PHY 472 - 2020

Quiz 6 / Solution

The states of the system are $\psi_{m_{s1}\, m_{s2}} = \frac{1}{\sqrt{2}}\left(|m_{s1} m_{s2}\rangle + |m_{s2}\, m_{s1}\rangle\right)$ if $m_{s1} \neq m_{s2}$. Here the first number gives the projection of the spin of the first particle on a given axis, and the second number gives the projection of the spin of the second particle. For example $\psi_{01} = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right) \equiv \psi_{10}$ (this is one state)

If $m_{s1} = m_{s2}$, we have $\psi_{m_{s1}\, m_{s1}} = |m_{s1}\, m_{s1}\rangle$

The system has 6 states: $|11\rangle$, $|00\rangle$, $|-1-1\rangle$, $\frac{1}{\sqrt{2}}\left(|10\rangle + |01\rangle\right)$, $\frac{1}{\sqrt{2}}\left(|1-1\rangle + |-1 1\rangle\right)$, $\frac{1}{\sqrt{2}}\left(|0-1\rangle + |-1 0\rangle\right)$

They are degenerate in the absence of the coupling.

Obviously, with the coupling on, the ground state is $\psi_g = |11\rangle$ or $\psi_g = |-1 -1\rangle$. Both are eigenstates of H_i with the energy $E_g = -a\hbar^2$

The highest-energy state is where the spins point in the opposite directions

$\psi_t = \frac{1}{\sqrt{2}}\left(|1-1\rangle + |-1 1\rangle\right)$, $E_t = \langle\psi_t|H_i|\psi_t\rangle = a\hbar^2$