Student No.: ____________________

Qualifying/Placement Exam, Part-A
9:30 – 11:30, August 20, 2013, 1400 BPS

Put your **Student Number** on every sheet of this 6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-A of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. Do not use the back of the previous page for this purpose!

You may need the following constants:

$k_e = 8.99\times10^9\ \mathrm{Nm^2/C^2}$	Coulomb force constant
$\sigma = 5.7\times10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$	Stefan-Boltzmann constant
$k = 1.4\times10^{-23}\ \mathrm{J/K}$	Boltzmann constant
$\hbar = 1.05\times10^{-34}\ \mathrm{J\cdot s}$	Planck's constant
$c = 3.0\times10^{8}\ \mathrm{m/s}$	speed of light
$e = 1.602\times10^{-19}\ \mathrm{C}$	charge of the electron

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1. [10 pts] A $10\mu F$ capacitor is charged to 20 volts, and a $20\mu F$ capacitor is charged to 10 volts. The negative terminals are connected together directly. The two positive terminals are connected by a 500Ω resistor at $t = 0$. Compute the current through the resistor as a function of time.

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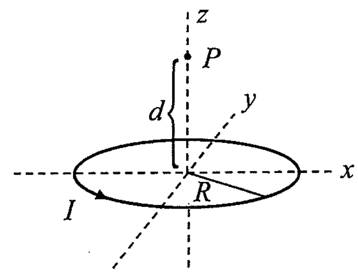
2. [10 pts] Calculate the magnetic field at point P, which is a distance d along the z-axis from a loop carrying current I. This loop has a radius R, lies in the x-y plane, and is centered at the origin as shown below.



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3. [10 pts] Beginning with Maxwell's equations,

a. [5 pts] DERIVE the wave equation for an electromagnetic wave.
b. [5 pts] Express the speed of light in terms of ε_0 and μ_0

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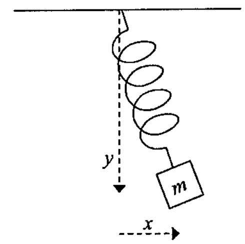
4. [10 pts] A mass m is attached to the end of a massless spring of spring constant k and allowed to move in the x-y plane with the other end of the spring connected to a fixed point in the ceiling. The equilibrium length of the spring is L_0 (when the spring is not being stretched by a weight).



a. [5 pts] Express the Lagrangian in terms of horizontal and vertical coordinates, x and y described in the picture.
b. [3 pts] Find Lagrange's equations of motion
c. [2 pts] From the Lagrangian, find the frequencies of the normal modes of small oscillations

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5. [10 pts] In a particular frame, the moment of inertia tensor of a rigid body is given (in some units) by

$$I = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 7 \end{pmatrix}.$$

Find the principal moments of inertia of the rigid body.

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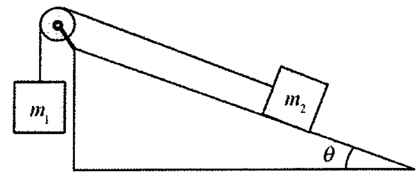
6. [10 pts] Two masses, m_1 and m_2, are attached together by a string of length ℓ that goes over a massless and frictionless pulley, attached to a ramp as shown in the figure. Mass-1 hangs from the string and mass-2 is on the ramp, and both are initially at rest. Apply Lagrangian Mechanics to obtain,



a) [7 pts] acceleration of the system of masses, and
b) [3 pts] the direction of the pulley's rotation.

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Qualifying/Placement Exam, Part-B
2:00 – 4:00, August 20, 2013, 1400 BPS

Put your **Student Number** on every sheet of this 6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-B of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. Do not use the back of the previous page for this purpose!

You may need the following constants:

$k_e = 8.99\times10^{9}\,\mathrm{Nm^2/C^2}$	Coulomb force constant
$\sigma = 5.7\times10^{-8}\,\mathrm{Wm^{-2}K^{-4}}$	Stefan - Boltzmann constant
$k = 1.4\times10^{-23}\,\mathrm{J/K}$	Boltzmann constant
$\hbar = 1.05\times10^{-34}\,\mathrm{J\cdot s}$	Planck's constant
$c = 3.0\times10^{8}\,\mathrm{m/s}$	speed of light
$e = 1.602\times10^{-19}\,\mathrm{C}$	charge of the electron

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1. [10 pts] The Hamiltonian H for a system and another observable L are represented by the matrices

$$H = \begin{pmatrix} \hbar\omega & 0 & 0 \\ 0 & 2\hbar\omega & 0 \\ 0 & 0 & 2\hbar\omega \end{pmatrix}, \qquad L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \hbar \\ 0 & \hbar & 0 \end{pmatrix}, \qquad \text{where } \omega > 0.$$

a. [2 pts] Find the eigenvalues of L.
b. [2 pts] Find the normalized eigenvectors of L, labeled in a clear way.
c. [1 pt] Show that these eigenvectors are also eigenvectors of H, and what are the corresponding eigenvalues?
d. [2 pts] If the initial state of the wavefunction Ψ is

$$|\Psi(0)\rangle = \begin{pmatrix} \sqrt{2/3} \\ 0 \\ \sqrt{1/3} \end{pmatrix}$$

write down an expression for $|\Psi(t)\rangle = \exp(-iHt/\hbar)|\Psi(0)\rangle$.

e. What is the probability that a measurement of the energy of $|\Psi(t)\rangle$ yields $\hbar\omega$?
f. Assuming that the result of measuring the energy was $\hbar\omega$, if I now measure the observable L, what result will I get and with what probability?

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2. [10 pts] The wavefunction for a particle in an infinite potential well of width a is

$$\psi(x) = Ax(a-x).$$

a. [3 pts] Normalize $\psi(x)$.
b. [3 pts] Using the normalized $\psi(x)$, calculate $\langle x \rangle$
c. [1 pt] What eigenstate of the infinite well system does $\psi(x)$ most nearly resemble?
d. [3 pts] Calculate $\langle H \rangle$

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3. [10 pts] An electron in the initial state

$$\psi(0) = \begin{pmatrix} \cos(\alpha/2) \\ \sin(\alpha/2) \end{pmatrix}$$

moves in a uniform magnetic field $\vec{B}$ oriented in the z direction

a. [5 pts] If $\omega = \gamma B$, where $H = -\gamma \vec{S} \cdot \vec{B}$, determine $\psi(\mathrm{t})$.

b. [5 pts] If S_x is measured, what is the probability that the result is $\hbar/2$ at time t?

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4. [10 pts] The B^0 meson has a mass (in GeV units) $Mc^2 = 5.3$ GeV and a mean lifetime of $\tau = 1.5\times10^{-12}$ seconds. Suppose that in the decay of a Higgs boson, a B^0 meson is produced with a total energy $E = 62$ GeV. What is the mean distance that the B^0 meson would travel from the point of production to the point of decay?

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5. [10 pt] A harmonic oscillator has energy levels $0, \hbar\omega, 2\hbar\omega, 3\hbar\omega, \cdots$. If the system is at temperature T, what is the probability for the system to be in the ground state?

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6. [10 pts] An electron is confined to a region of space the approximate size of an atom (~ 0.1 nm).

a) [5 pts] What is the uncertainty in the momentum of the electron?
b) [3 pts] What is the uncertainty in its velocity?
c) [2 pts] What is the kinetic energy (in eV) of an electron with a velocity equal to its uncertainty?