SUBJECT EXAM

PHYSICS 851/852, SPRING 1999

1. Consider a particle of mass m that feels an attractive one-dimensional delta function potential,

$$V(x) = -\beta\delta(x)$$

   (a) (5 pt.s) Derive the ground state energy.

   (b) (10 pt.s) Consider a particle in the ground state of the well. If the well suddenly dissolves, find the differential probability of observing an asymptotic momentum state p.

2. (10 pt.s) Express the state $|s = 1/2, \ell = 1, m_s = 1/2, m_\ell = 0\rangle$ as a linear combination of eigenstates of total angular momentum J and projection M.

3. Two types of spin-1/2 fermions, referred to as "*bob*"s and "*carol*"s, exist in a **TWO-DIMENSIONAL WORLD**. They may undergo reactions, *bob* $\leftrightarrow$ *carol* $+ \gamma$, where γ refers to a photon. The masses, m, of *bob*s and *carol*s are identical and the net density, $n = n_b + n_c$, is fixed. The *carol*s feel an additional attractive energy U, which lowers their energy relative to the *bob*s.

   (a) (10 pt.s) For an equilibrated system at zero temperature, what fraction of the particles are *bob*s? Give your answer in terms of n, m, U and $\hbar$.

   (b) (5 pt.s) Demonstrate that the fraction you gave as the answer above is dimensionless.

4. (a) (5 pt.s) Consider an operator A in the Schrödinger representation. Given the Hamiltonian, $H = H_0 + V$, write expressions for $A_H(t)$ and $A_{\text{int}}(t)$ which are the Heisenberg and interaction representations of A.

   (b) (5 pt.s) Show that in the interaction representation, the evolution operator,

$$U(t) \equiv e^{iH_0t}e^{-iHt} ,$$

   satisfies the equality,

$$\langle\psi|e^{iHt}Ae^{-iHt}|\phi\rangle = \langle\psi|U^\dagger(t)A_{\text{int}}(t)U(t)|\phi\rangle$$

   (c) (5 pt.s) Show that U satisfies the equation,

$$U(t_f - t_0) = 1 - \frac{i}{\hbar}\int_{t_0}^{t_f} dt'V(t')U(t' - t_0)$$

5. (15 pt.s) If an interaction has an explicit time dependence, $V_t = V\cos\omega t$, Fermi's golden rule becomes:

$$\Gamma_{f\to i} = \frac{2\pi}{4\hbar}|\langle f|V|i\rangle|^2\left\{\delta(\epsilon_f - \epsilon_i - \hbar\omega) + \delta(\epsilon_f - \epsilon_i + \hbar\omega)\right\}$$

Consider a particle of mass m in the ground state of a delta function potential, with the wave function:

$$\psi_0(x) = \sqrt{k}e^{-k|x|}.$$

An oscillating electric field is added that contributes a term,

$$V_t = Fx\cos(\omega t),$$

to the Hamiltonian. The frequency, ω, corresponds to an energy greater than the binding energy of the well, $\hbar\omega > \hbar^2k^2/(2m)$.

Estimate the rate at which the particle is ionized using Fermi's golden rule.

6. (15 pt.s) Consider eigenstates of the hydrogen atom whose angular wave functions are described by ℓ and m_ℓ. Which of the following matrix elements equal zero? All other information about the eigenstate (e.g. spin and radial wave functions) are referred to by α and β

   (a) $\langle\alpha,\ell=2,m_\ell=0|\ r^2\ |\beta,\ell=0,m_\ell=0\rangle$
   (b) $\langle\alpha,\ell=2,m_\ell=0|\ x^2+y^2\ |\beta,\ell=0,m_\ell=0\rangle$
   (c) $\langle\alpha,\ell=3,m_\ell=0|\ z\ |\beta,\ell=0,m_\ell=0\rangle$
   (d) $\langle\alpha,\ell=3,m_\ell=3|\ z^2\ |\beta,\ell=3,m_\ell=3\rangle$
   (e) $\langle\alpha,\ell=3,m_\ell=3|\ z^2\ |\beta,\ell=3,m_\ell=1\rangle$

7. An electron is placed in a constant magnetic field of strength B which lies along the z axis. Neglect the coupling of the spin to $\vec{B}$, and assume the electron is confined two-dimensionally to the $z=0$ plane.

   (a) (5 pt.s) Show that when using a gauge such that $\vec{A}$ lies purely along the y axis, that the operator $P_y \equiv -i\hbar\partial/\partial y$ commutes with the Hamiltonian.
   (b) (5 pt.s) Given that a wave function $\phi_{p_y}(x,y)$ is an eigenstate of P_y with eigenvalue p_y, and is also an eigenstate of the Hamiltonian, write an expression for the ground state wave function $\phi_{0,p_y}(x,y)$. (Do not concern yourself with the normalization.) What is the energy of the ground state?
   (c) (5 pt.s) Find the degeneracy of the ground state if the dimensions of the surface are L_x and L_y. Express your answer in term of e, c, B, m, L_x and L_y.

SUBJECT EXAM PHYSICS 851/852

September 1, 1999

1. Consider two orthogonal states $|\uparrow\rangle$ and $\downarrow$. Define the state,

$$|R\rangle \equiv \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + i|\downarrow\rangle\right)$$

If a particle is in the state $|R\rangle$ at time $t = 0$, and feels the interaction,

$$H = E\Theta(t)|\uparrow\rangle\langle\uparrow|,$$

find the probability for being in the state $|R\rangle$ as a function of time.

2. (a) Evaluate the matrix element

$$\langle 0|aaa^\dagger aa^\dagger a^\dagger|0\rangle.$$

where $a^\dagger$ and a are Bose creation and destruction operators.

(b) Consider $b^\dagger$, $d^\dagger$, b and d to be Fermi creation and destruction operators. Consider operators defined,

$$\tilde{b} \equiv b\cos\theta + d^\dagger \sin\theta, \quad \tilde{d} \equiv d\cos\theta - b^\dagger \sin\theta.$$

Find all anti-commutation relations between the operators, $\tilde{b}$, $\tilde{d}$, $\tilde{b}^\dagger$ and $\tilde{d}^\dagger$. Make sure the new vacuum is normalized.

(c) Write an expression for the vacuum $|\tilde{0}\rangle$, which is annihilated by $\tilde{b}$ and $\tilde{d}$, in terms of θ, $b^\dagger$, $d^\dagger$, and the original vaccum $|0\rangle$, which is annihilated by b and d.

3. Consider a particle of mass m that feels a three-dimensional potential

$$V(r) = \beta\delta(r - a), \quad \beta < 0.$$

In terms of β and m, find the minimum value of a that permits a bound state.

HINT: You need only cosider $\ell = 0$ states.

4. Write down all the allowed J, L, S combinations for 2 electrons in the 2-p shell of Carbon. Show that these values of J, L, S account for the 15 ways to put 2 electrons into the 6 p states.

5. The matrix element for the electromagnetic decay of an atomic d state with $m = 0$ to an p state with $m = 0$ is given by the matrix element,

$$\mathcal{M} \equiv \alpha\vec{\epsilon}\cdot \langle \ell = 1, m = 0|\vec{r}|\ell = 2, m = 0\rangle$$

where $\alpha\vec{\epsilon}\cdot\vec{r}$ is the interaction responsible for the decay, and $\vec{\epsilon}$ represents the polarization vector of the outgoing photon. The projection m is along the z axis. Assume that one has used this matrix element to calculate the decay rate for this reaction and the resulting rate is noted as Γ_{00}.

(a) What is the polarization of the outgoing photon in the reaction described above?

(b) In terms of Γ_{00} and Clebsch-Gordan coefficients, find the decay rates $\Gamma_{m1,m2}(\vec{\epsilon})$ for all five d states with projection m_1 into all three p states. Assume the photon is linearly polarized along the z axis, $\vec{\epsilon} = \hat{z}$. DO NOT EVALUATE CLEBSCH-GORDAN COEFFICIENTS.

HINT: The vector $\mathbf{r}$ can be written in terms of Lengendre Polynomials $Y_{\ell,m}(\theta,\phi)$,

$$z = \sqrt{\frac{4\pi}{3}}rY_{1,0}\ , \quad x + iy = \sqrt{\frac{8\pi}{3}}rY_{1,1}\ , \quad x - iy = \sqrt{\frac{8\pi}{3}}rY_{1,-1}$$

6. Consider a particle of mass m, confined to one dimension, and in the ground state of a delta function potential, with the wave function:

$$\psi_0(x) = \sqrt{k}e^{-k|x|}.$$

An oscillating electric field is added that contributes a term,

$$V_t = Fx\cos(\omega t),$$

to the Hamiltonian. The frequency, ω, corresponds to an energy greater than the binding energy of the well, $\hbar\omega > \hbar^2k^2/(2m)$.

Estimate the rate at which the particle is ionized using Fermi's golden rule.

HINT: If an interaction has an explicit time dependence, $V_t = V\cos\omega t$, Fermi's golden rule becomes:

$$\Gamma_{f\to i} = \frac{2\pi}{4\hbar}|\langle f|V|i\rangle|^2\left\{\delta(\epsilon_f - \epsilon_i - \hbar\omega) + \delta(\epsilon_f - \epsilon_i + \hbar\omega)\right\}$$

SUBJECT EXAM

PHYSICS 851/852, SEPTEMBER 1, 2000

Perform integrals unless specified otherwise

1. Consider a spin 1/2 system. The projection operator P_y projects the component of the wave function that has positive spin along the y axis.

$$\langle\eta|P_y|\eta\rangle = |\langle y,\uparrow|\eta\rangle|^2$$

   (a) (5 pt.s) Express P_y as a matrix in the basis where $(1/\sqrt{2})\begin{pmatrix} 1 \\ i \end{pmatrix}$ denotes a state with positive spin along the y axis.

   (b) (5 pt.s) Calculate P_y^2.

2. A resonance of type α and with angular momentum $j_\alpha = 2$ and projection m_α can decay to a resonance of type β with angular momentum $j_\beta = 1$ and m_β by radiating a massive spin-1 "spartan" particle. The matrix element describing the decay is

$$\mathcal{M} = \langle\beta|\vec{\epsilon}\cdot\vec{r}|\alpha\rangle,$$

   where $\vec{\epsilon}$ is the polarization of the spartan particle as described by one of three polarizaton vectors,

$$\vec{\epsilon}_0 = \hat{z},\ \vec{\epsilon}_+ = \frac{1}{\sqrt{2}}(\hat{x} + i\hat{y}),\ \vec{\epsilon}_- = \frac{1}{\sqrt{2}}(\hat{x} - i\hat{y}).$$

   The decay rate, $\Gamma_{m_\alpha=0,m_\beta=0}$, is measured for the decay of the state with $m_\alpha = 0$ to the state where $m_\beta = 0$.

   (a) (5 pt.s) What is the polarization of the spartan particle emitted in the decay $m_\alpha = 0$ and $m_\beta = 0$?

   (b) Assuming the initial state was prepared with the projection $m_\alpha = 2$, for each of the three values of $m_\beta = 1, 0, -1$:

      i. (5 pt.s) Find the decay rates $\Gamma_{m_\alpha=2,m_\beta}$ in terms of Γ_{00} and the ratio of Clebsch-Gordan coefficients. Do not evaluate the Clebsch-Gordan coefficients.

      ii. (5 pt.s) Describe the polarization of the spartan particles.

3. (10 pt.s) A particle of mass m initially populates the ground state of a **ONE-DIMENSIONAL** harmonic-oscillator potential.

$$V(x) = \frac{1}{2}m\omega^2x^2.$$

   At time $t = 0$ the potential is suddenly switched off. What is the differential probability, dN/dk, of populating a plane wave, $\psi_k(x) = e^{ikx}/\sqrt{L}$.

4. A particle of mass m initially populates the ground state of a **ONE-DIMENSIONAL** delta function potential

$$V(x) = -\beta\delta(x).$$

A time dependent perturbation is added,

$$\delta V(x,t) = \alpha\cos(\omega t).$$

   (a) (5 pt.s) Find the ground state wave function when the perturbation is neglected.

   (b) (15 pt.s) Using Fermi's golden rule, find the rate at which the particle is ionized.

5. Consider Fermi creation operators $a^\dagger$ and $b^\dagger$ where an initial Hamiltonian is of the form

$$H = \epsilon(a^\dagger a + b^\dagger b) + \gamma(a^\dagger b^\dagger + ab).$$

   (a) (10pt.s) In terms of ϵ and γ, find E, E_0 and θ such that two new operators

$$\alpha^\dagger \equiv a^\dagger\cos\theta + b\sin\theta, \quad \beta^\dagger \equiv b^\dagger\cos\theta - a\sin\theta,$$

   can be used to rewrite the Hamiltonian,

$$H = E(\alpha^\dagger\alpha + \beta^\dagger\beta) + E_0.$$

   (b) (5 pt.s) For the vacuum state $|\tilde{0}\rangle$ defined by

$$\alpha|\tilde{0}\rangle = 0, \quad \beta|\tilde{0}\rangle = 0,$$

   find the average number of a and b quanta, $\langle\tilde{0}|a^\dagger a + b^\dagger b|\tilde{0}\rangle$.

6. (20 pt.s) A proton and neutron with magnetic moments μ_p and μ_n sit in s-wave states of a nuclear well. The interaction with the magnetic field is then

$$V_B = \left(\mu_p\vec{S}_p + \mu_n\vec{S}_n\right)\cdot\vec{B}.$$

They exhibit a spin-spin attraction,

$$V_{s.o.} = \alpha\mathbf{S}_p\cdot\mathbf{S}_n$$

They are also placed in a constant magnetic field, B. Find the splitting of the four degenerate states due to spin-spin and magnetic interactions.

7. (10 pt.s) Consider a particle of mass m moving under the influence of a repulsive rotationally-symmetric potential,

$$V(r) = \begin{cases} V_0, & r < a \\ 0, & r > a \end{cases}$$

An $\ell = 0$ spherical wave is phase shifted by an amount δ.

$$\psi(r) \sim \frac{1}{r}\left(e^{-ikr} - e^{ikr+2i\delta}\right), \quad r > a.$$

Assuming $\hbar^2k^2/(2m) < V_0$, find δ as a function of k.

SUBJECT EXAM

PHYSICS 851/852, MAY 3, 2000

1. (10 pt.s) A magnetic field at 60° to the z axis is applied with the time dependence,

$$B(t) = \begin{cases} 0, & t < 0 \\ B_0, & t > 0 \end{cases}$$

If an at-rest electron is initially in a spin-up state (up being defined relative to the z axis), find the probability of the electron being in the spin-down state as a function of time t.

2. Consider the $\ell = 1$ basis where $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ represent the $m_\ell = 1, 0$ and -1 eigenstates, respectively, of L_z.

   (a) (10 pt.s) Write down matrices to represent the operators L_z, L_x and L_y in this basis.

   (b) (5 pt.s) Write down the matrix components of the operator that represents a rotation by an angle ϕ about the z axis.

3. (15 pt.s) Four types of spin-1/2 fermions, referred to as *bob*s and *carol*s, *ted*s and *alice*s, exist in a **TWO-DIMENSIONAL WORLD**. The net charge is conserved.

$$Q = eQ_{bob} - eQ_{carol} + 2eQ_{ted} - 2eQ_{alice}.$$

The overall baryon number is also conserved.

$$N_b = N_{bob} + N_{carol} + 2N_{ted} + 2N_{alice}.$$

They may undergo any reactions, e.g. $2 \cdot bob + 2 \cdot carol \leftrightarrow ted + alice$, that conserve charge and baryon number. The masses of the various species are:

$$m_{bob} = m_{carol} = m, \quad m_{ted} = m_{alice} = 2m.$$

Consider an isolated system of volume V with a net baryon number $N_b \neq 0$ and zero net charge $Q = 0$ which is in the lowest energy state.

Find the numbers of each species, N_{bob}, N_{carol}, N_{ted} and N_{alice} as a function of m, N_b and V.

4. An electron is placed in a constant magnetic field of strength B which lies along the z axis. The electron also experiences an electric field E which lies along the y axis. Neglect the coupling of the spin to the magnetic field.

   (a) (5 pt.s) Write down a vector potential $\mathbf{A}(\mathbf{r}, t)$ with $\mathbf{A}$ being solely along the y axis that results in the electromagnetic field described above.

   (b) (5 pt.s) Write the Hamiltonian for an electron in the field described above.

   (c) (5 pt.s) Assuming the wave function is of the form $\psi(\mathbf{r}, t) = e^{ik_y y + ik_z z}\phi_{k_y,k_z}(x, t)$, write the wave equation for $\phi_{k_y,k_z}(x, t)$ where $\hbar k_y$ and $\hbar k_z$ are the eigenvalues of P_y and P_z.

5. A particle of mass m, moving in a **ONE-DIMENSIONAL** world, is confined to the ground state of an infinite square well,

$$0 < x < L.$$

At a time $t = 0$, the well is suddenly expanded to twice the original size.

$$0 < x < 2L.$$

(a) (5 pt.s) What is the probability that the particle will be found in the ground state of the new well?

(b) (5 pt.s) What is the expectation of the energy, $\langle \psi(t)|H|\psi(t)\rangle$, for times after the expansion of the square well.

FYI:

$$\begin{aligned}
\int_0^{\pi/2} d\theta \sin m\theta \sin n\theta &= \frac{-1}{2(m+n)}\sin(m+n)\frac{\pi}{2} + \frac{1}{2(m-n)}\sin(m-n)\frac{\pi}{2} \\
\int_0^{\frac{\pi}{2}} d\theta \cos m\theta \cos n\theta &= \frac{1}{2(m+n)}\sin(m+n)\frac{\pi}{2} + \frac{1}{2(m-n)}\sin(m-n)\frac{\pi}{2} \\
\int_0^{\frac{\pi}{2}} d\theta \sin m\theta \cos n\theta &= \frac{-1}{2(m+n)}\cos(m+n)\frac{\pi}{2} + \frac{-1}{2(m-n)}\cos(m-n)\frac{\pi}{2}
\end{aligned}$$

6. (15 pt.s) Consider eigenstates of the hydrogen atom whose angular wave functions are described by ℓ and m_ℓ. Using the Wigner-Eckart theorem and conservation of parity, determine which of the following matrix elements must equal zero? All other information about the eigenstate (e.g. spin and radial wave functions) are referred to by α and β

(a) $\langle \alpha, \ell = 2, m_\ell = 0|\ r^2\ |\beta, \ell = 2, m_\ell = 1\rangle$
(b) $\langle \alpha, \ell = 2, m_\ell = 0|\ x^2 + y^2\ |\beta, \ell = 1, m_\ell = 0\rangle$
(c) $\langle \alpha, \ell = 3, m_\ell = 0|\ z\ |\beta, \ell = 0, m_\ell = 0\rangle$
(d) $\langle \alpha, \ell = 3, m_\ell = 3|\ (x+iy)^2 + (x-iy)^2\ |\beta, \ell = 3, m_\ell = 3\rangle$
(e) $\langle \alpha, \ell = 3, m_\ell = 3|\ (x+iy)^2 + (x-iy)^2\ |\beta, \ell = 3, m_\ell = 1\rangle$

7. A particle of mass m moving through normal **THREE-DIMENSIONAL** space feels a spherically symmetric attractive potential,

$$V(r) = -\beta\delta(r - R),$$

where r is the distance from the origin.

(a) (10 pt.s) For fixed R and m, find the minimum strength of the potential β that results in the existence of a bound state. Express $\beta_{\rm min}$ as a function of $\hbar$, R and m.

(b) (5 pt.s) A spherical s wave scatters off the potential, with asymptotic form

$$\psi(r) \sim \frac{1}{r}\left(e^{-ikr} - e^{ikr+2i\delta}\right).$$

For the potential above (with arbitrary β), find the phase shift δ as a function of k.

SUBJECT EXAM

PHYSICS 851/852, SPRING 2001

1. (10 points) Consider a two-component system described by the spinor,

$$\psi(t) = \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \end{pmatrix}$$

The system evolves under the influence of a Hamiltonian,

$$H = H_0 + \hbar\omega\sigma_x.$$

If the system begins life in the state

$$\psi(t=0) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix},$$

find $P_\uparrow(t)$, the probability of being in the state

$$\psi_\uparrow \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

2. A particle of mass m moves under the influence of a repulsive spherically symmetric potential,

$$V(r) = \begin{cases} V_0, & r < a \\ 0, & r > a \end{cases}$$

   (a) (10 points) Find the s-wave phase shift $\delta(E)$ for energies $E < V_0$.

   (b) (5 points) What is the cross section for scattering in the limit $E \to 0$.

3. (10 points) Express the state $|s = 1/2, \ell = 2, m_s = 1/2, m_\ell = 1\rangle$ as a linear combination of eigenstates of total angular momentum J and projection M.

4. (15 points) Two types (*ted* and *alice*) of non-relativistic spin-1/2 fermions have equal mass m and move in a **TWO-DIMENSIONAL** world. They can undergo a reaction $ted + \gamma \leftrightarrow alice + \gamma'$, where γ refers to a photon. A macroscopic number are placed in a large box of area A that conserves the net number $N = N_{ted} + N_{alice}$, but allows photons to escape. The particles feel different potentials within the box,

$$\begin{aligned} V_{ted}(x,y) &= V_{ted} \\ V_{alice}(x,y) &= 0. \end{aligned}$$

After equilibrating at zero temperature, find N_{ted} and N_{alice} in terms of N, A, m and V_{ted}. (Assume $V_{\rm ted}$ is much less than the Fermi energy.)

5. A *bob* particle of mass m is in the first excited state of a **ONE-DIMENSIONAL** harmonic oscillator characterized by frequency ω. It can decay to the ground state via the emission of a *carol* particle which is massless and spinless. The potential responsible for the decay is

$$V = g \int dx \;\Psi^\dagger(x)\Phi(x)\Psi(x),$$

where Ψ and Φ are field operators for *bob* and *carol* particles respectively,

$$\begin{aligned} \Psi(x) &= \frac{1}{\sqrt{L}} \sum_k b_k e^{-ikx} = \sum_n \phi_n(x) b_n \\ \Phi(x) &= \frac{1}{\sqrt{L}} \sum_k \frac{1}{\sqrt{kc}} \left(c_k^\dagger e^{ikx} + c_k e^{-ikx} \right), \end{aligned}$$

where $b_k^\dagger$ and $c_k^\dagger$ create *bob*s and *carol*s with momentum $\hbar k$, and $b_n^\dagger$ would create *bob*s into any state n which is part of an orthonormal basis described by wave functions $\phi_n(x)$.

(a) (5 points) What is the dimension of g?

(b) (10 points) Calculate $\langle k, 0|V|1\rangle$, the matrix element for decay of a *bob* from the first excited state into the ground state via emission of a *carol* with momentum k.

(c) (10 points) In terms of $\hbar$, m, ω and $\mathcal{M} \equiv \sqrt{L}\langle k, 0|V|1\rangle$, calculate the lifetime of the first excited state.

Potentially useful information:

$$\begin{aligned} \psi_0(x) &= \frac{1}{\pi^{1/4} a^{1/2}} e^{-x^2/(2a^2)}, \quad a^2 = \frac{\hbar}{m\omega} && (1) \\ \psi_1(x) &= x\frac{\sqrt{2}}{a}\psi_0(x) && (2) \\ E &= \hbar k c, \text{ for a massless particle.} && (3) \end{aligned}$$

6. (15 points) Consider eigenstates of the hydrogen atom whose angular wave functions are described by ℓ and m_ℓ. All other information about the eigenstate (e.g. spin and radial wave functions) are referred to by α and β. For each of the matrix elements below,

   (a) $\langle \alpha, \ell = 2, m_\ell = 0 |\ r^2\ | \beta, \ell = 0, m_\ell = 0 \rangle$
   (b) $\langle \alpha, \ell = 4, m_\ell = 0 |\ (x + iy)^2\ | \beta, \ell = 2, m_\ell = 0 \rangle$
   (c) $\langle \alpha, \ell = 2, m_\ell = 2 |\ z^2\ | \beta, \ell = 0, m_\ell = 0 \rangle$
   (d) $\langle \alpha, \ell = 3, m_\ell = 3 |\ z^2\ | \beta, \ell = 3, m_\ell = 3 \rangle$
   (e) $\langle \alpha, \ell = 3, m_\ell = 2 |\ x\ | \beta, \ell = 3, m_\ell = 1 \rangle$,

   choose one of the following statements.

   A. Might be non-zero.
   B. Must be zero due to parity.
   C. Must be zero due to time-reversal.
   D. Must be zero due angular momentum conservation, a.k.a. the Wigner Eckart theorem.
   E. Must be zero due to conservation of electric charge.

7. Consider the quantum state
$$|\eta\rangle \equiv e^{-\eta^*\eta/2} e^{\eta a^\dagger} |0\rangle.$$

   (a) (5 points) Calculate $\langle 0 | a | \eta \rangle$.
   (b) (5 points) Calculate $\langle \eta | (a^\dagger)^3 a^2 | \eta \rangle$

   (You can use the fact that $\langle \eta | \eta \rangle = 1$.