Problem 1. A photon with initial energy 0.46 MeV was scattered off a free electron (originally at rest, with mass 0.51 MeV/C^2) at an angle 120° with respect to the direction of the initial photon.

(4 points) (a) Find the energy of the scattered photon.
(4 points) (b) Find the recoil angle of the scattered electron.
(1 point) (c) Find the energy of the recoil electron.
(1 point) (d) Find the kinetic energy of the recoil electron.
Problem 2. (10 points) A hemisphere of radius R is charged over its volume with charge density $\rho = \rho_0 \exp(-\beta r)$ where $\rho_0$ and $\beta$ are constant parameters, and $r$ is the distance from the center of curvature of the hemisphere. Find the electrostatic field vector $\vec{E}$ at the center, assuming that no other external field is present.
Problem 3. A cylindrical cavity of radius $a$ is made off-center inside a long massive cylinder of radius $b > a$; the axis of the cavity and of the original cylinder are parallel. A steady electric current $I$, parallel to the axes, is uniformly distributed over the cross section of the remaining part of the cylinder.

(10 points) (a) Find the direction and the magnitude of the magnetic field inside the cavity.

(5 points) (b) When the above event is observed by an observer in an inertial frame (moving with a constant velocity), it is found that neither the electric field $\vec{E}$ nor the magnetic field $\vec{H}$ inside the cavity vanishes. Determine the angle between $\vec{E}$ and $\vec{H}$. Is the magnitude of $\vec{E}$ larger or smaller than that of $\vec{H}$, when taking the speed of light $C$ to be 1? Why?

(Hint: Use Lorentz invariants constructed by $\vec{E}$ and $\vec{H}$.)
Problem 4. (10 points) A monochromatic plane electromagnetic wave is described by the vector potential

$$\vec{A}(\vec{r}, t) = \vec{a} \cos(\omega t - \vec{k} \cdot \vec{r} + \alpha),$$

where $\vec{a}$ is a constant amplitude vector and $\alpha$ is a constant phase. A cylinder of radius $R$ and height $h$ is placed perpendicular to the direction of the wave propagation. The wave striking on the cylinder is totally absorbed by the cylinder surface. Neglecting all diffraction effects, determine the force applied to the cylinder (averaged over the period of the wave).
Problem 5. (10 points) A nonrelativistic particle with mass $m$, charge $e > 0$ and initial velocity $\vec{u}_0$ crosses the space between the plates of a capacitor where a constant voltage $V$ is applied. The distance between the plates is equal to $\ell$, the initial angle between $\vec{u}_0$ and the electric field $\vec{E}$ inside the capacitor is $\alpha < \pi/2$. Find the energy loss by dipole radiation during the time of flight inside the capacitor.
(Neglect the influence of radiation on the trajectory.)