Experiment 1: Thin Converging Lens PHY 431

This experiment is a classic exercise in geometric optics. The goal is to measure the radius of curvature and focal length of a single *converging* lens from which you can calculate the index of refraction *n*. We shall explicitly consider the errors that accompany any measurement and how errors are analyzed to yield a quantitative estimate of uncertainty. This includes quantities derived from measurements, such as the index of refraction here.

In the procedures for this lab, you are explicitly reminded to estimate the uncertainty several times. (In future labs these reminders may be omitted.) Please see Appendices (i)-(iii) for reference material and relevant equations. The questions, labeled **Q1**, **Q2**, … <u>should be directly addressed in your report in the Analysis & Discussion section</u>.

Procedure:

A. Choose a converging lens for the experiment. By definition, the focal length *f* of a lens is the image distance from the lens center for an infinitely distant object. To obtain a rough estimate of *f*, project an image of some distant object in the space outside the lab, e.g. a tree, onto white paper. ***Q1.*** *Why does the object appear upside down?*
B. Use a spherometer to measure the radius of curvature of both surfaces of your lens. See Appendix (i). You begin by finding the "zero" position, x_0, using a scratch-free spot on your bench (which is a good approximation to a flat surface). Then perform the measurement with your lens in place, getting x_1; the distance *h* is then $|x_0-x_1|$. (Always be sure to include the uncertainty. In this case you can repeat the measurements a few times to obtain an estimate for the spherometer's precision, σ_{x0}.) Consult the manual for the spherometer. You can use the tables there instead of the formulas in Appendix (i). ***Q2.*** *Which edge of the spherometer head rests on the measured convex spherical surface and which on concave? For which edge should you use the specified diameter or radius in determining the radius of curvature of your surfaces?* ***Q3.*** *Having estimated measurement uncertainties σ_{x0} and σ_{x1}, write an expression for σ_h and evaluate it using your data. For now you can ignore any error on b.*
C. Arrange an object (the T on the lamp window) and screen on the optical rail, with a separation greater than 4*f*. Locate the lens position which gives a sharp image on the screen. Record the object and image distances measured from the center of the lens. (Be sure to estimate the uncertainty for these distances.) Use the thin lens equation to calculate *f*. (Also calculate σ_f.) Repeat this for 4 positions of the screen increasing the object-screen separation in increments of about 2 cm. Find your best value for the focal length using the equation by the end of Appendix (ii) (the mean). ***Q4.*** *How does the best value for f compare to your original rough estimate?*
D. Insert a variable iris before/after the lens. Observe the image as the aperture size is changed. Specifically note how the aperture affects your ability to focus the image. ***Q5.*** *What is the meaning of the term "depth of field" in this context?*
E. Place the light source a distance less than *f* from the lens. Try to position the screen to bring the object into focus. ***Q6.*** *Are you encountering any difficulties? What is going on here?*
F. Calculate the index of refraction (including uncertainty) for the glass of your lens using the lensmaker's equation. (Remember that your report should include comments as to whether or not your value is reasonable).

Appendix (i): Miscellaneous Equations

Thin Lens Equation:

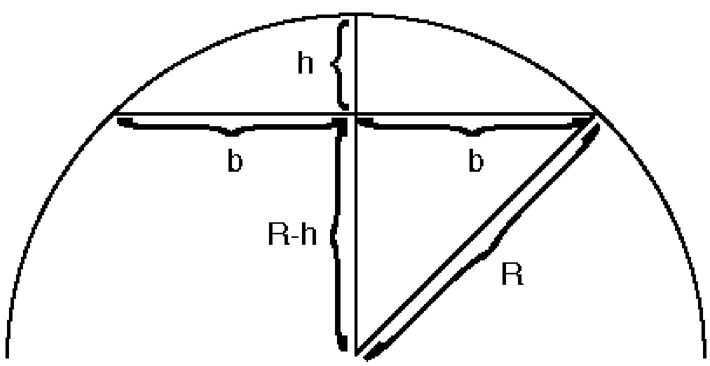
$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

Lensmaker's Equation:

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$, assuming that the index of refraction for the surrounding medium (air) is 1.000.

Spherometer Equation:

$$R=\frac{b^2}{2h}+\frac{h}{2}$$



From Pythagoras' Theorem: $R^2 = (R-h)^2 + b^2$

$$R^2 = R^2 - 2Rh + h^2 + b^2$$
$$R=\frac{b^2}{2h}+\frac{h}{2}$$

Appendix (ii): Random Errors & Error Propagation

Random fluctuations in the measurement process lead to a Gaussian distribution about the true value. This distribution gives us a parameter, σ, called the "standard deviation". (Systematic errors can give a non-Gaussian distribution.) Essentially, if many measurements are taken, 68% of the data points lie within $x_0 \pm \sigma_x$, where x_0 is true value.

Now, suppose an arbitrary function f(x,y) depends on the variables x and y assumed to be independent of each other. How do we compute the uncertainty in f, σ_f, given σ_y and σ_x? Under the assumption that the uncertainties are small compared to the range over which f significantly varies, the following expression works:

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \sigma_x{}^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y{}^2}$$

For exemplary specific functions this yields:

$$f = ax + by$$
$$\sigma_f = \sqrt{a^2\sigma_x^2 + b^2\sigma_y^2}$$

$$f = cxy$$
$$\frac{\sigma_f}{f} = \sqrt{\left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2}$$

$$f = cx^a y^b$$
$$\frac{\sigma_f}{f} = \sqrt{\left(\frac{a\sigma_x}{x}\right)^2 + \left(\frac{b\sigma_y}{y}\right)^2}$$

$$f = ce^{bx}$$
$$\frac{\sigma_f}{f} = b\sigma_x$$

$$f = ca^{bx}$$
$$\frac{\sigma_f}{f} = (b\ln a)\sigma_x$$

Lastly, we address the situation where we make N measurements of the same quantity x, each with an uncertainty of σ_x. Intuitively, we expect that combination of a number of measurements will yield uncertainty smaller than σ_x. In fact, if the fluctuations of measurements around the true value are uncorrelated, the estimated uncertainty in the average over measurements is reduced by $1/\sqrt{N}$ compared to individual measurements, when N is large:

$$\bar{x} = \frac{(x_1 + x_2 + \cdots + x_N)}{N} \rightarrow \sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}}$$

If σ_x is unknown, it can be estimated from the spread in measurements and formulaically from

$$\sigma_x^2 \approx \frac{\overline{x^2} - \bar{x}^2}{N - 1}$$

for large N, with the approximation improving as $N \rightarrow \infty$

Appendix (iii): More on Errors

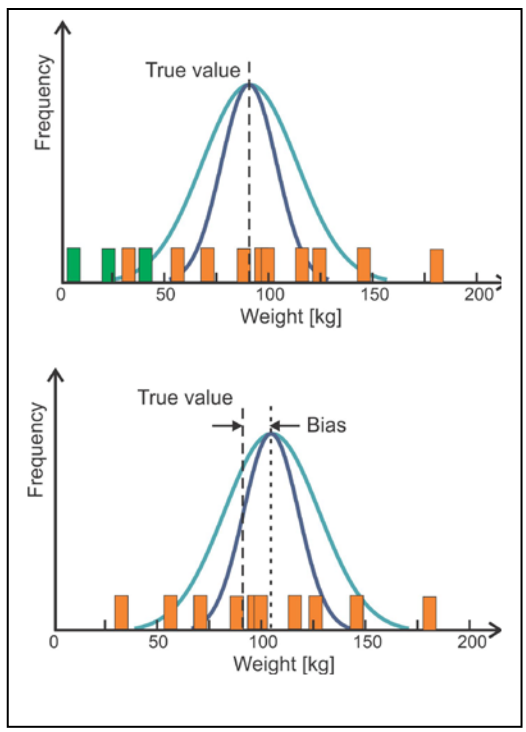
Random & Systematic Errors

Random and systematic errors are error types related, respectively, to precision and accuracy. Random errors vary between successive measurements. They are equally likely to be positive and negative. They tend to be always there in an experiment. Their presence is obvious from distribution of values obtained. Their impact can be minimized by performing multiple measurements of the same quantity.

Systematic errors are generally constant throughout a set of measurements. They may result from calibration of equipment or from methodology behind the measurements. They cause the mean of measured values to depart from the correct value. They can be difficult to estimate. At times the manufacturers provide accuracy of the instruments they supply. In absence of such information, one can assume that the systematic error is at least half of the last digit that the instrument provides in a measurement.

When few measurements are carried out, random (also called reading) errors tend to have more impact on the outcome of the measurements. However, if many measurements are carried out and the impact of random errors diminishes, the systematic errors begin to dominate the overall error.

As an example illuminating systematic errors, let us consider a situation where researchers are to determine an average weight of some population. If they select representative samples of the population, illustrated with boxes in the upper panel of the adjacent figure, errors are purely random. Following central-limit theorem, the distribution of the average weight for samples, Gaussian curves there, narrows as sample size increases, eventually becoming very narrow around the true average for the population. The width of the distribution can be estimated using the distribution of weight values for a sample. However, if the researchers make local arrangements such that they can only weigh those above the age of 10, illustrated with boxes in the lower panel of the figure, their samples begin to be biased towards higher values of weight. When their samples become very large, the distribution of the average weight for a sample becomes narrow, but it peaks around a value that is systematically in excess of the true average for the population. Increasing the size of a sample does not help to eliminate the bias. It is impossible to estimate the bias on the basis of a sample alone.



Understanding the nature of the bias, one can try to correct for it, e.g. using data for another population. However, even after such correction some residual systematic error will remain that the size of a sample will not help with. Other systematic errors may be due to the weighted individuals being in clothes, scale calibration etc.

For functions systematic errors are propagated in the same fashion as random errors, using partial derivatives when errors are small. For net final error, the one with random origin is added in quadrature to the systematic error:

$(\text{net error})^2 = (\text{random error})^2 + (\text{systematic error})^2$.

In the experiments done today, the systematic errors stem e.g. from accuracy of the ruler and from approximating a thick lens with a thin lens. If error of random origin is large, the systematic error may be disregarded, but not if the random error for good or wrong reasons is small.

Stating Results and Errors

Generally state errors to 1-2 significant digits. Two digits are advisable, if the leading digit is low. Quote result to the same significance as error. When using scientific notation, quote value and error with the same exponent.

- Value 33, error 11 → 33±11
- Value 72, error 36 → 70±40
- Value 5.6×10^3, error 6×10^2 → $(5.6 \pm 0.6) \times 10^3$

Incorrectly stated results:

- 36±0.7
- 36.06±0.7