Physics 472 - 2020

Quantum Mechanics

Problem Set 6

1. (a) Suppose you know the roots $x_1^{(0)}, x_2^{(0)}, \ldots$ of the equation $F_0(x) = 0$. Find, to the second order in λ, the roots $x_1, x_2, \ldots$ of the equation $F_0(x) + \lambda F_1(x) = 0$, assuming that $|\lambda| \ll 1$. Assume that the functions F_0, F_1 are smooth near $x_i^{(0)}$ and that the roots are nondegenerate, i.e. $|x_i - x_i^{(0)}| \ll |x_i^{(0)} - x_j^{(0)}|$ for $i \neq j$.

   (b) Calculate by the perturbation theory the first 3 digits of $\sqrt{18}$ and of $\log_2 5$.

2. With lasers, you can cool atoms down to a very low temperature where their motion is quantized. Cold atoms can be trapped in an (almost) harmonic potential. Consider three identical atoms with mass M that do not interact with each other and are trapped in a potential $U(x) = kx^2/2$. Find the energies of the lowest and the first excited state of the atoms assuming first that they are (a) bosons and then that they are (b) spin-1/2 fermions.

3. Write the wave function of the system of three bosons, assuming that the single-particle states are $\psi_a(\mathbf{r}, m_s), \psi_b(\mathbf{r}, m_s)$, and $\psi_c(\mathbf{r}, m_s)$. Keep in mind that the wave function should not change when you interchange any particles.

4. Consider a particle of mass M in the potential $U(x) = \frac{1}{2}kx^2 + \frac{1}{4}\gamma x^4$. Find the energy levels to the first order in γ.

Each problem is 10 pt.