## Physics 472 - 2020

# Quantum Mechanics

### Problem Set 7

1. Write the wave function of the system of three fermions, assuming that the single-particle states are $\psi_a(\mathbf{r}, m_s), \psi_b(\mathbf{r}, m_s)$, and $\psi_c(\mathbf{r}, m_s)$. Keep in mind that the wave function should change sign when you interchange any two particles.

2. Suppose you know the roots $x_1^{(0)}$ and $x_2^{(0)}$ of the equation $F_0(x) = 0$, and suppose $x_1^{(0)} = x_2^{(0)}$ (a double root). Find, to the lowest nontrivial order in $\lambda$, the roots $x_1, x_2$ of the equation $F_0(x) + \lambda F_1(x) = 0$, assuming that $|\lambda| \ll 1$. Assume that the functions $F_0, F_1$ are smooth near $x_{1,2}^{(0)}$ and $F_1(x_{1,2}^{(0)}) \neq 0$.

3. Two identical spin-3/2 fermions, mass $m$, are confined to move in one direction, along the $x$-axis. Additionally, they are placed into a parabolic potential well with potential energy $U(x) = kx^2/2$. There is no magnetic field. The fermions are interacting with the potential energy $V(x_1, x_2) = -a\delta(x_1 - x_2)$. Assuming that $a$ is small, find the ground-state energy of the fermions to the first order in $a$. What is the parameter, compared to which $a$ has been assumed small?

4. The same as in the previous problem, but assume that a magnetic field $B$ is applied to the system along the $x$-axis, so that the spins prefer to be aligned along the field. Set the gyromagnetic ratio to be $\gamma$ and assume that $|\gamma\hbar B - \hbar(k/m)^{1/2}| \gg |a|$. There are two cases to consider in this problem, you have to identify them.

5. *Extra credit* What does "confined" in the previous problems imply?

Each problem is 10 pt. You need 35 pt out of 50 to get full credit.

**The solution is due on Friday, Feb. 28, but will be accepted earlier, too. There will be no quiz on Feb. 28**