments of this quantity are

$$\alpha_s(f^{(n)}_{G,DY} - f^{(n)}_{G,2}) = \frac{\alpha_s}{2\pi}\frac{1}{2}\left[\frac{2}{(n+1)^2} - \frac{2}{(n+2)^2} - \frac{n^2+n+2}{n(n+1)(n+2)}\sum_{j=1}^{n}\frac{1}{j}\right.$$

* This equation corrects eq. (93) of ref. [1]. This is the content of the erratum to ref. [1].

$$+\frac{13}{2}\frac{1}{n+2}-\frac{7}{n+1}+\frac{3}{2n}\bigg]\,. \tag{96}$$

This completes our calculation of the corrections to the Drell-Yan total cross section.

In order to make a preliminary estimate of the size of these corrections we plot the moments of these corrective terms in fig. 5. We have plotted the quantities in curly brackets in eqs. (92) and (96), so that in the case of the $q\bar{q}$ terms the scale of the corrections is given by multiplying the quantity plotted by $\alpha_s/2\pi$. Taking a notational value of $\alpha_s/2\pi \sim \frac{1}{20}$ appropriate for $Q^2 \sim 100$ GeV^2 we see that for this value of α_s the $q\bar{q}$ corrections are by no means small compared to 1. The gluon corrections (expanded scale) are negative and small. Whilst it is true that in the evaluation of the cross section the gluon corrections will be convoluted with a substantial gluon distribution tending to increase their effect, for reasonable parametrisations

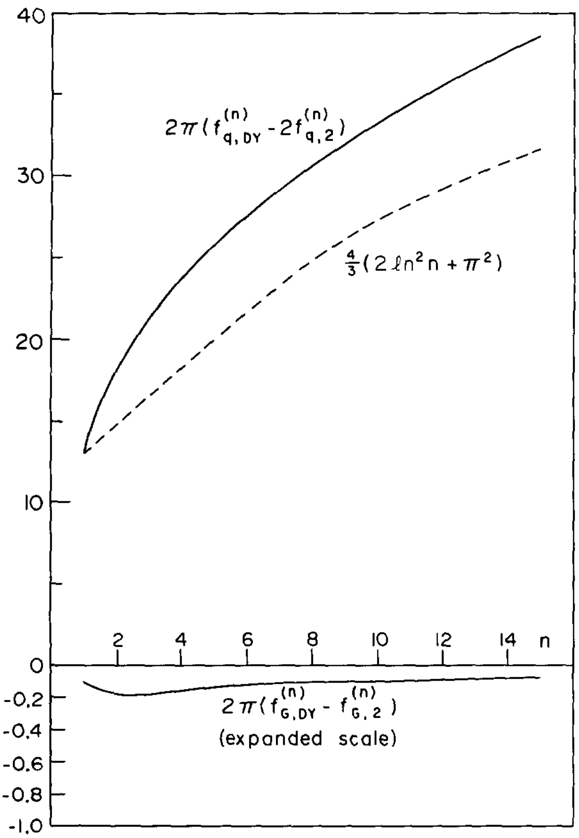

Fig. 5. Plot of the moments of the Drell-Yan correction terms $2\pi(f^{(n)}_{q,DY} - 2f^{(n)}_{q,2})$ and $2\pi(f^{(n)}_{q,DY} - f^{(n)}_{G,2})$ as a function of n. Note that the scale of the ordinate has been multiplied by ten for negative values. Also plotted is the function $\frac{4}{3}(2\ln^2 n + \pi^2)$ which gives a large contribution to $2\pi(f^{(n)}_{q,DY} - 2f^{(n)}_{q,2})$.

of the gluon distribution they will remain small [1]. We therefore concentrate our attention on the quark-antiquark terms.

The terms in eq. (91) which are giving large corrections are the last two. Would it have been possible to predict the form of these terms before doing the calculation? The answer is that in large measure we could have. Consider first of all the logarithmic distribution $2(1+z^2)(\ln(1-z)/(1-z))_+$. The calculation of the logarithmic terms in the real gluon emission graphs for the leptoproduction and the Drell-Yan process can be represented, in a slightly schematic notation, as

$$\mathcal{F}_2 \sim \delta(1-z) + \frac{\alpha_s}{2\pi}\frac{4}{3}\frac{1+z^2}{1-z}\int_{t_{\min}}^{t_{\max}} \frac{\mathrm{d}t}{t}, \tag{97}$$

$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}Q^2} \sim \delta(1-z) + \frac{\alpha_s}{2\pi}\frac{4}{3}2\frac{1+z^2}{1-z}\int_{t_{\min}}^{t_{\max}} \frac{\mathrm{d}t}{t}, \tag{98}$$

where the integration over the final two-particle phase space is represented as an integral over the four-momentum transfer squared. The value of $t_{\min}$ in eqs. (97) and (98) is common to both integrals and dependent on the particular method chosen to regulate the collinear divergences. The values of $t_{\max}$ on the other hand depend on the particular process and are, for leptoproduction

$$t_{\max} = -\frac{Q^2}{z}, \qquad z = \frac{Q^2}{2p\cdot q}, \tag{99}$$

whereas for the Drell-Yan process we have

$$t_{\max} = -\frac{Q^2(1-z)}{z}, \qquad z = \frac{Q^2}{s}. \tag{100}$$

Performing the integration we obtain, in this extended leading logarithmic approximation,

$$\alpha_s(f_{q,\mathrm{DY}} - 2f_{q,2}) \sim \frac{\alpha_s}{2\pi}\frac{4}{3}2\frac{1+z^2}{1-z}\ln(1-z). \tag{101}$$

The above derivation is valid for values of $z < 1$. In order to see how the expression in eq. (101) becomes the distribution $(1+z^2)(\ln(1-z)/(1-z))_+$ (a result connected with the cancellation of the soft singularities) we have to consider the also virtual graphs.

The other large term in eq. (91) is the delta function at $z = 1$ with coefficient $1 + \frac{4}{3}\pi^2$. As already noted in sect. 3, a term of magnitude π^2 in this expression comes from the mismatch of the space-like values of q^2 appropriate for electroproduction and the time-like values appropriate for lepton-pair production. The coefficient of this factor of π^2 is determined by the strength of the soft singularity. Details of how it arose in our method of regularisation have been given in sect. 3. In fig. 5 we have also plotted the quantity $\frac{4}{3}(2\ln^2 n + \pi^2)$ which is the approximate form of the moments of those correction terms for which we have given a simple explanation. The presence of both these terms is related to the existence of soft gluon singularities in the theory.

5. The differential cross section $d^2\sigma/dQ^2 dx_F$

The data for the Drell-Yan process come from the observation of muon pairs over a limited solid angle and are often presented in the form of a cross-section differential in Feynman x_F, where x_F is related to the momentum of the muon pair along the beam direction in the hadron-hadron c.m.s.:

$$x_F = \frac{2q_z}{\sqrt{S}}, \qquad S = \frac{s}{x_1 x_2}. \tag{102}$$

For the lowest-order process $q + \bar{q} \to \gamma^*$, the z component of the virtual-photon momentum equals the z component of the momentum of the annihilating partons:

$$\tfrac{1}{2}\sqrt{S}\,\delta(q_z - q_z^0) = \delta(x_1 - x_2 - x_F)\,. \tag{103}$$

The differential cross section in the naive parton model is hence given by ($A = 4\pi\alpha^2/9S$)

$$\frac{Q^2 d^2\sigma}{dQ^2 dx_F} = \frac{A}{x_1^0 + x_2^0}\left[\sum_f e_f^2 q_{0f}^{[1]}(x_1^0)\,\bar{q}_{0f}^{[2]}(x_2^0) + (1 \leftrightarrow 2)\right]. \tag{104}$$

The parton densities are evaluated at the points

$$x_1^0 = \tfrac{1}{2}(x_F + \sqrt{x_F^2 + 4\tau})\,,$$

$$x_2^0 = \tau/x_1^0 = \tfrac{1}{2}(-x_F + \sqrt{x_F^2 + 4\tau})\,. \tag{105}$$

When higher-order corrections are included this simple form is no longer maintained. Partons initially having momenta $x_1(x_2)$ greater than $x_1^0(x_2^0)$ can degrade their longitudinal momentum by the emission of partons. The z component of the virtual photon momentum is given in this case by

$$q_z = \left(\frac{s - Q^2}{2\sqrt{s}}\cos\theta + \frac{x_1 - x_2}{x_1 + x_2}\frac{s + Q^2}{2\sqrt{s}}\right)\frac{x_1 + x_2}{2\sqrt{x_1 x_2}}\,, \tag{106}$$

so that x_F is fixed in terms of the *parton* c.m. scattering angle θ to be:

$$\tfrac{1}{2}\sqrt{S}\,\delta(q_z - q_z^0) = \delta\left[(x_1 + x_2)\frac{1 - z}{2}\cos\theta + (x_1 - x_2)\frac{1 + z}{2} - x_F\right]. \tag{107}$$

Inserting this delta function into the sum of eqs. (81) and (83) we obtain an expression for the $q\bar{q}$ contribution to the cross section:

$$\frac{Q^2 d^2\sigma}{dQ^2 dx_F} = A \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} H(x_1, x_2) \Bigg\{ 1 + \frac{\alpha_s}{2\pi} \frac{4}{3} \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)}$$

$$\times \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} + \pi^2 - 8 \right) \delta(1-z)\, \delta(x_1 - x_2 - x_F) + \frac{\alpha_s}{2\pi} \frac{4}{3} \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{\theta(1-z)}{\Gamma(1-\epsilon)}$$

$$\times \int_0^1 dy\, \delta[(x_1 + x_2)(1-z)y + x_1 z - x_2 - x_F]\, z^\epsilon (1-z)^{-2\epsilon} y^{-\epsilon} (1-y)^{-\epsilon}$$

$$\times \left[(1-\epsilon)(1-z) \left(-2 + \frac{1}{1-y} + \frac{1}{y} \right) + 2 \left(\frac{1}{1-z} - 1 \right) \left(\frac{1}{1-y} + \frac{1}{y} \right) \right] \Bigg\}, \qquad (108)$$

where for (relative) compactness of notation we have set:

$$H(x_1, x_2) = \sum_f e_f^2 \,[q_{0f}^{[1]}(x_1)\, \bar{q}_{0f}^{[2]}(x_2) + (1 \leftrightarrow 2)] \,. \qquad (109)$$

Directing our attention temporarily to the real graph contributions in eq. (108) we use the identities

$$(1-z)^{-1-\epsilon} = -\frac{1}{\epsilon} \delta(1-z) + \frac{1}{(1-z)_+} - \epsilon \left(\frac{\ln(1-z)}{1-z} \right)_+ ,$$

$$y^{-1-\epsilon} = -\frac{1}{\epsilon} \delta(y) + \frac{1}{y_+} - \epsilon \left(\frac{\ln y}{y} \right)_+ , \qquad (110)$$

to write

$$z^\epsilon (1-z)^{-2\epsilon} y^{-\epsilon} (1-y)^{-\epsilon} \left[(1-\epsilon)(1-z) \left(-2 + \frac{1}{1-y} + \frac{1}{y} \right) + 2 \left(\frac{1}{1-z} - 1 \right) \left(\frac{1}{1-y} + \frac{1}{y} \right) \right]$$

$$= \Bigg\{ \left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} \right) \delta(1-z)(\delta(1-y) + \delta(y))$$

$$+ \delta(1-z) \left[-\frac{1}{\epsilon} \left(\frac{1}{(1-y)_+} + \frac{1}{y_+} \right) + \frac{\ln y}{1-y} + \left(\frac{\ln(1-y)}{1-y} \right)_+ + \frac{\ln(1-y)}{y} + \left(\frac{\ln y}{y} \right)_+ \right]$$

$$+ (\delta(1-y) + \delta(y)) \left[-\frac{1}{\epsilon} \left(\frac{1+z^2}{(1-z)_+} + \tfrac{3}{2} \delta(1-z) \right) + (1-z)(1 + 2\ln(1-z) - \ln z) \right.$$

$$\left. - 2z \frac{\ln z}{1-z} + 4z \left(\frac{\ln(1-z)}{1-z} \right)_+ \right] + \frac{1+z^2}{(1-z)_+} \left(\frac{1}{(1-y)_+} + \frac{1}{y_+} \right) - 2(1-z) \Bigg\} . \qquad (111)$$

Performing the integral over y and adding in the virtual terms we obtain *:

$$\frac{Q^2 \mathrm{d}^2\sigma}{\mathrm{d}Q^2 \mathrm{d}x_\mathrm{F}} = A \int \frac{\mathrm{d}x_1}{x_1} \frac{\mathrm{d}x_2}{x_2} H(x_1,x_2) \Bigg\{ \delta(1-z)\,\delta(x_1-x_2-x_\mathrm{F}) + \frac{\alpha_s}{2\pi}\frac{4}{3}\left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \frac{1}{\Gamma(1-\epsilon)}$$

$$\times \Bigg[(\tfrac{2}{3}\pi^2 - 8)\,\delta(1-z)\,\delta(x_1-x_2-x_\mathrm{F}) + \theta(1-z)[\delta(x_1-zx_2-x_\mathrm{F}) + \delta(zx_1-x_2-x_\mathrm{F})]$$

$$\times \left[-\frac{1}{\epsilon}\left(\frac{1+z^2}{(1-z)_+} + \tfrac{3}{2}\delta(1-z)\right) + (1-z)\left(1 + 2\ln(1-z) - \ln z - 2z\frac{\ln z}{1-z} + 4z\left(\frac{\ln(1-z)}{1-z}\right)_+\right)\right]$$

$$+ \frac{\theta(1-z)\,\theta(y^*)\,\theta(1-y^*)}{(1-z)(x_1+x_2)}\left[-2(1-z) + \frac{1+z^2}{(1-z)_+}\left(\frac{1}{(1-y^*)_+} + \frac{1}{y^*_+}\right)\right]\Bigg]\Bigg\}, \tag{112}$$

where

$$y^* = \frac{(x_2 - x_2^0)(x_2 + x_1^0)}{x_2(1-z)(x_1+x_2)}, \qquad 1 - y^* = \frac{(x_1 - x_1^0)(x_1 + x_2^0)}{x_1(1-z)(x_1+x_2)}. \tag{113}$$

The interchange $y^* \leftrightarrow (1 - y^*)$ occurs when the replacements $(x_\mathrm{F} \leftrightarrow -x_\mathrm{F})$, $(x_1 \leftrightarrow x_2)$ are made. Evaluating the integrals constrained by delta functions we have:

$$\frac{Q^2 \mathrm{d}^2\sigma}{\mathrm{d}Q^2 \mathrm{d}x_\mathrm{F}} = A \Bigg\{ \frac{1}{\sqrt{x_\mathrm{F}^2 + 4\tau}} \Bigg[H(x_1^0, x_2^0)\left[1 + \frac{\alpha_s}{2\pi}\tfrac{4}{3}(\tfrac{2}{3}\pi^2 - 8)\right]$$

$$+ \alpha_s \int_{x_1^0}^1 \frac{\mathrm{d}x_1}{x_1} H(x_1, x_2^0) f\left(\frac{x_1^0}{x_1}\right) + \alpha_s \int_{x_2^0}^1 \frac{\mathrm{d}x_2}{x_2} H(x_1^0, x_2) f\left(\frac{x_2^0}{x_2}\right)\Bigg] \tag{114}$$

$$+ \frac{\alpha_s}{2\pi}\frac{4}{3} \int_{x_1^0}^1 \frac{\mathrm{d}x_1}{x_1} \int_{x_2^0}^1 \frac{\mathrm{d}x_2}{x_2} \frac{H(x_1,x_2)}{(1-z)(x_1+x_2)} \left[-2(1-z) + \frac{1+z^2}{(1-z)_+}\left(\frac{1}{(1-y^*)_+} + \frac{1}{y^*_+}\right)\right]\Bigg\},$$

where

$$\alpha_s f(z) = \frac{\alpha_s}{2\pi}\frac{4}{3}\left\{\left(-\frac{1}{\epsilon} + \ln\frac{Q^2}{\mu^2} + \gamma_\mathrm{E} - \ln 4\pi\right)\left(\frac{1+z^2}{(1-z)_+} + \tfrac{3}{2}\delta(1-z)\right)\right.$$

* When multiplied by a delta function $\delta(1 - z)$, the x_F fixing delta function $\delta[(x_1 + x_2)(1 - z)\,y + x_1 z - x_2 - x_\mathrm{F}]$ reduces to $\delta(x_1 - x_2 - x_\mathrm{F})$ times the integral in dy which is zero over the distributions

$$\frac{1}{y_+}, \quad \frac{1}{(1-y)_+}, \quad \left(\frac{\ln y}{y}\right)_+$$

and $-\frac{1}{3}\pi^2$ over

$$\frac{\ln y}{1-y} + \frac{\ln(1-y)}{y}.$$

$$+ (1-z)(1+2\ln(1-z)-\ln z) - 2z\frac{\ln z}{1-z} + 4z\left(\frac{\ln(1-z)}{1-z}\right)_+ \Bigg\} . \tag{115}$$

The final step in the calculation of the $q\bar{q}$ correction is to express the product of distribution functions $H(x_1, x_2)$ in terms of $\tilde{H}(x_1, x_2, t)$ containing quark densities defined through $\mathcal{F}_2$, eq. (30). After renormalisation-group improvement of the α_s to $\alpha_s(t)$ we obtain

$$\frac{Q^2 d^2\sigma}{dQ^2 dx_F} = A \left\{ \frac{1}{\sqrt{x_F^2 + 4\tau}} \left[\tilde{H}(x_1^0, x_2^0, t)\left[1 + \frac{\alpha_s(t)}{2\pi}\,\tfrac{4}{3}(\tfrac{4}{3}\pi^2 + 1)\right] \right. \right.$$

$$+ \alpha_s(t) \int_{x_1^0}^{1} \frac{dx_1}{x_1} \tilde{H}(x_1, x_2^0, t)\, g\left(\frac{x_1^0}{x_1}\right) + \alpha_s(t) \int_{x_2^0}^{1} \frac{dx_2}{x_2} \tilde{H}(x_1^0, x_2, t)\, g\left(\frac{x_2^0}{x_2}\right) \Bigg] \tag{116}$$

$$\left. + \frac{\alpha_s(t)}{2\pi}\frac{4}{3} \int_{x_1^0}^{1} \frac{dx_1}{x_1} \int_{x_2^0}^{1} \frac{dx_2}{x_2} \frac{\tilde{H}(x_1, x_2, t)}{(1-z)(x_1+x_2)} \left[-2(1-z) + \frac{1+z^2}{(1-z)_+}\left(\frac{1}{(1-y^*)_+} + \frac{1}{y_+^*}\right)\right] \right\}$$

where

$$\alpha_s g(z) = \frac{\alpha_s}{2\pi}\frac{4}{3}\left[-2 - 3z + (1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ + \frac{3}{2}\frac{1}{(1-z)_+}\right] . \tag{117}$$

The first line in eq. (116) is simply a scale correction to the parton-model result. We will refer to this as the delta-function contribution for obvious reasons. The subsequent lines contain the effects of partons cascading down from $x_1, (x_2) > x_1^0, (x_2^0)$. Integrating eq. (116) over x_F we obtain, after some simple manipulations, the $q\bar{q}$ contribution to the total cross section $Q^2 d\sigma/dQ^2$.

The last term in eq. (116) contains the product of two distributions. The explicit method of handling this product is as follows. We first change the variables of integration x_1, x_2 to z and $u = y^*$ using the Jacobian:

$$\frac{dx_1}{x_1}\frac{dx_2}{x_2}\frac{1}{(1-z)(x_1+x_2)} = \frac{dz}{z}\frac{du}{J(z,u)} , \tag{118}$$

where

$$J(z,u) = \left(x_2 + \frac{\tau}{x_2}\right)(1-u) + \left(x_1 + \frac{\tau}{x_1}\right)u , \tag{119}$$

and x_1 and x_2 are expressed in terms of z and u according to

$$x_2 = \frac{-x_F + [x_F^2 + 4\tau(1-(1-z)u(1+((1-z)/z)u)]^{1/2}}{2[1-(1-z)u]} ,$$

$$x_1 = \frac{\tau}{zx_2} . \tag{120}$$

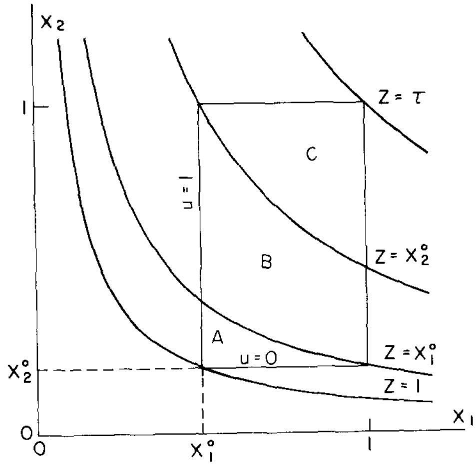

Fig. 6. The region of integration in the x_1, x_2 plane shown for the case $x_1^0 > x_2^0$ is given by the rectangle A + B + C. Note that only in region A does the integration range extend to $z = 1$ or $u = 0$.

Consider, for example, the integral

$$\int_{x_1^0}^{1} \frac{dx_1}{x_1} \int_{x_2^0}^{1} \frac{dx_2}{x_2} \frac{H(x_1, x_2)}{(1-z)(x_1+x_2)} \frac{1+z^2}{(1-z)_+} \frac{1}{y_+^*}$$

$$= \iint_{A+B+C} \frac{dz}{z} \frac{1+z^2}{(1-z)_+} \frac{du}{u_+} \frac{H(z,u)}{J(z,u)} . \tag{121}$$

The domain of integration (fig. 6) splits into three regions A, B and C. Assuming for the moment, $x_F > 0$, (the case shown in fig. 6), it is only in region A that the poles at $z = 1$ and $u = 0$ can be reached. In region A, $x_1^0 \leqslant z \leqslant 1$ and $0 \leqslant u \leqslant 1$ so that

$$\iint_{A} \frac{dz}{z} \frac{1+z^2}{(1-z)_+} \frac{du}{u_+} \frac{H(z,u)}{J(z,u)} = \int_{x_1^0}^{1} \frac{dz}{z} \frac{1+z^2}{(1-z)_+} \int_0^1 \frac{du}{u} \left(\frac{H(z,u)}{J(z,u)} - \frac{H(z,0)}{J(z,0)} \right) . \tag{122}$$

Note that the difference appearing in the u integration vanishes at $z = 1$. This is because $z = 1$ and $0 \leqslant u \leqslant 1$ imply $x_{1,2} = x_{1,2}^0$ for arbitrary u. The whole integral, eq. (122), therefore has no pole at $z = 1$ and no further subtraction at $z = 1$ is required:

$$\iint_{A} \frac{dz}{z} \frac{1+z^2}{(1-z)_+} \frac{du}{u_+} \frac{H(z,u)}{J(z,u)} = \int_{x_1^0}^{1} \frac{dz}{z} \frac{1+z^2}{1-z} \int_0^1 \frac{du}{u} \left(\frac{H(z,u)}{J(z,u)} - \frac{H(z,0)}{J(z,0)} \right) . \tag{123}$$

The points $z = 1$ and $u = 0$ are excluded from regions B and C so we may write

$$\iint\limits_{\mathrm{B+C}} \frac{\mathrm{d}z}{z} \frac{1+z^2}{(1-z)_+} \frac{\mathrm{d}u}{u_+} \frac{H(z,u)}{J(z,u)} = \iint\limits_{\mathrm{B+C}} \frac{\mathrm{d}z}{z} \frac{1+z^2}{1-z} \frac{\mathrm{d}u}{u} \frac{H(z,u)}{J(z,u)} \,. \tag{124}$$

The final result over the whole region of integration may *formally* be written as

$$\iint\limits_{\mathrm{A+B+C}} \frac{\mathrm{d}z}{z} \frac{1+z^2}{(1-z)_+} \frac{\mathrm{d}u}{u_+} \frac{H(z,u)}{J(z,u)}$$

$$= \iint\limits_{\mathrm{A+B+C}} \frac{\mathrm{d}z}{z} \frac{1+z^2}{1-z} \frac{\mathrm{d}u}{u} \frac{H(z,u)}{J(z,u)} - \int_{x_1^0}^{1} \frac{\mathrm{d}z}{z} \frac{1+z^2}{1-z} \int_0^1 \frac{\mathrm{d}u}{u} \frac{H(z,0)}{J(z,0)} \,. \tag{125}$$

It may easily be checked that eq. (125) is also valid for $x_{\mathrm{F}} \leqslant 0$. The companion integrals with $y^* \to 1 - y^*$ can be obtained by the substitutions $x_{\mathrm{F}} \to -x_{\mathrm{F}}$ and $H(x_1, x_2) \to H(x_2, x_1)$.

The quark-gluon contribution to the differential cross section can be obtained from eq. (86) using an essentially identical procedure ★:

$$\frac{Q^2 \mathrm{d}^2\sigma}{\mathrm{d}Q^2 \mathrm{d}x_{\mathrm{F}}} = A \int \frac{\mathrm{d}x_1}{x_1} \int \frac{\mathrm{d}x_2}{x_2} \left\{ K(x_1,x_2) \frac{\alpha_{\mathrm{s}}}{2\pi} \frac{1}{2} \left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon} \frac{\theta(1-z)}{\Gamma(1-\epsilon)} \right.$$

$$\times \left[\delta(x_1 - zx_2 - x_{\mathrm{F}}) \left[-\frac{1}{\epsilon}(z^2 + (1-z)^2) + (z^2 + (1-z)^2) \frac{\ln(1-z)^2}{z} + 1 \right] \right.$$

$$\left. + \frac{\theta(y^*)\,\theta(1-y^*)}{(1-z)(x_1+x_2)} \left[2z(1-z) + (1-z)^2(1-y^*) + (z^2+(1-z)^2) \frac{1}{(1-y^*)_+} \right] \right]$$

$$+ K(x_2,x_1) \frac{\alpha_{\mathrm{s}}}{2\pi} \frac{1}{2} \left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon} \frac{\theta(1-z)}{\Gamma(1-\epsilon)}$$

$$\times \left[\delta(x_1 z - x_2 - x_{\mathrm{F}}) \left[-\frac{1}{\epsilon}(z^2+(1-z)^2) + (z^2+(1-z)^2) \ln \frac{(1-z)^2}{z} + 1 \right] \right.$$

$$\left.\left. + \frac{\theta(y^*)\,\theta(1-y^*)}{(1-z)(x_1+x_2)} \left[2z(1-z) + (1-z)^2 y^* + (z^2+(1-z)^2) \frac{1}{y^*_+} \right] \right] \right\}, \tag{126}$$

where

$$K(x_1,x_2) = \sum_f e_f^2 (q_{0f}^{[1]}(x_1) + \bar{q}_{0f}^{[1]}(x_1))\, G_0^{[2]}(x_2) \,. \tag{127}$$

Defining the quark densities as usual in terms of $\mathcal{F}_2$, so that $K(x_1, x_2)$ becomes

★ The quark-gluon corrections to $\mathrm{d}^2\sigma/\mathrm{d}Q^2\mathrm{d}y_{\mathrm{R}}$ have also been considered in ref. [22].

$\tilde{K}(x_1, x_2, t)$ and performing the integrals where possible we obtain

$$\frac{Q^2 \mathrm{d}^2\sigma}{\mathrm{d}Q^2 \mathrm{d}x_F} = A \left\{ \frac{1}{(x_F^2 + 4\tau)^{1/2}} \left[\alpha_s(t) \int_{x_2^0}^{1} \frac{\mathrm{d}x_2}{x_2} \tilde{K}(x_1^0, x_2, t)\, h\left(\frac{x_2^0}{x_2}\right) \right. \right.$$

$$\left. + \alpha_s(t) \int_{x_1^0}^{1} \frac{\mathrm{d}x_1}{x_1} \tilde{K}(x_2^0, x_1, t)\, h\left(\frac{x_1^0}{x_1}\right) \right]$$

$$+ \frac{\alpha_s(t)}{2\pi} \frac{1}{2} \int_{x_1^0}^{1} \frac{\mathrm{d}x_1}{x_1} \int_{x_2^0}^{1} \frac{\mathrm{d}x_2}{x_2} \frac{\tilde{K}(x_1, x_2, t)}{(1-z)(x_1+x_2)}$$

$$\times \left[2z(1-z) + (1-z)^2(1-y^*) + (z^2 + (1-z)^2) \frac{1}{(1-y^*)_+} \right]$$

$$+ \frac{\alpha_s(t)}{2\pi} \frac{1}{2} \int_{x_1^0}^{1} \frac{\mathrm{d}x_1}{x_1} \int_{x_2^0}^{1} \frac{\mathrm{d}x_2}{x_2} \frac{\tilde{K}(x_2, x_1, t)}{(1-z)(x_1+x_2)}$$

$$\left. \times \left[2z(1-z) + (1-z)^2 y^* + (z^2 + (1-z)^2) \frac{1}{y_+^*} \right] \right\}, \tag{128}$$

where,

$$\alpha_s h(z) = \frac{\alpha_s}{2\pi} \tfrac{1}{2} \left[(z^2 + (1-z)^2) \ln(1-z) + 1 - 6z(1-z) \right] . \tag{129}$$

The sum of eqs. (116) and (128) gives our final result for the differential cross section.

By simple modifications one could also obtain the rapidity distribution. The rapidity is defined as

$$y_R = \tfrac{1}{2} \ln\left(\frac{E + q_z}{E - q_z}\right), \tag{130}$$

where $E = (q_z^2 + q_T^2 + Q^2)^{1/2}$ and q_z are the γ^* energy and momentum along z in the hadron-hadron c.m.s. For example $Q^2 \mathrm{d}^2\sigma/\mathrm{d}Q^2 \mathrm{d}y_R|_{y_R=0}$, a quantity often used in plotting the data, is obtained from our eqs. (116) and (128) by setting $x_F = 0$ and replacing $\tilde{H}(x_1, x_2, t)$ and $\tilde{K}(x_1, x_2, t)$ by

$$\tilde{H}(x_1, x_2, t) \frac{2(x_1 x_2 + \tau)}{(x_1 + x_2)} \quad \text{and} \quad \tilde{K}(x_1, x_2, t) \frac{2(x_1 x_2 + \tau)}{(x_1 + x_2)} .$$

6. Numerical calculations

Preliminary estimates presented in sect. 4 indicated that the quark-antiquark annihilation terms in the total cross section received large corrections when the effect of gluon radiation was included. We now present detailed calculations for the differential cross section which will confirm those conclusions.

In all calculations we set $\sqrt{S} = 27$ GeV and consider incident protons on nucleons as in the data of ref. [23]. The protons are directed along the positive z direction. Our input parton densities at a reference momentum $Q^2 = Q_0^2 = 75$ GeV^2 are (SU(4) symmetric sea)

$$\begin{aligned} u_V(x) &= u - \bar{u} = 1.78(1-x)^3(1+2.3\,x)/\sqrt{x}\,, \\ d_V(x) &= d - \bar{d} = 0.993(1-x)^3/\sqrt{x}\,, \\ s(x) &= 0.21(1-x)^7/x\,, \qquad \text{(4 flavours)}\,, \\ G(x) &= 2.54(1-x)^5/x\,. \end{aligned} \tag{131}$$

With this parametrisation the momentum carried by the various components ($n = 2$ moment) is

$$\begin{aligned} &u_V^{(2)}(Q_0^2) \simeq 0.30\,, &&d_V^{(2)}(Q_0^2) \simeq 0.10\,, \\ &s^{(2)}(Q_0^2) \simeq 0.026\,, &&G^{(2)}(Q_0^2) \simeq 0.40\,. \end{aligned} \tag{132}$$

The running coupling constant is parametrised as usual by

$$\alpha_s(Q^2) = \frac{12\pi}{25\ln(Q^2/\Lambda^2)}\,, \tag{133}$$

with $\Lambda = 0.5$ GeV. The renormalisation-group improvement implicit in the above formula has been inserted by hand in the formula of the previous sections, since calculations performed in $O(\alpha_s)$ are insensitive to the change with scale of the coupling constant. By the same token, we have no way of knowing (short of actually performing calculations in higher orders) whether Q^2 or some other large variable is the correct variable to describe the fall-off of the running coupling constant. This represents a theoretical uncertainty in our estimate of the size of the correction terms.

The evolution of the parton densities (eqs. (23), (24)) with changing Q^2 is calculated as follows. Firstly, the moments of the densities are performed analytically. These moments can then be calculated at the appropriate value of Q^2 using the well-known eigenvalue matrix of the logarithmic exponents. Finally at each Q^2 of interest inverse Mellin transforms are taken. A check on the inversion procedure at every value of Q^2 is obtained by recalculation of the moments from the final x-dependent parton densities and comparison with the original moments at that value of Q^2. The errors are less than 2% throughout the Q^2 range of interest for all moments with $n \leqslant 10$.

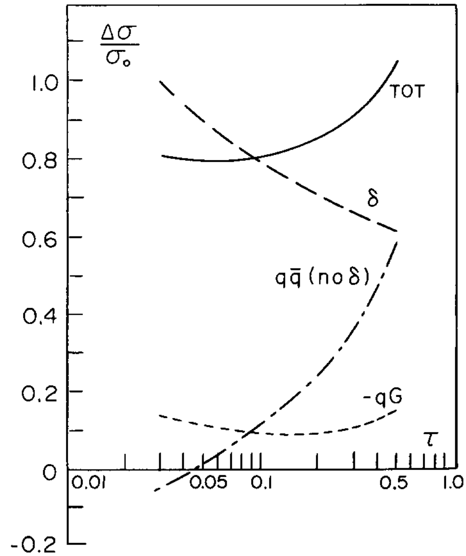

Fig. 7. Plot of the changes in the cross section, $\sigma = Q^2 d^2\sigma/dQ\,dy_R|_{y_R=0}$ due to the correction terms. The quantity $\Delta\sigma/\sigma_0$ (where σ_0 is the naive prediction with scaling parton densities) is plotted against τ. The different curves refer to different values of $\Delta\sigma = (\sigma - \sigma_0)$. (a) $\sigma = \sigma_{TOT}$ includes all the correction terms ($q\bar{q}$ and qG) in eqs. (116) and (128). (b) $\sigma = \sigma_\delta$ includes only the delta function contribution. The variation of this piece is due to the running coupling constant. The correction is large at all values of τ. (c) $\sigma = \sigma_{q\bar{q}}$ includes all the $\bar{q}q$ corrections other than the delta function. (d) $\sigma = \sigma_{qG}$ shows the quark-gluon corrections. This correction term is negative and is shown changed in sign.

In fig. 7 we plot the changes in $Q^2 d^2\sigma/dQ^2 dy_R|_{y_R=0}$ due to the corrections. The total corrections together with various components are shown plotted as a fractional change with reference to σ_0, the naive scaling parton model results. The total correction (TOT) spans the range from 80% to 100% in the range of τ investigated. At low values of τ this correction is almost entirely due to the delta-function contribution (δ) proportional to $\frac{4}{3}\pi^2 + 1$ whereas at the highest value of τ, both this term and the other $q\bar{q}$ corrections ($q\bar{q}$ (no δ)) play an equal role. The quark-gluon correction (−qG) is negative and does not exceed 15% in the range investigated.

The size of the delta-function correction is independent of the form of the input q and $\bar{q}$ distributions and is determined by the running coupling constant. Since the fall-off of the coupling constant is only logarithmic, these corrections can only be made small by increasing Q^2 by several orders of magnitude. The other $q\bar{q}$ corrections (in particular the term whose moments grow like $\ln^2 n$) depend on the form of the input distributions. In valence-valence quark-antiquark annihilation processes (e.g., πN, $\bar{\text{P}}$N) these latter terms will be slightly less significant at lower values of τ.

In fig. 8 we plot $Q^2 d^2\sigma/dQ^2 dy_R|_{y_R=0}$ as a function of τ. The dotted curve shows the cross section with Q^2 dependent parton densities. The solid curve has all the

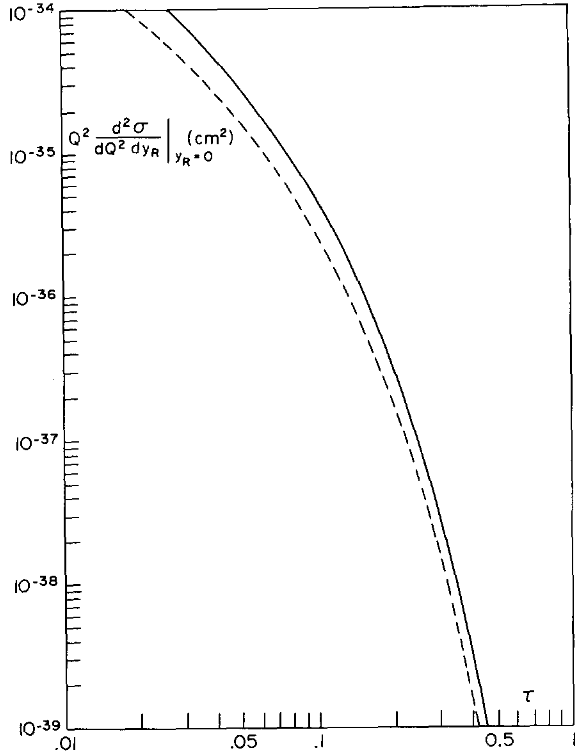

Fig. 8. $Q^2 d^2\sigma/dQ^2 dy_R|_{y_R=0}$ in cm^2 plotted as a function of τ. The dashed curve is the parton-model prediction with Q^2 dependent parton densities. The solid curve is the prediction including all the corrections in order $\alpha_s(Q^2)$.

$\alpha_s(Q^2)$ corrections included. The scale is logarithmic.

Lastly in fig. 9 we show the shape of the rapidity cross section as a function of x_F for several values of τ. Only the relative magnitudes of the curves at each value of τ are significant. The total corrections are large at all values of x_F and τ, so that there is no special configuration in which the correction can be ignored. The quark-gluon correction is small and negative at all values of x_F.

7. Conclusions

The numerical analysis of sect. 6 has shown that the corrections to Drell-Yan processes expressed in terms of leptoproduction parton densities are so large at present values of Q^2 that the lowest-order formula for the Drell-Yan process with scale-breaking parton densities is unreliable. A correct theoretical description will require the inclusion of the significant terms appearing in higher orders in the perturbation

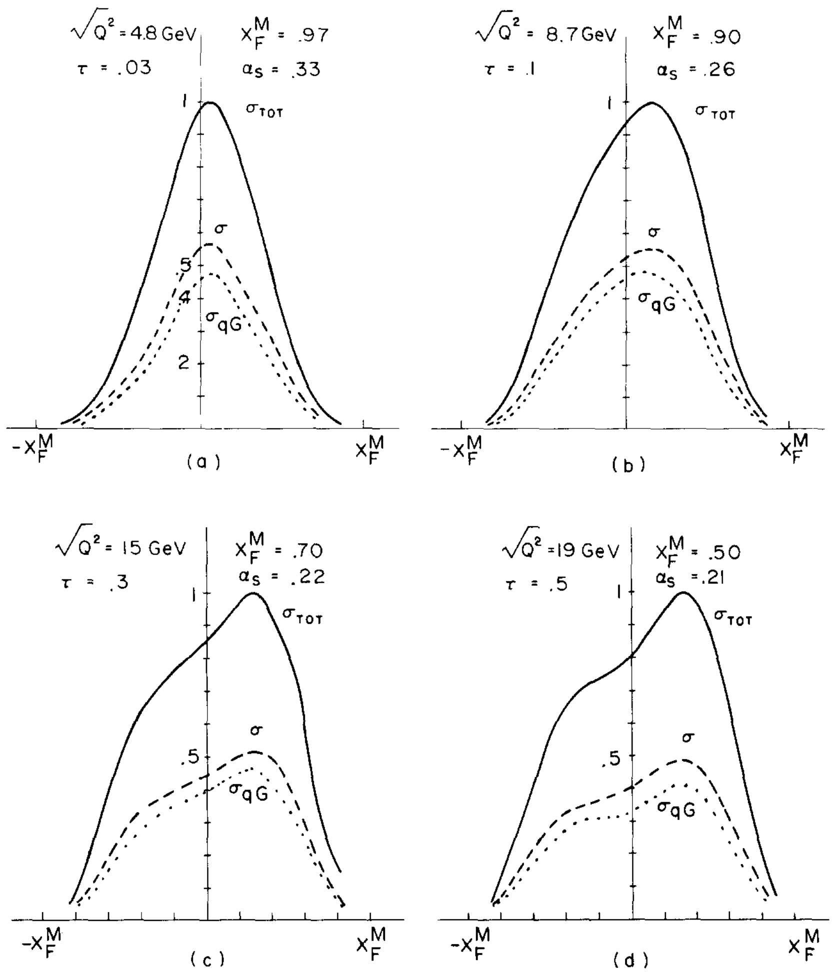

Fig. 9. Plot of $\sigma = Q^2 d^2\sigma/dQ^2 dx_F$ *versus* x_F for various values of Q^2 and τ. The scale for each graph is arbitrary and is obtained by dividing all curves by the maximum value of the completely corrected cross section $Q^2 d^2\sigma_{TOT}/dQ^2 dx_F$. The relative size of the various curves in each diagram is significant. σ_{TOT} is the cross section including all the correction terms. σ is the parton-model prediction with parton densities evolved to the appropriate value of Q^2. σ_{qG} only includes the effects of the quark-gluon correction. On each graph the appropriate values of Q^2, τ, α_s, and x_F^M (the maximum value of x_F) are given.

series. A possible clue to the identification of such terms in higher orders is the fact that in order α_s the large terms are the vestiges of the cancelled soft singularity.

Our analysis leaves open the possibility of describing Drell-Yan type processes in terms of Drell-Yan parton densities alone. At the present stage of experimental information this implies a substantial loss of predictive power.

Appendix

In this appendix we present a series of results connected with the moments of the functions encountered in the text. We define the moments of a function $f(z)$ to be,

$$f^{(n)} = \int_0^1 dz\, z^{n-1} f(z)\,. \tag{A.1}$$

In table 1 we list the moments $f^{(n)}$ corresponding to the functions $f(z)$. The distributions $1/(1-z)_+$ and $(\ln(1-z)/(1-z))_+$ are defined in terms of their integrals with an arbitrary function $h(z)$

$$\int_0^1 dz\, h(z) \frac{1}{(1-z)_+} \equiv \int_0^1 dz \frac{h(z)-h(1)}{1-z}\,, \tag{A.2}$$

$$\int_0^1 dz\, h(z) \left(\frac{\ln(1-z)}{1-z}\right)_+ \equiv \int_0^1 dz (h(z)-h(1)) \frac{\ln(1-z)}{1-z}\,. \tag{A.3}$$

The above distributions differ from the normal functions $1/(1-z)$ and

Table 1
The moments $f^{(n)}$ of the function $f(z)$ defined by the relation $f^{(n)} = \int_0^1 dz\, z^{n-1} f(z)$

$f(z)$	$f^{(n)}$
1	$\frac{1}{n}$
$\ln z$	$-\frac{1}{n^2}$
$\frac{\ln z}{(1-z)}$	$-\frac{1}{6}\pi^2 + \sum_{j=1}^{n-1} \frac{1}{j^2}$
$\ln(1-z)$	$-\frac{1}{n} \sum_{j=1}^{n} \frac{1}{j}$
$\delta(1-z)$	1
$\frac{1}{(1-z)_+}$	$-\sum_{j=1}^{n-1} \frac{1}{j}$
$\left(\frac{\ln(1-z)}{1-z}\right)_+$	$\sum_{k=1}^{n-1} \frac{1}{k} \sum_{j=1}^{k} \frac{1}{j}$

$\ln(1-z)/(1-z)$ by functions which have support only at $z = 1$. It therefore follows that the integral

$$\int_x^1 \mathrm{d}z\, h(z) \frac{1}{(1-z)_+} = \int_0^1 \mathrm{d}z\, h(z) \frac{1}{(1-z)_+} - \int_0^x \mathrm{d}z\, h(z) \frac{1}{1-z}\,, \tag{A.4}$$

and similarly for the distribution $(\ln(1-z)/(1-z))_+$.

The moments of these distributions are readily derived using the identity

$$(z^{n-1} - 1) = (z-1) \sum_{j=1}^{n-1} z^{j-1}\,. \tag{A.5}$$

In generalizing table 1 it is useful to remember the convolution theorem for Mellin transforms

$$f(z) = \int_z^1 \frac{\mathrm{d}x}{x} g(x)\, h\left(\frac{z}{x}\right), \qquad f^{(n)} = g^{(n)} h^{(n)}\,. \tag{A.6}$$

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