In this lab you will explore several types of aberrations of a real lens. The techniques you will use are similar to those you used before for finding the focal length of a lens. However, here you will explore how the focal length changes with incidence of the light rays and with color. Since the effects are small, you will need to make a series of measurements and to plot the results and search for a trend. This strategy should also facilitate comparisons of your measurements to theory. You will need to utilize a plotting program (such as Kaleidagraph) that will let you fit a least-squares line to your data. Remember to include the error bars for your measured values on your plot.
A. Choose a converging lens with two different radii of curvature, preferably a plano-convex lens, with a diameter of at least 2 inches. Measure the focal length of your lens following the thin lens formula. Further measure the radii of the lens using a spherometer. Using a representative value of $n$ from the Appendix, compute $f$ from the lens-maker formula. Determine uncertainties and compare your two results for $f$.
B. Place the lens about $1.25 f$ away from the object discussed below, with the flat (or less convex) side facing the object. Instead of the T-lines on a window covering the lamp, as your object use an illuminated piece of paper mounted on a frame, with a pattern drawn or printed on it. The goal is to eliminate changes in $s_{o}$, a potential source of error, when the lamp used for illumination is tilted. Place the smallest aperture you will be using (if necessary just make a small hole in cardboard) immediately in front of the lens. Be careful to position the aperture on the optical axis. Focus the image and measure the image distance $s_{i}$. With this aperture, assume that your measurement is aberration-free.
C. Replace the aperture with different ring stops by the lens and measure new si. In each case measure and record the radius $h$ of the stop.
D. Plot $h$ vs shift $\Delta\left(1 / s_{i}\right)$ from aberration-free finding and fit a parabola to your results as suggested by the equations in the Appendix. If your plotting program is not capable of fitting a parabola, plot $h^{2}$ vs $\Delta\left(1 / s_{i}\right)$ and fit a straight line. Q1. What measure could you use to quantify the goodness of the fit?
E. Rotate the lens (less convex side away from the object) and repeat the steps above in BD, but first read the following remark. Note: A thick lens does not work like a thin. In fact, after you flip the lens, the image that was sharp before the flip will usually not remain sharp after the flip, as $s_{o}$ and $s_{i}$ change due to the thickness of the lens. After the flip determine $s_{o}$ and the aberration-free $s_{i}$ in B again. You may also choose to bring conditions after the flip as close as possible to those before with such steps: Set the image in focus in B for the original lens orientation. Thereafter flip the lens orientation and, with the image screen position fixed, move the lens a bit back and forth a bit to get the image into the best possible focus. Measure $s_{o}$ and $s_{i}$. After getting this done for B do not move the lens (nor object) anymore for the subsequent operations.
F. Using the equations in the Appendix, calculate the parameter $S$ for your setup, employing the radii and image and object distances from your measurements. Consider both orientations of the lens. Q2. What is the change in $\sigma$ when you flip the lens? Q3. How do your calculations compare to the values of $S$ following from the least-squares fit parameters to measurements? Q4. Do some of your points significantly deviate from a parabola? Why do you think this happens? Is the validity of the expansion leading to the equations in the Appendix likely to be the same for small as for large $h$ ?
G. Place one of the colored filters in front of the lens and carry out measurements to determine the focal length (error included) following the thin lens equation. Repeat actions with the other filters. Your measurements will be easiest to compare to each other, if you do not change $s_{o}$, just $s_{i}$.
H. Plot $1 / f$ vs $n$ using the values of the refractive index given in the Appendix and fit a straight line to your results. Q5. Compare the proportionality coefficient in your fit to that deduced from the radii measured with the spherometer. What might be the sources of any discrepancy between the coefficient values?

## APPENDIX

See the textbook by the Pedrottis for both qualitative and qualitative discussions of the aberrations, possibly at different locations there depending on edition. A qualitative discussion of the aberrations can be moreover found in the Technical Library at cvilaseroptics.com under Performance Factors.

## Spherical Aberration

The derivation of an equation for spherical aberration is too lengthy to be given here. For a thin lens, expansion of pertinent equations under the assumption of small effects yields

$$
\Delta \frac{1}{s_{i}}=S \frac{h^{2}}{f^{3}}
$$

where $\Delta \frac{1}{s_{i}}$ is the shift in inverse image position relative to that formed by paraxial rays, $h$ is the distance away from the axis at which an oblique ray traverses the lens and $f$ is the paraxial focal length. The factor $S$ depends on characteristics of the lens and on object position:

$$
S=\frac{1}{8 n(n-1)}\left[\frac{n+2}{n-1} \sigma^{2}+4(n+1) p \sigma+(3 n+2)(n-1) p^{2}+\frac{n^{3}}{n-1}\right] .
$$

Here, $n$ is the index of refraction for the lens and $\sigma$ is the so-called Coddington shape-factor defined with

$$
\sigma=\frac{R_{2}+R_{1}}{R_{2}-R_{1}},
$$

where the radius is positive if the center of curvature points to the side of (real) object and negative if away. Finally, $p$ is position factor

$$
p=\frac{s_{i}-s_{o}}{s_{i}+s_{o}},
$$

where $s_{i}$ is position for paraxial rays. Assume $n=1.52$ for these equations.

## Chromatic Aberration

For chromatic aberration, you can use the lensmaker's equation and the values of $n$ for glass at the filter transmission maxima given in the table below.

| Color | $\lambda(\mathbf{n m})$ | $\mathbf{n}$ |
| :--- | :--- | :--- |
| Violet | 420 | 1.5318 |
| Blue | 460 | 1.5265 |
| Cyan | 485 | 1.5240 |
| Green | 540 | 1.5196 |
| Yellow | 580 | 1.5172 |
| Red | 640 | 1.5145 |

