Physics 472 - 2020

Quantum Mechanics

Problem Set 11

1. Consider a system with 2 states with equal energies, $H^{(0)}|\psi_1\rangle = E^{(0)}|\psi_1\rangle$ and $H^{(0)}|\psi_2\rangle = E^{(0)}|\psi_2\rangle$. At $t = 0$ there is turned on a perturbation $H^{(1)} = V$ with matrix elements $V_{12} = V_{21}^*$ and $V_{11} = V_{22} = 0$. Solve the time-dependent Schrödinger equation and find the time-dependent wave function $|\Psi(t)\rangle$ assuming that at $t = 0$ the wave function of the system is $|\psi_1\rangle$.

2. The expressions for the δ-function below are written as the limits for $T \to \infty$. Make plots of these expressions before you go to this limit; set $T = 1, 10, 25$.

$$\delta(x) = \frac{1}{2\pi} \lim_{T\to\infty} \int_{-T}^{T} dt\, e^{ixt}, \quad \lim_{T\to\infty} \frac{\sin xT}{x} = \pi\delta(x), \qquad \lim_{T\to\infty} \frac{1-\cos xT}{Tx^2} = \pi\delta(x)$$

   What has to be done to prove these relations?

3. Consider a one-dimensional particle of mass m in a potential $U(x) = -\alpha\delta(x)$ with $\alpha > 0$. The particle is driven by a field at frequency $\omega > m\alpha^2/2\hbar^3$, with $H^{(1)} = -Fx\cos\omega t$. Calculate the rate of transitions from the ground state to the states in the continuous spectrum.

4. Show that $\nabla^2|\mathbf{r} - \mathbf{r}'|^{-1} = -4\pi\delta(\mathbf{r} - \mathbf{r}')$