

Physics 472 - 2020

# Quantum Mechanics

## Problem Set 9

1. Consider an electron with the total angular momentum quantum number  $j = 3/2$ . The electron is placed into a weak magnetic field  $\mathbf{B}$ . Find the energy spectrum to the first order in  $B$
2. Consider a weakly nonlinear oscillator with Hamiltonian

$$H = \hbar\omega(a^\dagger a + 1/2) + \beta(a^\dagger a^2 + (a^\dagger)^2 a)$$

Find corrections to the energies of the ground state and the first excited state of the oscillator to the second order in  $\beta$ . The term  $\propto \beta$  (along with other terms that are cubic in  $a, a^\dagger$ ) plays an important role in the dynamics of vibrational systems studied in different areas of physics.

3. Show how the energy levels of an electron with  $l = 2$  and  $l = 3$  are split due to the spin-orbit coupling.
4. Use the explicit form of the wave functions of the states of the hydrogen atom with the principal quantum number  $n = 2$  to find, to the first order in the electric field  $E$ , the Stark shifts of these states.