Physics 472 - 2020

Quantum Mechanics

Problem Set 10

1. Using the direct variational method, find the lowest energy of the electron in the hydrogen atom.

2. Using the direct variational method with one variational parameter, find the approximate ground state energy of a particle with mass m in a triangular potential well, $U(x) \to \infty$ for $x \leq 0$ and $U(x) = Ax$ for $x > 0$, with $A > 0$. This model plays an important role in the theory of field effect transistors.

3. Consider a spin with the wave functions $|\uparrow\rangle$ and $|\downarrow\rangle$ and assume that $\psi(t=0) = |\downarrow\rangle$. The Hamiltonian is $H = H^{(0)} + H^{(1)}$ with

$$H^{(0)} = \frac{1}{2}\hbar\omega\sigma_z; \quad H^{(1)} = \frac{1}{2}\sigma_x V\Theta(t)$$

   where $\Theta(t)$ is the step function. Assume that $|V| \ll \hbar\omega$. Find the population of the state $|\uparrow\rangle$ as a function of time.

4. Consider a spin with the wave functions $|\uparrow\rangle$ and $|\downarrow\rangle$ and describe the time evolution of the system assuming that $\psi(t=0) = |\downarrow\rangle$. The Hamiltonian is $H = H_0 + H_1$ with

$$H_0 = \frac{1}{2}\hbar\omega\sigma_z; \quad H_1 = \frac{1}{2}\sigma_x V\cos\omega t.$$

   Assume that $|V| \ll \hbar\omega$. Can you use the simple perturbation theory in V? In what time range, if so?

5. *Extra credit 10 pt* What happens in the previous problem beyond the time range where the simple perturbation theory applies?