Physics 472 - 2020

Quantum Mechanics

Problem Set 12

1. Consider the case where the potential $U(\mathbf{r})$ is small and is localized in a small region of space. Assume that it can be analyzed using perturbation theory. The wave function of the zeroth order approximation is $\psi^{(0)}(\mathbf{r}) = \exp(i\mathbf{k}\cdot\mathbf{r})$, with $\mathbf{k} = \mathbf{p}/\hbar$. Show that the Schrödinger equation for the first order correction to the wave function $\psi^{(1)}$ *with the same energy* has the form

$$\nabla^2\psi^{(1)} + k^2\psi^{(1)} = \frac{2mU(\mathbf{r})}{\hbar^2}\psi^{(0)}$$

2. Relate to each other two delta-functions: $\delta\big(f(x)\big)$ and $\delta(x)$.

3. Consider the correction to the theory of Rabi oscillations. In this theory, it is assumed that a two-state system with the transition frequency $\omega_{21} = (E_2^{(0)} - E_1^{(0)})/\hbar$ is driven by a perturbation $H^{(1)} = \mathcal{V}\cos\omega t$ with frequency $\omega \approx \omega_{21}$, i.e., $|\omega - \omega_{21}| \ll \omega$. The matrix element $V = \langle 2|\mathcal{V}|1\rangle$ was assumed small in the absolute value. Specify with respect to what quantity it is small and consider a modification of the theory when the corresponding correction is taken into account.

4. A system in a bound state $|1\rangle$ is subject to a periodic perturbation which causes transitions from this state into a continuous spectrum. The transition rate is W. Calculate how the population of the state $|1\rangle$ evolves in time, if initially the system occupies this state, $P_1 = 1$.