QM Subject Exam, Spring 2015

Read all of the following information before starting the exam:

- Make sure your secret student number is written at the top of every page. Do not write your name on the exam.
- Show all work (neatly as possible and in logical order) to maximize your credit. Circle or otherwise indicate your final answers.
- All work should be shown in the space after each question. If you need extra space, use the blank pages that are attached and indicate clearly what problem it corresponds to. You are not allowed to use your own scrap paper.
- This test has 5 problems for a total of 100 points. Please make sure that you have all of the pages.
- Good luck!

1. (*20 points*) Consider a charged particle in a uniform magnetic field in the z-direction.

a. (*15 pts*) Find the energy eigenvalues and eigenfunctions. Make sure you indicate which gauge (i.e., choice of $\mathbf{A}(\mathbf{x})$) you are working in. Use the cyclotron frequency $\omega \equiv \frac{|e|B}{mc}$ in your expressions.

b. **(***5 pts***)** Write down an alternative vector potential $\mathbf{A}'(\mathbf{x})$ and the corresponding gauge transformation that relates it to your original choice $\mathbf{A}' = \mathbf{A} + \nabla f$. What are the energy eigenvalues and eigenfunctions for this choice of gauge?

2. (*20 points*) Consider an electron ($e < 0$) in a uniform magnetic field B in the z-direction. At $t = 0$, the electron is in the $S_x = \hbar/2$ eigenstate. In the following, you may neglect all spatial degrees of freedom (i.e., $H = \frac{|e|B}{mc} S_z$).

a. (*10 pts*) Find the probability for the electron to be found in the $S_x = \hbar/2$ eigenstate at some later time t. Express your answers in terms of $\omega \equiv \frac{|e|B}{mc}$

b. (*10 pts*) At time $t > 0$, find $\langle S_y \rangle$ and $\langle S_z \rangle$.

3. (*20 points*) A spinless particle undergoes elastic scattering from a spherically symmetric potential.

a. (*10 pts*) Find the $l = 0$ phase shift when the potential is an impenetrable sphere of radius R (i.e., $V(r > R) = 0$ and $V(r < R) = \infty$).

b. (*10 pts*) Find the $l = 0$ phase shift for a delta shell potential $V(r) = g\frac{\hbar^2}{2m}\delta(r-R)$.

4. (*20 points*) Consider a spinless particle of mass m in one dimension subjected to an attractive potential $V = -g\frac{\hbar^2}{2m}\delta(x)$.

a. (*10 pts*) Find the ground state energy E_0 and normalized wavefunction ψ_0 for this potential.

b. (*10 pts*) The particle is subjected to a periodic perturbation $W = -xW_0 \sin \omega t$ where $\hbar\omega >> |E_0|$ so that a transition to the continuum (i.e., ionization) is possible. Use Fermi's Golden Rule to find the transition rate for the particle to be ejected with energy $E_k = \frac{\hbar^2 k^2}{2m} > 0$. To simplify the algebra, set $\hbar = 1 = 2m$ and express your final answer in terms of E_0, W$_0$, and ω. You may use

$$\int_{-\infty}^{\infty} e^{-ikx-\kappa|x|} x dx = -\frac{4ik\kappa}{(k^2+\kappa^2)^2}$$

5. (*20 points*) Quick questions.

a. (*2 pts*) The low-lying states of the ^{18}O nucleus can be modeled as two valence neutrons in the $d_{5/2}$ shell outside an inert ^{16}O core. What are the possible values of total angular momentum?

b. (*3 pts*) Consider an electron in a central potential $V(r)$. Show whether or not L_z is a constant of motion for a relativistic electron described by the Dirac equation.

c. (*3 pts*) Using symmetry arguments, identify which of the following matrix elements are identically zero. Give a reason for each one you identify. Here, α and β denote all non-angular momentum quantum numbers.

1. $\langle \alpha, l = 2, m_l = 0 | r^2 | \beta, l = 2, m_l = 1 \rangle$
2. $\langle \alpha, l = 2, m_l = 0 | x^2 + y^2 | \beta, l = 1, m_l = 0 \rangle$
3. $\langle \alpha, l = 3, m_l = 0 | z | \beta, l = 0, m_l = 0 \rangle$
4. $\langle \alpha, l = 3, m_l = 3 | (x + iy)^2 + (x - iy)^2 | \beta, l = 3, m_l = 3 \rangle$
5. $\langle \alpha, l = 3, m_l = 3 | (x + iy)^2 + (x - iy)^2 | \beta, l = 3, m_l = 1 \rangle$

d. (*2 pts*) In second quantization, a normalized two particle state can be written as $|\alpha\beta\rangle = a^\dagger_\alpha a^\dagger_\beta|0\rangle$. Evaluate $a_\beta|\alpha\beta\rangle$ for i) identical fermions and ii) identical bosons assuming $\alpha \neq \beta$.

e. (*3 pts*) Using rotation operators applied to a S_z eigenstate $|+\rangle$, find $|\mathbf{S}\cdot\hat{n};+\rangle$ where $\hat{n}$ makes an angle β with the z-axis and α with the x-axis.

f. (*4 pts*) Assume the valence electron in an Alkali atom experiences a perturbation due to the weak interactions of the form $V = g[\delta^3(\mathbf{x})\mathbf{S}\cdot\mathbf{p} + \mathbf{S}\cdot\mathbf{p}\delta^3(\mathbf{x})]$. *Using symmetry properties alone,* identify any restrictions on the $n'l'j'm'$ quantum numbers that appear in the first-order perturbation theory correction to the wave function $|nljm\rangle$.

g. (*3 pts*) Evaluate $a_\alpha e^{\lambda a_\alpha^\dagger}|0\rangle$ for the case of i)bosons and ii) fermions.

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