Physics 472 - 2020

Quantum Mechanics

Problem Set 13

1. Consider a spin with the wave functions $|\uparrow\rangle$ and $|\downarrow\rangle$ and assume that $\psi(t=0) = |\uparrow\rangle$. The Hamiltonian is $H = H^{(0)} + H^{(1)}$ with
$$H^{(0)} = \frac{1}{2}\hbar\omega\sigma_z; \quad H^{(1)} = \frac{1}{2}\sigma_y V \Theta(t)$$
where $\Theta(t)$ is the step function. Assume that $|V|$ can be of the same order as $\hbar\omega$. Find the population of the state $|\downarrow\rangle$ as a function of time.

2. Consider a particle with the wave function $\psi(\mathbf{r}) = f(\theta)e^{ikr}/r$, where θ is the polar angle and r is the distance from the origin, whereas $f(\theta)$ is an arbitrary smooth and at least twice differentiable function. Show that, for large r, the function $\psi(\mathbf{r})$ is a solution of the Schrödinger equation in free space with energy $E = \hbar^2 k^2/2m$.

3. Find the differential scattering cross-section in the Born approximation for a screened Coulomb potential $U(r) = -(Ze^2/4\pi\epsilon_0 r)\exp(-br)$

4. Find the total scattering cross-section for the screened Coulomb potential in the previous problem. What happens when $b \to 0$?

5. *Extra credit:* Consider a one-dimensional particle of mass m in a potential $U(x) = -\alpha\delta(x)$ with $\alpha > 0$. Find the wave functions of the continuous spectrum, i.e., with energy $E > 0$. In other words, find how the wave functions $\psi(x) = \exp(ikx)$ are modified by the potential.