PHY 472 - 2020

Quiz 7 - solution

$H = g\mu_B \frac{\vec{S}}{\hbar}\cdot\vec{B}$;

The wave function of the stationary state is

$\psi_j = \alpha_j \chi_+ + \beta_j \chi_-$, $j = 1, 2$.

$\frac{g\mu_B}{\hbar}(\vec{S}\cdot\vec{B})\psi_j = E_j \psi_j$; We know $E_j = (-1)^j \frac{1}{2} g\mu_B B$,

$B = \sqrt{B_x^2 + B_y^2}$

$s_x \chi_+ = \frac{1}{2}\chi_-$; $s_x \chi_- = \frac{1}{2}\chi_+$; $s_y \chi_+ = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{2} i \chi_-$

$s_y \chi_- = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = -\frac{1}{2} i \chi_+$

Then

$$g\mu_B \left(\frac{1}{2} B_x \alpha_j \chi_- + \frac{1}{2} B_x \beta_j \chi_+ + \frac{1}{2} i B_y \alpha_j \chi_- - \frac{1}{2} i B_y \beta_j \chi_+\right) = E_j (\alpha_j \chi_+ + \beta_j \chi_-)$$

Set the coefficients at χ_+ equal to each other, the same for χ_-:

$$\frac{1}{2} g\mu_B (B_x \beta_j - i B_y \beta_j) = E_j \alpha_j$$

$$\frac{1}{2} g\mu_B (B_x \alpha_j + i B_y \alpha_j) = E_j \beta_j$$

Check the energy from the secular equation:

$$\begin{vmatrix} E_j & -\frac{1}{2} g\mu_B (B_x - iB_y) \\ -\frac{1}{2} g\mu_B (B_x + iB_y) & E_j \end{vmatrix} = 0, \quad E_j^2 - \left(\frac{1}{2} g\mu_B\right)^2 (B_x^2 + B_y^2) = 0$$

$$E_j = \pm \frac{1}{2} g\mu_B B$$

$$\alpha_j = \frac{B_x - iB_y}{B}\beta_j (-1)^j, \quad |\alpha_j|^2 + |\beta_j|^2 = 1$$

Orthogonality: $\alpha_1^* \alpha_2 + \beta_1^* \beta_2 = \beta_1^* \beta_2 \left[-\frac{(B_x + iB_y)(B_x - iB_y)}{B^2} + 1\right] = 0$