Physics 472 - 2020

Quantum Mechanics

https://web.pa.msu.edu/courses/2020spring/PHY472/desc_PHY472.html

Problem Set 1

1. Show that

$$\left(A^\dagger\right)^\dagger = A; \quad (AB)^\dagger = B^\dagger A^\dagger, \quad [A, BC] = [A, B]C + B[A, C].$$

   Express $[AB, C]^\dagger$ in terms of $A^\dagger, B^\dagger$ and $C^\dagger$

2. Use the result of the previous problem to calculate the commutator of the operators of the x-component of the momentum operator $\mathbf{p}$ and the y-component of the angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p}$. Evaluate also $[L_x, x]$, $[L_x, y]$, $[L_x, z]$

3. Consider a particle of mass m which moves along the z-axis in a potential $U(z) = \infty$ for $z \leq 0$, $U(z) = -\Lambda/z$ for $z > 0$ ($\Lambda > 0$).

   - In what energy range does the particle have bound states?
   - Find the behavior of the wave functions of the bound states for large and small positive z
   - Is that set of eigenfunctions of the Hamiltonian $H = (p_z^2/2m) + U(z)$ with the chosen $U(z)$ complete for the set of continuous functions $\psi(z)$ defined for $-\infty < z < \infty$?

4. For the previous problem, find the energy levels of the bound states.

Each problems is 10 pt.