Physics 472 - 2020

Quantum Mechanics

Problem Set 8

1. Consider two harmonic oscillators with identical masses and equal angular frequencies ω. The oscillators are weakly coupled, so that the total Hamiltonian of the system reads
$$H = \frac{p_1^2}{2m} + \frac{1}{2}m\omega^2 q_1^2 + \frac{p_2^2}{2m} + \frac{1}{2}m\omega^2 q_2^2 + mVq_1q_2$$
Assume that $|V| \ll \omega^2$. Find the energy levels of the oscillators. There are two ways to solve this problem, both are acceptable. Those who find the both ways will get extra 5 points.

2. Consider an electron spin, $s = 1/2$, in a magnetic field with the x- and z- components, $\mathbf{B} = (B_x, 0, B_z)$. Starting from the eigenstates $\chi_\pm$ of the operator s_z, find the energies and the eigenfunctions of the spin states.

3. Assume that the Hamiltonian of the system H depends on a parameter λ (this is not a small parameter!), $H \equiv H(\lambda)$. Respectively, the eigenvalues of the Hamiltonian depend on λ, too, $E_n \equiv E_n(\lambda)$. Do the eigenfunctions ψ_n depend on λ? Show that
$$\frac{\partial E_n}{\partial \lambda} = \left\langle \psi_n \middle| \frac{\partial H}{\partial \lambda} \middle| \psi_n \right\rangle$$
(the Feynman-Hellmann theorem).

4. The kinetic energy of a relativistic particle with rest mass m as a function of the particle momentum is $T = c\sqrt{p^2 + m^2c^2} - mc^2$. Use this relation to express T in terms of the particle velocity. *Hint:* $v = \partial T/\partial p$.

Each problem is 10 pt.